International Journal of Science and Research (IJSR)
ISSN (Online): 2319-7064
Index Copernicus Value (2016): 79.57 | Impact Factor (2017): 7.296

Estimating the Hurst Parameter of the Nigerian All-Share Index (1990-2007)

Maurice Nnamdi ANNORZIE

Department of Mathematics, Imo State University, P.M.B. 2000, Owerri, Nigeria

Abstract: ***In this work, the Rescaled Range Statistics R/S was used to analyze the Nigerian All-Share Index (NASI) of the Nigerian Stock Market from 1990 to 2007. The times series of NASI driven by a four-quarter moving average over 72 observations was classified. The Hurst parameter $H \in (0,1)$as a dimensionless estimator, was obtained, in order to characterize the historical market's trend. The value of H obtained showed the long-range dependence (LRD) of the NASI.***

Keywords: Rescaled Range Statistics, Nigerian Stock Market, Hurst Parameter, Long-range Dependence

1. Introduction

Hurst (1951) gave a long term storage capacity of water reservoirs. The Hurst parameter, H, is an estimator (without dimension) for the self-similarity of a times series initially defined by Harold Edwin Hurst to develop a law for regularities of the Nile's river water level, which had useful application in medicine and finance.

The *R/S* statistic has the ability to detect long range dependence (LRD). Its sensitivity to short range was discussed by Mandelbrot and Wallis (1968), while Mandelbrot (1975) highlighted on fractional Gaussian noise. Anis and Lloyd (1976) discussed rescaled Hurst range of independent normal summands, whereas Davies and Harte (1987) gave some tests for Hurst effects.

Aydogan and Booth (1988)examined long circles in common stock returns. Lo (1991) analysed long term memory in stock market prices by proposing a modified *R/S* to overcome any shortcoming. Also, in Peters(1991), the *S&P* 500 index was analysed using *R/S* but afterwards revealed some characteristic LRD of return series. In Bayraktar et al. (2003), *S&P* 500 index data were sampled and analysed at every minute intervals for 11.5 years (January 1989-May 2000). $H \in (0,1)$ was estimated by employing an asymptotically Gaussian estimator through the log-scale spectrum. Also, the estimated variance of the N points data segments had order$\frac{1}{N}$.

Matos et al. (2008), used a notable approach of understanding time and scale-dependent H, while emphasis on H estimation having long memory effect was given by Chronopoulou and Viens (2009). Pfaff(2006) highlighted on times series. Gloter and Hoffman (2007) estimated H for discrete data with noise. A new insight into financial market dynamics was presented by Annorzie(2015).

2. Self-Similar Processes

Self-similarity of a process $X = (X_t, -\infty < t < \infty)$ can be defined through its distribution. Assume that $(X_{at})$and $a^k(X_t)$are identically finite-dimensional distributions for all $a > 0$, then X is self-similar with H (Taqqu,1988).

Let $X = (X_t, t = 0,1,2,\dots)$ be a covariance stationary stochastic process with $\mu, \sigma^2, and r(k), k \geq 0$ as mean, variance and autocorrelation function respectively. Suppose X has an autocorrelation function of the form $r(k) \sim k^{-\beta}L(k),\ as k \to \infty$ such that $0 < \beta < 1$ and $L$is slowly varying at infinity. Also, let L be asymptotically constant. For each $u = 1,2,\dots$, let $X^{(u)} = (X_k^{(u)}, k = 1,2,3,\dots)$ denote the time series obtained by averaging the original series Xover non-overlapping blocks of size u, i.e. $X(u)$is given by

$$X_k^{(u)} = \frac{1}{u}\left(X_{(k-1)u} + \cdots + X_{ku-1}\right), k \geq 1 \quad (2.1)$$

Definition 2.1: A process X is called (exactly) second-order self-similar with self-similarity parameter $H = 1 - \frac{\beta}{2}$ $if, for all u = 1,2,\dots, \mathrm{VAR}[X^{(u)}] = a^2u^{-\beta} and$

$$r^{(u)}(k) = r(k) = \frac{1}{2}((k+1)^{2H} - 2k^{2H} + |k-1|^{2H}),\ \ k \geq 0 \quad (2.2)$$

Where *r(u)* denotes the autocorrelation function of $X^{(u)}$

Definition 2.2: A process X is called (asymptotically) second-order self-similar with self-similarity parameter $H = 1 - \beta/2$ if, for all k large enough,

$$r^{(u)}(k) \to r(k),\ \ as u \to \infty \quad (2.3)$$

2.1 Properties of Self-similar processes

$H$is a self-similar process whose properties are equivalent to the following:

(a) Hurst effect: The *R/S* is characterized by a power law

$$E[R(u)/S(u)] \sim a_1 u^H \ \ as\ u \to \infty\ \ with\ \frac{1}{2} < H < 1$$

(b) Slowly decaying variances: The variances of the sample mean are decaying more slowly than the reciprocal of the sample size, i.e.

$$VAR[X^{(u)}] \sim a_2 u^{2H-2}\ as\ u \to \infty\ with\ \frac{1}{2} < H < 1.$$

(c) Long-range dependence (LRD): The autocorrelations decay hyperbolically rather than exponentially in order to show a non-summable autocorrelation function $\Sigma r(k) = \infty$.This implies that though the $r(k)$'sare individually small for large lag, their cumulative effect is important.

(d) L/f-noise. The spectral density *f(·)* obeys a power law near the origin, i.e.,

$$f(\lambda) \backsim \alpha_3 \lambda^{1-2H}, as\ \lambda \rightarrow 0, \quad with\ \frac{1}{2} < H < 1,$$
$$where\ f(\lambda) = \sum_K r(k)e^{ik\lambda}.$$

2.2 Estimation Methods

We present an overview of some estimation methods for a long-range dependence parameter, H. Most of the methods below are also described by Taqquet al. (1995).

2.2.1 Aggregated Variance Method

The aggregated variance method is based on the self-similarity of the sample as shown in Definition (2.1). Similarly, due to the asymptotically self-similarity of the aggregated processes $X^{(m)}$given by

$$X_k^{(m)} = \frac{1}{m}\left(X_{km} + \ldots + X_{(k+1)m-1}\right), \text{ for } k = 0,1,\ldots, \quad (2.4)$$

$X^{(m)}$has the same finite dimensional distribution as $m^{H-1}$for large m. Inparticular, $Var(X_k^{(m)}) = m^{2H-2h-2}Var(X_k)$.The variance of $X_k^{(m)}$)is equal for every k and a plausible estimator is

$$\widehat{Var(X_k^{(m)})} = \frac{1}{M}\sum_{i=0}^{M-1}(X_i^{(m)} - \bar{X}^{(m)})^2 \quad (2.5)$$

where $\bar{X}^{(m)}$ denotes the sample average of $X^{(m)}$:

$$\bar{X}^{(m)} = \frac{1}{M}\sum_{i=0}^{M-1} X_i^{(m)}$$

3. R/S Analysis: A Simulation Experiment

In 1908, the Einstein's *T*to the one-half rule is given by

$$R = T^{\frac{1}{2}} \quad (3.1)$$

where*R* represents the displacement in time *T*. It has been useful both in Statistics and Financial economics to annualize volatility or standard deviations, Edgar (1994:p.55).

From Table 1, we present the analysis of the NASI data as follows (Annorzie, 2015). Let $y_r = (NASI)_r$ and $x_r = log(y_r)$ $where$ $x_r = (x_1,\ldots,x_n)$ represent n consecutive terms of a time series with mean, $\bar{x}$, defined as

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \quad (3.2)$$

The standard deviation, S_d, is estimated as

$$S_d = \sqrt{\frac{\sum_{r=1}^{n}(x_r - \bar{x})^2}{n}} \quad (3.3)$$

which is merely the standard normal formula for standard deviation. If $w_r = (x_r - \bar{x})$, $r = 1, 2, \ldots, n$, then the data can be normalized as:

$$\bar{w} = \frac{\sum_{r=1}^{n} w_r}{n} = \frac{\sum_{r=1}^{n}(x_r - \bar{x})}{n} = \bar{x} - \frac{n\bar{x}}{n} = 0 \quad (3.4)$$

The cumulative time series Y_n can be written as:

$$\left.\begin{aligned} Y_n &= \sum_{r=1}^{n} z_r \\ \Rightarrow Y_n &= \sum_{r=1}^{n}(x_r - \bar{x}) = \sum_{r=1}^{n} x_r - n\bar{x} = 0 \end{aligned}\right\} \quad (3.5)$$

where $Y_1 = w_1 + w_r$, $r = 2, \ldots, n$, that is, Y_n will always be zero because ω has a mean of zero. The adjusted range, R_n, is:

$$R_n = \max(Y_1, \ldots, Y_n) - min(Y_1, \ldots, Y_n) \quad (3.6)$$

which is the (non-negative) distance travelled by the system.

Deviation from the mean is given by $w_r = x_r - \bar{X}$, as shown in Equation (3.4) and Table 1. The first value of the cumulative sum is the same as that of deviation (from the mean) while the next value of cumulative sum is the sum of its first and second values of the deviation. Thus, current value of the cumulative sum is the sum of its previous value and the current value of the deviation, see Table 1,and Equations (3.4) and (3.5).

The supremum (sup) of the cumulative sum occurs at the fourth quarter of 2007 while the infimum (inf) of the cumulative sum occurs at the first quarter of 1996, see Table 1. Hurst found a more general form of Equation (3.1) to be:

$$(R/S)_N = C_n{}^H \quad (3.7)$$

where the subscript, n, for $(R/S)_n$refers to the *R/S* value for $x_1, \ldots, x_n$ and C is a constant. Equation (3.7) becomes

$$log(R/S)_n = log(C) + Hlog(n) \quad (3.8)$$

The Hurst parameter can be approximated by plotting the $log(R/S)_n$ versus the log(*n*) and using an ordinary least squares regression to solve for the slope through.

3.1 Systematic Guide to *R/S* Analysis

R/S method is one of the processes that is highly data-intensive. This section breaks Equations (3.2)-(3.8) into a series of executable steps. These include the following:

1) Begin with a time series of length *M*. Convert this into a time series of length $N = M - 1$ of logarithmic ratios:

$$N_i = log(M_{i+1}/Mi), i = 1,2,3,\ldots, M-1 \quad (3.9)$$

2) Divide this time period into *A*contiguous sub-periods of length n, such that $nA = N$. Label each sub-period I_a, with $a = 1,2,3,\ldots, A$. Each element in $I_a$is labeled $N_{k,a}$such that $k = 1,2,3, \ldots, n$. For each I_a; of length *n*, the average value is defined as:

$$e_a = \frac{1}{n}\sum_{k=1}^{n} N_{k,a} \quad (3.10)$$

wheree_a= average value of the N_i, contained in sub-period $I_a$of length *n*.

3) For each sub-period I_a, the accumulated departures $(X_{k,a})$from the mean value have their time series defined as:

$$X_{k,a} = \frac{1}{n}\sum_{k=1}^{n}(N_{i,a} - e_a), \qquad k = 1,2,3,\ldots,n \quad (3.11)$$

4) The range within each sub-period I_a is defined as:

$$R_{I_a} = \max(X_{k,a}) - \min(X_{k,a}), \qquad k \in [1,n] \quad (3.12)$$

5) For each sub-period I_a, the definition of the calculation of sample standard deviation is given by:

$$S_{I_a} = \left(n^{-1}\sum_{k=1}^{n}(N_{k,a} - e_a^2) \right)^{\frac{1}{2}} \quad (3.13)$$

6) Each range, R_{I_a},is now normalized by dividing by the S_{I_a},corresponding to it. The rescaled range for each $I_a$sub-period is equal to R_{I_a}/S_{I_a}. Observe in Equation (3.10), we had Acontiguous sub-periods of length *n*, so that the average *R/S* for length n becomes:

(3.14)

7) The length *n* is increased to the next higher value while $(M-1)/n$is an integer value. We use values of *n*that includes the beginning and ending points of the time series such that steps 1 through 6 are repeated until $n = (M-1)/2$. We can now apply Equations (3.7) and (3.8) by performing an ordinary least squares regression on $\log n$ as the independent variable and $\log(R/S)_n$ as the dependent variable. The intercept is the estimate for $\log C$, the constant. The slope of the equation is the estimate of the Hurst parameter, *H*, Edgar(1994).

The descriptive *R/S* statistic measure is given by:

$$R/S = \frac{1}{S_T}\left[\max_{1 \le k \le T} \sum_{j=1}^{k}(y_j - \bar{y}) - \min_{1 \le k \le T} \sum_{j=1}^{k}(y_j - \bar{y}) \right] \quad (3.15)$$

where$S_T$is the usual maximum likelihood standard deviation estimator given by:

$$S_T = \left[\frac{1}{T}\sum_{j=1}^{T}(y_j - \bar{y})^2 \right]^{\frac{1}{2}} \quad (3.16)$$

This measure is always non-negative because the deviations from the sample mean$\bar{y}$sum up to zero. The Hurst coefficient is then estimated as:

$$H = \frac{\log(R/S)}{\log(T)} \quad (3.17)$$

$H = \frac{1}{2}$represents a short memory process while $H > \frac{1}{2}$shows a long memory behaviour, Pfaff(2006: p.35).

4. Results

The mean of $\bar{y}$ is given by $\bar{x} = 4.21874$. The supremum (Sup) of the cumulative sum is its maximum (or greatest) value, $sup = -4.1E-06$ which occurs at the fourth quarter of 2007. The infimum (Inf)of the cumulative sum is its minimum (or least) value,i.e. $Inf = -16.6344$ which occurs at the first quarter of 1996. The adjusted range R_n,is given by Equation (3.6) while the standard deviation is given by Equation (3.3) as $S_d = 0.58023$. Equation (3.15) gives *R/S* value while the Hurst parameter of NASI (1990-2007) is given by $H = 0.78468 \simeq 0.78$.

5. Conclusion

The above procedures show that a given stock market can be characterized to know the nature (behaviour) of its trading activities by obtaining its Hurst parameter, H,which could be useful in interpreting the market as being anti-persistent if $H = 0.5$, a Brownian motion, if $H = 0.5$ and persistent or stable if $H > 0.5$. $H$is self-similar with LRD phenomenon. Also, the effect of fractional noises could be determined.

The market was stable. Figure 1 showed the nature of the Nigerian stock market (1990-2007) as an emerging, stable and persistent market, which also agrees with the value of Hobtained.

This result of obtaining Hcan be applied to certain real life's problems involving certain sectors of a given economy with enough accurate information of usually regular or seasonal activities such as financial institutions; electricity network, distribution and consumption; transport systems; health; population studies; disease epidemics, governance; geometry of nature, self- similar processes, chaos and dynamical systems; and stochastic processes with corresponding model(s) where applicable.

Paper ID: ART20183884 DOI: 10.21275/ART20183884

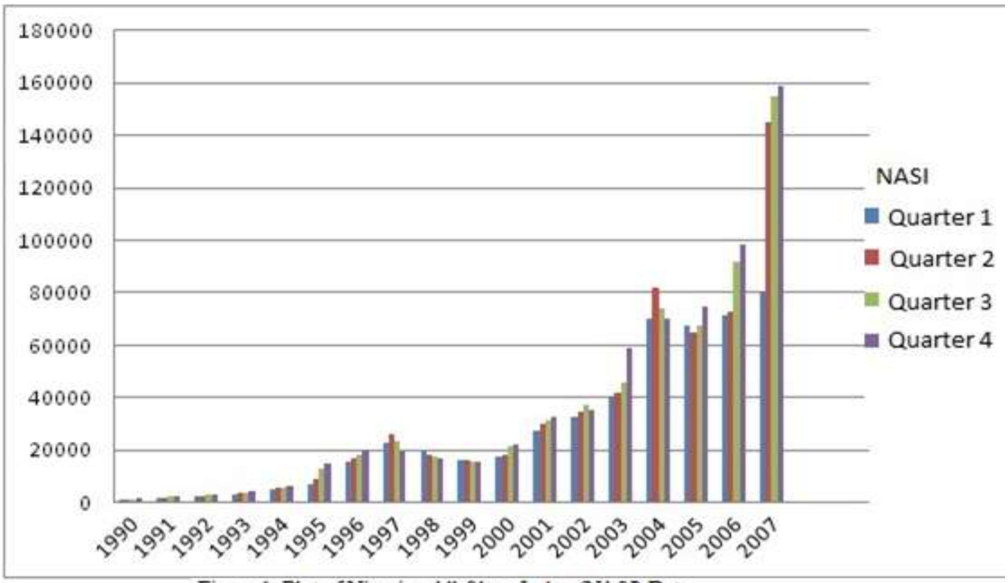

Figure 1: Plot of Nigerian All-Share Index (NASI) Data

Table 1: *R/S* Estimates of *H*

Year-t	y_r	x_r	w_r	c_r	Year-t	y_r	x_r	w_r	c_r
1990-1	1036.5	3.015569	-1.20317	-1.20317	1999-1	16404.8	4.214971	-0.00377	-15.80205
1990-2	1129	3.052G94	-1. 16G04	-2.3G921	1999-2	15959.4	4.203017	-0.01572	-15.81777
1990-3	1351. 7	3.13088	-1.08786	-3.45707	1999-3	15290.9	4.184434	-0.0343	-15.85208
1990-4	1476	3.169086	-1.04965	-4.50672	1999-4	15233	4.182784	-0.03595	-15.88803
1991-1	1647.2	3.216746	-1.00199	-5.50871	2000-1	17481.3	4.242574	0.02384	-15.86419
1991-2	1897.3	3.27813G	-0.940G	-6.44931	2000-2	18210.1	4.260312	0.04158	-15.82261
1991-3	2098.1	3.321826	-0.89691	-7.34622	2000-3	21366.1	4.329725	0.11099	-15.71163
1991-4	2288.8	3.359608	-0.85913	-8.20535	2000-4	22179.4	4.34595	0.12721	-15.58441
1992-1	2433.1	3.38616	-0.83258	-9.03793	2001-1	27067.9	4.432454	0.21372	-15.37070
1992-2	2575.4	3.410845	-0.80789	-9.84582	2001-2	29884.9	4.475452	0.25671	-15.11398
1992-3	2869.06	3.457738	-0.761	-10.60681	2001-3	31363.3	4.496422	0.27768	-14.83630
1992-4	3270.9	3.514GG7	-0.70407	-11.31088	2001-4	32877.4	4.516898	0.29816	-14.53814
1993-1	3352.5	3.525369	-0.69337	-12.00425	2002-1	32427	4.510907	0.29217	-14.24597
1993-2	3497	3.543696	-0.67504	-12.67929	2002-2	34939.9	4.543322	0.32459	-13.92138
1993-3	3579.4	3.55381	-0.66493	-13.34422	2002-3	37307.1	4.571791	0.35305	-13.56833
1993-4	4113.G	3.614222	-0.00451	-13.94873	2002-4	35025.1	4.54438	0.32564	-13.24269
1994-1	5053.4	3.703584	-0.51515	-14.46389	2003-1	40596	4.608483	0.38975	-12.85294
1994-2	5580.1	3.746642	-0.4721	-14.93598	2003-2	41930.1	4.622526	0.40379	-12.44915
1994-3	5781.4	3.762033	-0.4567	-15.39269	2003-3	46058.6	4.663311	0.44457	-12.00458
1994-4	6227.2	3.794293	-0.42444	-15.81713	2003-4	59085.2	4.771479	0.55274	-11.45183
1995-1	7055.4	3.848522	-0.37022	-16.18735	2004-1	70431.3	4.847766	0.62903	-10.82281
1995-2	8970.6	3.952821	-0.26592	-16.45326	2004-2	82166.5	4.914695	0.69596	-10.12685
1995-3	12712.6	4.104234	-0.1145	-16.56776	2004-3	74044.3	4.869491	0.G5075	-9.47609
1995-4	15156.5	4.180599	-0.03814	-16.60590	2004-4	70391.5	4.84752	0.62878	-8.84731
1996-1	15495.7	4.190211	-0.02853	-16.63443	2005-1	67334.4	4.828237	0.6095	-8.23781
1996-2	16656.5	4.221584	-0.00285	-16.63158	2005-2	64832.2	4.811791	0.59305	-7.64476
1996-3	18216.3	4.26046	-0.04172	-16.58986	2005-3	67748.4	4.830899	0.61216	-7.0
1996-4	20166.4	4.304628	-0.08589	-16.50397	2005-4	74694.7	4.87329	0.65455	-6.37804
1997-1	22768.5	4.357334	-0.1386	-16.36537	2006-1	71350.8	4.853399	0.63466	-5.74338
1997-2	25876.3	4.412902	-0.19416	-16.17120	2006-2	72722.6	4.861676	0.64294	-5.10045
1997-3	23610.9	4.373112	-0.15438	-16.01683	2006-3	916837	4.962292	0.74356	-4.35689
1997-4	19666.3	4.293722	-0.07498	-15.94184	2006-4	98254.7	4.992353	0.77362	-3.58328
1998-1	19250.4	4.28444	0.0657	-15.87614	2007-1	80458.6	4.905573	0.68684	-2.89644
1998-2	18249.9	4.261261	0.04252	-15.83362	2007-2	144952	5.161223	0.94249	-1.95395
1998-3	17411.1	4.249827	0.02209	-15.81153	2007-3	155127	5.190687	0.97195	-1.0
1998-4	17060	4.231979	0.01324	-15.79829	2007-4	158758	5.200734	0.982	-4.1E-06

References

[1] Anis, A.A. and Llyod, E. H. 1976.The expected value of the adjusted rescaled Hurst range of independent normal summands.Biometrika 63: 111-116.

[2] Annorzie, M. N. 2015. A Mathematical Model of the

Stochastic Dynamics of the Nigerian Stock Market.A Ph.D Thesis, Mathematics Department, University of Ibadan, Ibadan, Nigeria.

[3] Aydogan, K. and Booth, G. G. 1988. Are there long cycles in common stock returns? Southern Economic Journal 55:141-149.

[4] Bayraktar, E., Poor, H. V. and Sircar, K. R. 2003.Estimating the fractal dimension of the S& P 500 Index using Wavelet analysis.

[5] Chronopoulou, A. and Viens, F. 2009. Hurst index estimation for self-similar processes with long-memory.In Recent Development in Stochastic Dynamics and Stochastic Analysis.Duan, J., Luo, S. and Wang, C. (Eds.) 91118, World Scientific.

[6] Davies, R. B. and Harte, D. S. 1987. Tests for Hurst effects.Biometrika 74:95-102.

[7] Edgar, E. P. 1994. Fractal market analysis.Applying chaos theory to investment and economics. John Wiley, New York.

[8] Gloter, A. and Hoffman, M. 2007. Estimation of the Hurst parameter from discrete noisy data. Annals of Statistics, 35(5): 19947-1974.

[9] Lo, A. W. 1991. Long-term memory in Stock market prices.Econometrica 59:12791313.

[10] Hurst, H. 1951. Long term storage capacity of reservoirs. Transactions of the American Society of Civil Engineers 116:770-799.

[11] Mandelbrot, B. B. and Wallis, J. 1968. Noah, Joseph and operational hydrology. Water Resources Research 4:909-918.

[12] Mandelbrot, B. B. 1975. A fast fractional Gaussian noise generator. Water Resources Research. 7:543-553.

[13] Matos, J. A. 0.,Gama, S. M. A., Ruskin, H. J., Sharkasi, A. A. and Crane, M. 2008. Time and scale Hurst parameter analysis for financial markets. Physical A: Statistical Mechanics and its application, NorthHolland. 387.15:3910-3915.

[14] Peters, E. 1991. Chaos and order in the Capital Markets. John Wiley and Sons, New York.

[15] Pfaff, B. 2006.Analysis of Integrated and Cointegrated Time Series with R. Springer, New York.

[16] Taqqu, M. S., Teverovsky. V. and Willinger, W. 1995. Estimators for long-range dependence: an empirical study. Fractals 3.4:785-798.