International Journal of Science and Research (IJSR)
ISSN (Online): 2319-7064
Index Copernicus Value (2016): 79.57 | Impact Factor (2017): 7.296

Skin Friction Analysis of MHD Flow Past a Parabolic Started Infinite Isothermal Vertical Plate in the Presence of Thermal Radiation and Chemical Reaction

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Abstract: ***In this paper the unsteady laminar free-convection flow of a viscous incompressible fluid, past parabolic started infinite isothermal vertical plate in the presence of thermal radiation and chemical reaction is considered. The Laplace transform method is used to obtain the expression for skin-friction, Nusselt number and Sherwood number. The effect of velocity profiles are studied for different physical parameters like Prandt number, thermal Grashof number, mass Grashof number, Schmidt number, magnetic parameter, chemical reaction parameter, radiation parameter and time.***

Keywords: Parabolic, skin friction, infinite vertical plate, Thermal Radiation

1. Introduction

The study of magneto hydro-dynamics with mass and heat transfer in the presence of radiation and diffusion has attracted the attention of a large number of scholars due to diverse applications. The study of heat and mass transfer problems with chemical reaction is of great practical importance to engineers and scientists because of its almost universal occurrence in many branches of science and engineering. A few representative fields of interest in which combined heat and mass transfer along with chemical reaction play an important role are found in chemical process industries such as food processing and polymer production. Chambre and Young (1958) analyzed a first-order chemical reaction in the neighborhood of a horizontal plate. Das *et al.* (1994) studied the effect of a homogeneous first-order chemical reaction on the flow past an impulsively started infinite vertical plate with uniform heat flux and mass transfer. Again, a mass transfer effect on a moving isothermal vertical plate in the presence of chemical reaction was studied by Das *et al*. (1999). Muthucumaraswamy and Meenakshisundaram (2006) studied chemical reaction effects on a vertical oscillating plate with variable temperature. Natural convection on flow past an linearly accelerated vertical plate in the presence of viscous dissipative heat using perturbation method by Gupta *et al* (1979). Kafousias and Raptis (1981) extended this problem to include mass transfer effects subjected to variable suction or injection. Soundalgekar (1982) studied the mass transfer effects on flow past a uniformly accelerated vertical plate. Mass transfer effects on flow past an accelerated vertical plate with uniform heat flux was analyzed by Singh and Singh (1983). Free convection effects on flow past an exponentially accelerated vertical plate was studied by Singh and Naveen Kumar (1984). The skin friction for accelerated vertical plate has been studied analytically by Hossain and Shayo (1986). Mass transfer effects on exponentially accelerated infinite vertical plate with constant heat flux and uniform mass diffusion was studied by Basant Kumar Jha *et al* (1991). Agrawal et al (1998) studied free convection due to thermal and mass diffusion in laminar flow of an accelerated infinite vertical plate in the presence of magnetic field. Agrawal et al (1999) further extended the problem of unsteady free convective flow and mass diffusion of an electrically conducting elasto-viscous fluid past a parabolic starting motion of the infinite vertical plate with transverse magnetic plate. The governing equations are tackled using Laplace transform technique.

Skin friction analysis of parabolic started infinite vertical plate with variable temperature and variable mass diffusion in the presence of magnetic field was studied by Indira Priyatharshini *et al* (2017). The solutions for velocity, temperature concentrations fields, Sherwood number and Nusselt number are derived in terms of exponential and complementary error functions.

Effect of parabolic motion of isothermal vertical plate with constant mass flux was discussed by Muthucumaraswamy and Geetha (2014). The effects of skin friction were also discussed.

The object of the present paper is to study the effects of skin friction of MHD flow past an infinite isothermal vertical plate subjected to parabolic motion in the presence of thermal radiation and chemical reaction. The dimensionless governing equations are solved using the Laplace-transform technique. The solutions are in terms of exponential and complementary error function.

2. Mathematical Analysis

The unsteady flow of a viscous incompressible fluid past an infinite isothermal vertical plate with uniform diffusion, in the presence of thermal radiation and chemical reaction of first order has been considered. The x'-axis is taken along the plate in the vertically upward direction and the y -axis is taken normal to the plate. At time $t' \leq 0$, the plate and fluid are at the same temperature T_∞ and concentration C'_∞ .At time $t' > 0$, the plate is started with a velocity $u = u_0 . t'^2$ in its own plane against gravitational field and the temperature from the plate is raised to T_w and the concentration to C'_w. A chemically reactive species which transforms according to a simple reaction involving the concentration is emitted from the plate and diffuses into the fluid. The plate is also subjected to a uniform magnetic field of strength B_0 which is assumed to be applied normal to the plate. The reaction is assumed to take place entirely in the stream. Then under usual Boussinesq's approximation for unsteady parabolic starting motion is governed by the following equations:

$$\frac{\partial U}{\partial t} = Gr\theta + GcC + \frac{\partial^2 U}{\partial Y^2} - MU \qquad (1)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial Y^2} - \frac{R}{\Pr}\theta \qquad (2)$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial Y^2} - KC \qquad (3)$$

The corresponding initial and boundary conditions in dimensionless form are as follows:

$$\begin{aligned} &U = 0,\ \theta = 0,\ C = 0 \ \text{for all } Y, t \le 0 \\ &t > 0: U = t^2,\ \theta = 1,\ C = 1 \ \text{at } Y = 0 \\ &U \to 0,\ \theta \to 0,\ C \to 0 \ \text{as } Y \to \infty \end{aligned} \qquad (4)$$

On introducing the following non-dimensional quantities:

$$U = u\left(\frac{u_0}{\nu^2}\right)^{1/3},\quad t = \left(\frac{u_0^{\,2}}{\nu}\right)^{1/3} t',\quad Y = y\left(\frac{u_0}{\nu^2}\right)^{1/3},$$

$$\theta = \frac{T - T_\infty}{T_w - T_\infty},\ C = \frac{C' - C'_\infty}{C'_w - C'_\infty}$$

$$Gr = \frac{g\beta(T_w - T_\infty)}{k\,u_0^{1/3}},\ Gc = \frac{g\beta^*(C'_w - C'_\infty)}{(\nu . u_0)^{1/3}},$$

$$\Pr = \frac{\mu C_p}{k},\ Sc = \frac{\nu}{D}$$

$$R = \frac{16 a^* \sigma T_\infty^3}{k}\left(\frac{\nu^2}{u_0}\right)^{2/3},\ M = \frac{\sigma B_0^2}{\rho}\left(\frac{\nu}{u_0^{\,2}}\right)^{1/3}, K = K_l\left(\frac{\nu}{u_0^{\,2}}\right)^{1/3} \qquad (5)$$

The solutions of equations under the boundary condition have been obtained by Muthucumaraswamy and SivaKumar. The solutions are in terms of exponential and complementary error function.

$$\theta = \frac{1}{2}\left[\exp(2\eta\sqrt{\Pr at})\,erfc(\eta\sqrt{\Pr} + \sqrt{at}) + \exp(-2\eta\sqrt{\Pr at})\,erfc(\eta\sqrt{\Pr} - \sqrt{at})\right] \qquad (6)$$

$$C = \frac{1}{2}\left[\exp(2\eta\sqrt{Sc\,kt})\,erfc(\eta\sqrt{Sc} + \sqrt{kt}) + \exp(-2\eta\sqrt{Sc\,kt})\,erfc(\eta\sqrt{Sc} - \sqrt{kt})\right] \qquad (7)$$

$$\begin{aligned}
U = 2\Bigg(\Bigg[&\frac{(\eta^2 + Mt)t}{4M}\left[\exp(2\eta\sqrt{Mt})\,erfc(\eta + \sqrt{Mt}) + \exp(-2\eta\sqrt{Mt})\,erfc(\eta - \sqrt{Mt})\right]\Bigg] + \\
&\frac{\eta\sqrt{t}(1 - 4Mt)}{8M^{3/2}}\left[\exp(-2\eta\sqrt{Mt})\,erfc(\eta - \sqrt{Mt}) - \exp(2\eta\sqrt{Mt})\,erfc(\eta + \sqrt{Mt})\right] \\
&- \frac{\eta t}{2M\sqrt{\pi}}\exp(-(\eta^2 + Mt)) + \\
&+ d\Bigg(\frac{1}{2}\left[\exp(2\eta\sqrt{Mt})\,erfc(\eta + \sqrt{Mt}) + \exp(-2\eta\sqrt{Mt})\,erfc(\eta - \sqrt{Mt})\right] + \\
&- \frac{\exp(bt)}{2}\left[\exp(2\eta\sqrt{(M+b)t})\,erfc(\eta + \sqrt{(M+b)t}) + \exp(-2\eta\sqrt{(M+b)t})\,erfc(\eta - \sqrt{(M+b)t})\right] + \\
&- \frac{1}{2}\left[\exp(2\eta\sqrt{\Pr at})\,erfc(\eta\sqrt{\Pr} + \sqrt{at}) + \exp(-2\eta\sqrt{\Pr at})\,erfc(\eta\sqrt{\Pr} - \sqrt{at})\right] + \\
&+ \frac{\exp(bt)}{2}\left[\exp(2\eta\sqrt{\Pr(a+b)t})\,erfc(\eta\sqrt{\Pr} + \sqrt{(a+b)t}) + \exp(-2\eta\sqrt{\Pr(a+b)t})\,erfc(\eta\sqrt{\Pr} - \sqrt{(a+b)t})\right]\Bigg) \\
&+ e\Bigg(\frac{1}{2}\left[\exp(2\eta\sqrt{Mt})\,erfc(\eta + \sqrt{Mt}) + \exp(-2\eta\sqrt{Mt})\,erfc(\eta - \sqrt{Mt})\right] + \\
&- \frac{\exp(ct)}{2}\left[\exp(2\eta\sqrt{(M+c)t})\,erfc(\eta + \sqrt{(M+c)t}) + \exp(-2\eta\sqrt{(M+c)t})\,erfc(\eta - \sqrt{(M+c)t})\right] + \\
&- \frac{1}{2}\left[\exp(2\eta\sqrt{Sckt})\,erfc(\eta\sqrt{Sc} + \sqrt{kt}) + \exp(-2\eta\sqrt{Sckt})\,erfc(\eta\sqrt{Sc} - \sqrt{kt})\right] + \\
&+ \frac{\exp(ct)}{2}\left[\exp(2\eta\sqrt{Sc(k+c)t})\,erfc(\eta\sqrt{Sc} + \sqrt{(k+c)t}) + \exp(-2\eta\sqrt{Sc(k+c)t})\,erfc(\eta\sqrt{Sc} - \sqrt{(k+c)t})\right]\Bigg)
\end{aligned} \qquad (8)$$

By the expression (8) the skin-friction at the plate is given by

$$\tau = -\left(\frac{dU}{dy}\right)_{y=0}$$

$$= -\frac{1}{2\sqrt{t}}\left(\frac{dU}{d\eta}\right)_{\eta=0}$$

$$= \frac{t^{3/2}}{4}\left[\frac{4}{\sqrt{\pi}}e^{-Mt} + erf(\sqrt{Mt}) - 2\sqrt{Mt}\right]$$

$$+ \frac{(1-4Mt)}{8M^{3/2}}\left[1 - erf(\sqrt{Mt}\right] + \frac{1}{2M}\sqrt{\frac{t}{\pi}}e^{-Mt} +$$

$$+ d\left\{\left[\frac{1}{4\sqrt{t}}\left(\frac{4}{\sqrt{\pi}}e^{-Mt} + erf(\sqrt{Mt}) - 2\sqrt{Mt}\right)\right] +\right.$$

$$-\frac{e^{-bt}}{4\sqrt{t}}\left[\frac{4}{\sqrt{\pi}}e^{-(M+b)t} + erf(\sqrt{(M+b)t}) - 2\sqrt{(M+b)t}\right] +$$

$$-\frac{1}{4\sqrt{t}}\left[\frac{4}{\sqrt{\pi}}e^{-at} + erf(\sqrt{at}) - 2\sqrt{\Pr at}\right] +$$

$$\left. + \frac{e^{bt}}{4\sqrt{t}}\left[\frac{4}{\sqrt{\pi}}e^{-(a+b)t} + erf(\sqrt{(a+b)t}) - 2\sqrt{\Pr(a+b)t}\right]\right\} +$$

$$+ e\left\{\left[\frac{1}{4\sqrt{t}}\left(\frac{4}{\sqrt{\pi}}e^{-Mt} + erf(\sqrt{Mt}) - 2\sqrt{Mt}\right)\right] +\right.$$

$$-\frac{e^{-ct}}{4\sqrt{t}}\left[\frac{4}{\sqrt{\pi}}e^{-(M+c)t} + erf(\sqrt{(M+c)t}) - 2\sqrt{(M+c)t}\right] +$$

$$-\frac{1}{4\sqrt{t}}\left[\frac{4}{\sqrt{\pi}}e^{-kt} + erf(\sqrt{kt}) - 2\sqrt{Sckt}\right] +$$

$$\left. + \frac{e^{ct}}{4\sqrt{t}}\left[\frac{4}{\sqrt{\pi}}e^{-(k+c)t} + erf(\sqrt{(k+c)t}) - 2\sqrt{Sc(k+c)t}\right]\right\} \qquad (9)$$

By the expression (6), the rate of heat transfer in terms of Nusselt number in non dimensional form is given by

$$Nu = -\left(\frac{d\theta}{dy}\right)_{y=0}$$

$$= -\frac{1}{2\sqrt{t}}\left(\frac{d\theta}{d\eta}\right)_{\eta=0}$$

$$= \frac{e^{-at}}{\sqrt{\pi t}} + \sqrt{\Pr a}\, erf(\sqrt{at}) \qquad (10)$$

By the expression (7), the rate of mass transfer in terms of Sherwood number in non dimensional form is given by

$$Sh = -\left(\frac{dC}{dy}\right)_{y=0}$$

$$= -\frac{1}{2\sqrt{t}}\left(\frac{dC}{d\eta}\right)_{\eta=0}$$

$$= \frac{e^{-kt}}{\sqrt{\pi t}} + \sqrt{Sck}\, erf(\sqrt{kt}) \qquad (11)$$

Table I: Skin friction profiles for air (Pr=0.71)

t	Gr	Gc	Sc	R	M	K	Ʈ
0.2	2	5	0.16	6	5	5	-0.318806
0.3	2	5	0.16	6	5	5	-0.277565
0.2	2	5	0.16	6	4	5	-0.111409
0.3	2	5	0.16	6	4	5	-0.123268
0.2	2	5	0.16	6	3	5	0.088066
0.3	2	5	0.16	6	3	5	0.066489
0.4	2	5	0.16	6	3	5	0.080440
0.2	2	5	0.16	4	3	5	0.088420
0.3	2	5	0.16	4	3	5	0.067045
0.4	2	5	0.16	4	3	5	0.081229
0.2	10	5	0.16	6	5	5	-0.305786
0.3	10	5	0.16	6	5	5	-0.257096
0.2	2	10	0.16	4	3	5	0.204559
0.3	2	10	0.16	4	3	5	0.249617

Table II: Skin friction profiles for Water (Pr=7.0)

t	Gr	Gc	Sc	R	M	K	τ
0.2	2	5	0.16	5	3	5	1.699979
0.3	2	5	0.16	5	3	5	2.520924
0.2	10	5	0.16	5	3	6	8.157140
0.3	10	5	0.16	5	3	6	12.354703
0.2	2	10	0.16	5	4	6	1.499890
0.3	2	10	0.16	5	4	6	2.339947
0.4	2	10	0.16	5	4	6	3.249205
0.2	5	5	0.16	6	5	6	3.113200
0.3	5	5	0.16	6	5	6	4.971677
0.4	5	5	0.16	6	5	6	6.914775

Table III: Numerical Values of Sherwood Number

Sc	t	k	Sh
0.16	0.2	5	1.217839
0.16	0.3	5	1.049792
0.16	0.4	5	0.974458
0.6	0.2	5	1.923705
0.6	0.3	5	1.817671
0.6	0.4	5	1.773970
0.16	0.2	6	1.240889
0.16	0.3	6	1.093452
0.16	0.4	6	1.032837
0.6	0.2	6	2.047126
0.6	0.3	6	1.958006
0.6	0.4	6	1.924294

Table IV: Numerical Values of Nusselt Number

R	t	Pr	Nu
4	0.2	7.0	1.860151
4	0.3	7.0	1.751419
4	0.4	7.0	1.711862
5	0.2	7.0	2.003748
5	0.3	7.0	1.921040
5	0.4	7.0	1.900891
4	0.2	0.71	2.142229
4	0.3	0.71	2.058079
4	0.4	0.71	2.026181
5	0.2	0.71	2.335976
5	0.3	0.71	2.271567
5	0.4	0.71	2.250012

3. Results and Discussion

For physical understanding of the problem numerical computations are carried out for different physical parameters *Gr*, *Gc*, *Sc* and *t* upon the nature of the flow and transport. The value of the Schmidt number Sc is taken to be 0.6 which corresponds to water-vapor. Also, the values of Prandtl number Pr are chosen such that they represent air (Pr = 0.71). The numerical values of the velocity are computed for different physical parameters like Prandtl number, magnetic field parameter, radiation parameter, chemical reaction parameter, thermal Grashof number, mass Grashof number, Schmidt number and time.

The skin friction is tabulated in Table I and Table II. The effect of skin friction for the different values of thermal Grashof number Gr, mass Grashof number Gc, Prandtl number Pr, Schmidt number Sc, radiation parameter R, chemical reaction parameter K, magnetic field parameter M and time t was analyzed. Table I shows the effect of skin friction in the presence of air (Pr=0.71) and Table II displays the effect of skin friction in the presence of water (Pr=7.0). It is also observed that skin-friction increases with increase of Prandtl number. As time t advances the value of skin-friction decreases. Moreover the value of the skin friction increases with increasing thermal Grashof number or mass Grashof number. It is also clear that the value of skin friction increases with increasing radiation parameter, magnetic field parameter and chemical reaction parameter.

Table III depicts the Sherwood number (rate of mass transfer) for different values of Schmidt number (Sc), chemical reaction parameter (k) and time (t). It shows that Sherwood number enhances with increasing values of Schmidt number, chemical reaction parameter and time.

Table IV depicts the Nusselt number which is the ratio of heat transfer by convection to heat transfer by conduction. The result shows that the rate of heat transfer increases with increasing value of radiation parameter and the trend is reversed with respect to time t. Also from the value of Prandtl number it is clear that rate of heat transfer is more in air than in water.

4. Conclusion

The skin friction analysis of MHD flow past a parabolic started infinite isothermal vertical plate in the presence of thermal radiation and chemical reaction has been studied. The dimensionless governing equations are solved by the usual Laplace transform technique. The effect of the temperature, the concentration and the velocity fields for different physical parameters like Prandtl number, Schmidt number, thermal Grashof number and mass Grashof number, magnetic field parameter, radiation parameter, chemical reaction parameter and time are studied graphically. The conclusions of the study are as follows:

1) The effect of skin friction increases with increases values of Gr, Gc, Sc, M, K, R, Pr and the trend is reversed with respect to time.
2) The Sherwood number increases with increasing values of Schmidt number(Sc), chemical reaction parameter (K) and time
3) The rate of heat transfer increases with increasing value of radiation parameter.

References

[1] Chambre P.L. and Young J.D. (1958): On the diffusion of a chemically reactive species in a laminar boundary layer flow. – *The Physics of Fluids*, vol.1, pp.48-54.

[2] Das U.N., Deka R.K. and Soundalgekar V.M. (1994): Effects of mass transfer on flow past an impulsively started infinite vertical plate with constant heat flux and chemical reaction. –*Forschung im Ingenieurwesen*, vol.60, pp.284-287.

[3] Das U.N., Deka R.K. and Soundalgekar V.M. (1999): Effects of mass transfer on flow past an impulsively started infinite vertical plate with chemical reaction. – *The Bulletin of GUMA*, vol.5 pp.13-20.

[4] Muthucumaraswamy R. and Meenakshisundaram S. (2006): Theoretical study of chemical reaction effects on vertical oscillating plate with variable temperature, *Theoret. – Journal of Applied Mechanics*, vol.33, pp.245-257.

[5] Hossain MA and Shayo L K(1986). The skin friction in the unsteady free convection flow past an accelerated plate. *Astrophysics and Space Science* 125, pp.315-324.
[6] Hossain, M.A. and Takhar HS (1996). Radiation effect on mixed convection along a vertical plate with uniform surface temperature. *Heat and Mass Transfer* 31, pp.243-248.
[7] Basant Kumar Jha, Ravindra Prasad and Surendra Rai (1991). Mass transfer effects on the flow past an exponentially accelerated vertical plate with constant heat flux. *Astrophysics and Space Science* 181, pp.125-134.
[8] Das UN, Deka RK and Soundalgekar VM(1996). Radiation effects on flow past an impulsively started vertical infinite plate. *Journal of Theoretical Mechanics*1, pp.111-115.
[9] England WG and Emery AF(1969). Thermal radiation effects on the laminar free convection boundarylayer of an absorbing gas. *Journal of Heat Transfer* 91, pp.37-44.
[10] Raptis A and Perdikis C (1999, "Radiation and free convection flow past a moving plate", *International Journal of Applied Mechanics and Enginnering* 4 ,pp.817-821.
[11] Singh A.K. and Naveen Kumar(1984). Free convection flow past an exponentially accelerated vertical plate. *Astrophysics and Space Science* 98, pp.245-258.
[12] Agrawal A K, Samria N K, and Gupta S N (1999), Study of heat and mass transfer past a parabolic started infinite vertical plate, *Journal of Heat and mass transfer*, 21 pp. 67-75.
[13] Muthucumaraswamy R and Geetha E (2014), Effects of parabolic motion on an isothermal vertical plate with constant mass flux, *Ain Shams Engineering Journal* 5, pp. 1317-1323
[14] Indira Priyatharshini S and Neel Armstrong A (2017), Skin friction analysis of parabolic started infinite vertical plate with variable temperature and variable mass diffusion in the presence of magnetic field, *International Journal of Pure and Applied Mathematics* 4, pp. 81-92

Nomenclature, Greek Symbols

A	constant
D	mass diffusion coefficient
Gc	mass Grash of number
Gr	thermal Grash of number
g	acceleration due to gravity
q	heat flux per unit area at the plate
k	thermal conductivity of the fluid
Pr	Prandtl number
Sc	Schmidt number
T	temperature of the fluid near the plate
T_w	temperature of the plate
T_∞	temperature of the fluid far away from the plate
t'	time
t	dimensionless time
u	velocity of the fluid in the x-direction
u_0	velocity of the plate
U	dimensionless velocity component in *x*-direction
y	coordinate axis normal to the plate
Y	dimensionless coordinate axis normal to the plate
β	volumetric coefficient of thermal expansion
β *	volumetric coefficient of expansion with concentration
μ	coefficient of viscosity
υ	kinematic viscosity
ρ	density of the fluid
θ	dimensionless temperature
η	similarity parameter
erfc	complementary error function
M	magnetic field parameter
K	chemical reaction parameter
R	radiation parameter
C'	species concentration in the fluid
C	dimensionless concentration
C'_w	wall concentration
C'_∞	Concentration in the fluid far away from the plate