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Theory of Conservation of Photon Mechanical Energy, in the Transition between Two Middles, in Rotational Kinetic Energy

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Abstract: ***In this work, an analysis of the light transition between two middles was performed, considering the photon-matter interaction according to the literature, verifying the predominant effects for low energy photons. It was considered the possibility of the expansion of the predominance of the Compton effect to low energy photons in processes such as the phenomenon of refraction. The conservation of photon mechanical energy in rotational kinetic energy was introduced, which presented implications such as the conservation of angular momentum and linear momentum in the photon-electron collision, predicting that the state of motion of the electron before the collision would not be null as idealized by Compton, presenting linear and angular moments. A relation was established between the relative refractive index between the two middles and the possible quantum states allowed to the electron, for the occurrence of collisions. It has been found that the wavelength shift is directly proportional to the relative constant between two middles given by*** $(1-\frac{n_1^2}{n_2^2})^{-\frac{1}{2}}$.

Keywords: Photon rotational energy, refraction, photon-matter interaction

1. Introduction

The effects of the transition between two middles are widely known in the literature, as well as the interaction between photons and matter that present three well-known effects, as we will discuss in this work. The effects of transition of light between two middles, for example between air / vacuum and liquids, appear, apparently, as a discussion exhausted in the literature.

In these particular transitions, the effects of light scattering and refraction, among other aspects, are characterized by variations in velocity and deviations of electromagnetic, monochromatic or polychromatic waves trajectories, from the interface between the two middles. These effects are easily understood through waving analysis, in which the effects of variations in velocity are attributed to variations in wavelength, since according to [7], the frequency is a constant when a ray travels between two middles. The deviations of the trajectories are characterized by the Snell-Descartes Law.

Recently, experiments with interferometers have shown that light presents an angular momentum beyond the intrinsic, in an orbital motion around the propagation axis [1].

In this work, this discussion is resumed in the perspective of the corpuscular behavior, considering the photon-matter interaction and the conservation of linear momentum and relativistic mechanical energy. The conservation of the mechanical energy of the photon in rotational kinetic energy is introduced, discussing the aspects that involve the transition of the light between two middles under the light of the angular momentum and its rate.

2. Theoretical Reference

The refraction of light at the interface between two middles presents one of visual effects more admired, the decomposition of light in different length of waves. This effect is due to the fact that the polychromatic light is composed of monochromatic waves, which present refractions with different deviations from the original trajectory, because in agreement with the law of Snell-Descartes [4]:

$$n_1 sen\theta_1 = n_2 sen\theta_2 \quad (1)$$

Where the velocity in the second propagation middles is attenuated in the ratio:

$$v_2 = \frac{n_1}{n_2} v_1 \quad (2)$$

This for $n_1 < n_2$, or the opposite for $n_1 > n_2$.The ratio between the wavelengths:

$$\lambda_2 = \frac{n_1}{n_2} \lambda_1 \quad (3)$$

In agreement with [2], one can find three well-known forms of photon interactions: Photoelectric effect, Compton effect and pair production, disregarding those that for example involve pair annihilation or radiation production. According to [6], we can identify the effect that best represents a photon-matter interaction, knowing the chemical elements that constitute the sample, through a predominance analysis that can be verified in figure 1.

According to [8], the kinetic energy of relativistic translational associated with the photon is described by:

$$E_{translação} = \gamma mc^2 + mc^2 \quad (4)$$

For the photon its kinetic energy of translation will be described by the first term, since its energy of rest is null $(E_0 = mc^2 = 0)$.

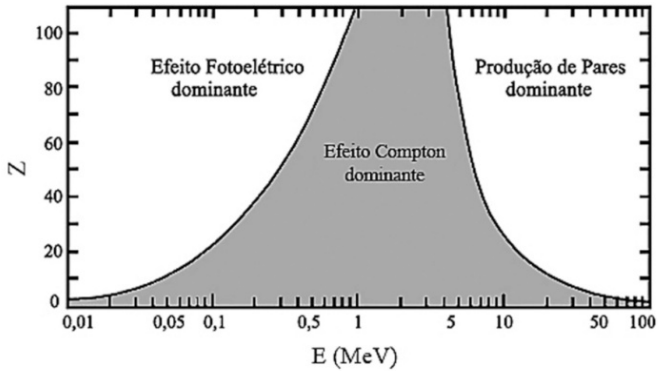

Figure 1: Diagram of the predominant effect on the photon-matter interaction **[6]**

According to [3], in the collisions in which the conservation of energy does not occur, it is observed that another quantity is being conserved, the linear momentum. Generally speaking, for "N" particles colliding:

$$\sum m_j v_j = const. \tag{5}$$

The rotational kinetic energy, according to [8], can be written in terms of the product between the angular momentum (L) and the angular velocity (ω), given by:

$$E_{rot} = \frac{1}{2}\omega L \tag{6}$$

In the microscopic description of the state of motion of an electron, for an atom of an electron, according to [2], the angular momentum of the electron is given by the quantum numbers "j", given by:

$$J = \sqrt{j(j+1)}\hbar \tag{7}$$

According to [2], Schroedinger's equation is consistent with relativistic energy, especially in the description of the free particle. In this sense, the linear momentum of the electron can be expressed in terms of the uncertainty of the variation of its translational kinetic energy, in the form of:

$$(\Delta p)^2 \approx 2m\Delta E_{trans.} \tag{8}$$

Heisenberg's Principle of Uncertainty apud [2], for an electron orbiting around the atomic nucleus, points out that the minimum values for the areas of the coordinates in the phase space, are given by the relation of uncertainty:

$$\Delta r \Delta p = 2\pi r \Delta p \geq \frac{\hbar}{2} \tag{9}$$

3. Materials and Methods

Considering the possible effects of the photon-matter interaction, discussed in the previous section, and for a molecular set such as water consisting of hydrogen and oxygen atomic numbers 1 and 8, respectively, According to figure 1, the photoelectric effect is predominant for low energy photons, such as light in the visible band. However, the photoelectric effect is due to characteristics that pass through the ejection of an electron, in which leftover only for photon the energy expenditure for work performed for the ejection and for new energy state of the electron. In this sense, we will verify a possible expansion of the predominance of the Compton effect, in the light of this study.

For this, the adhesion of the Compton effect to the corpuscular behavior of the light through transition between two middles was verified. According to Compton apud [2], a portion of the energy of the photon is transferred to the receiving electron and the remainder is spread in the form of another photon, with different wavelength, such that:

$$\Delta\lambda(\theta) = \frac{h}{mc}(1 - \cos\theta) \tag{10}$$

In the analysis of conservation of mechanical energy, the photon was considered as a free particle, except at the interface between two middles. The photon was allowed to conserve its energy in rotational kinetic energy. The Energy Conservation Principle was considered to verify the energy associated with the rotational movement.

To identify, in the photon-electron collision, the total quantum angular momentum of the electron from the wavelength shift, the angular momentum of the photon was quantized. Is assigned to the electron before the collision, the linear and angular moments. Consistent, especially with regard to the description of the free particle, for the photon is attributed a relativistic energy.

4. Results and Discussion

Considering the successive interactions with matter, which are often characterized by the absorption of photons in the electron clouds, there will be attenuation of beam intensity:

$$E_2 = n' h\nu < E_1 \tag{11}$$

Being:

$$E_1 = nh\nu \tag{12}$$

Although we may experience the decrease in beam intensity, it does not mean that it causes a decharacterization in the energetic package of each photon, because for the survivors $E_{f\acute{o}ton} = h\nu$.

However, the speed variation according to the Snell-Descartes Law refers to an attenuation of the translational kinetic energy, which can be restored if the photon returns to the original middle. In this sense, it is presumed here that somehow there is conservation of mechanical energy or linear momentum in this process. The following assumptions are made, one that verifies the expansion of the predominance of Compton effect (section 4.1) and another that assumes that the photon conserves its energy in rotational kinetic energy in the transition between two middles (section 4.2).

4.1 Trajectory deviation in accordance with Compton displacement

The Compton effect occurs in the interaction of an x-ray photon with matter, represented by a graphite target. It can be seen from equation (10) that the Compton displacement considers that the photon will interact with only one electron as it traverses the thickness of the plate. In this work, we will consider that the second light propagation middle can be sectioned on blades of thickness similar to the Compton experiment, where for **n** blades, there will be **n** interactions with electrons, such that equation (10) can be written as:

$$\lambda_n - \lambda_{n-1} = \frac{h}{nm_e c}(1 - \cos\theta_n), \tag{13}$$

at where n = 1, 2, 3, ..., N.

Considering that the wavelength is a function of the scattering angle, we can differentiate:

$$\frac{d\lambda(\theta)}{d\theta} = \frac{h}{nm_e c} sen\theta \tag{14}$$

The number of electrons associated to the interactions corresponds to the number of blades, such that:

$$n = \frac{x}{dx} \tag{15}$$

Substituting the ratio given by equation (15) into (14), one can present sum the a continuous:

$$\int_{\lambda_{n-1}}^{\lambda_n} d\lambda = \frac{h}{m_e c}\int_0^{\theta} sen\theta d\theta \int_0^{x} \frac{dx}{x} \tag{16}$$

Which allows us to find an expression for the angle of deviation of the trajectory as a function of the thickness of the material, given by:

$$\theta = arc\cos\left[\frac{-m_e c\Delta\lambda}{h ln(x)}\right] \tag{17}$$

In figure (2), a simulation for equation (17) is presented, considering only the orders of magnitude for simplicity, for any material of up to 100 meters of thickness.

Although the first layers (blades) are responsible for the largest deviations of the trajectory, according to figure 2, agreeing with the effects of refraction at the interface between two middles, where the deviation of the trajectory is determinant for the new trajectory of the beam. It be seen that with increasing thickness the deviation remains around the mean. It implies that the deviation will increase gradually as the photon moves from one blade to the other.

The theoretical result seems to agree with intuitive expectations that if each layer were capable of producing a deviation in accordance with the Compton's displacement, a curve motion would present to the photon, which does not actually happen. In these terms, it is ruled out here that the Compton effect can explain the deviations of trajectories arising from the refraction of light between two middles, either through the photon-matter interaction. From this reading, the hypothesis that conserves the mechanical energy of the photon in rotational kinetic energy is verified, neglecting the need for comparisons between the Compton displacement and the displacement that can be obtained through the Snell-Descartes Law.

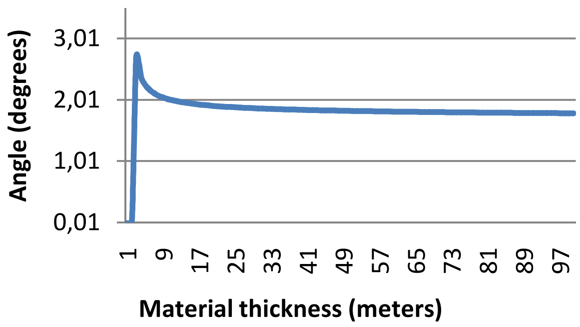

Figure 2: Variation of the deviation of the angle of refraction as a function of the thickness of the material, from the displacement of Compton[1]

4.2 Conservation of energy in rotational kinetic energy

Considering the photon as a free particle, which moves in the absence of forces, we have that associated potential energy:

$$U(x) = \int F(x)dx = 0 \tag{18}$$

Thus mechanical energy presents itself only through its kinetic energy. Let us assume that the kinetic energy of the photon can be written in the relativistic form:

$$E_c = \gamma mc^2 - mc^2 \tag{19}$$

Considering that the photon does not have resting energy, we have:

$$E_c = \gamma mc^2 \tag{20}$$

Being the relativistic constant:

$$\gamma = \frac{1}{\sqrt{\left(1 - \frac{V_{rel}^2}{C^2}\right)}} \tag{21}$$

It remains to attribute to the photon a kinetic energy of translation in the vacuum given by:

$$E_{fóton} = \gamma mc^2 \tag{22}$$

Introducing the rotation movement in the 2nd propagation middle:

$$E_{cinética} = E_{translação} + E_{rotação} \tag{23}$$

[1] To simulate the deviation of the angle, we considered only the orders of magnitude for the electron mass, the speed of light, the wavelength shift and the Planck constant.

Considering eq. (22) and assuming that the kinetic energy in vacuum (approaching air) is purely translational, the conservation of kinetic energy can be written by:

$$E_{fóton} = E_{translação} + E_{rotação} \quad (24)$$

Such that, considering that in the 2nd middle of propagation $v \cong c$, we can write:

$$\gamma mc^2 = \gamma mv^2 + E_{rotação} \quad (25)$$

The kinetic energy of rotation:

$$E_{rot} = \gamma mc^2 (1 - \frac{v^2}{c^2}) \quad (26)$$

It is possible to see that if v = 0, either by confinement, does not imply the absence of movement, since there will be a rotation movement, in its largest module:

$$|E_{rot}|_{máx.} = \gamma mc^2 \quad (26)$$

Using the Snell-Descartes Law, we have:

$$v = \frac{n_1}{n_2} c \quad (27)$$

Where n_1 and n_2 are the refractive indices in middles 1 and 2, respectively. It can be write the equation (27) in at form:

$$E_{rot} = \gamma mc^2 (1 - \frac{n_1^2}{n_2^2}) \quad (28)$$

For $n_1 = n_2$, the rotational kinetic energy is zero and photon has only translational motion.

Considering the mismatch between the relativistic constant and the mass of the photon, which according to [5], both numerator and denominator tend for zero, it adopted be write:

$$E_{rot} = h\nu (1 - \frac{n_1^2}{n_2^2}) \quad (30)$$

4.2.1 Possible causes for variations in original trajectory

The result of equation (30) allows us to relate its angular velocity and angular momentum to the relative refractive index between two middles, such that:

$$\frac{1}{2} L\omega = h\nu (1 - \frac{n_1^2}{n_2^2}) \quad (31)$$

We can rewrite equation (31), where the angular momentum is given by:

$$L = 2\hbar (1 - \frac{n_1^2}{n_2^2}) \quad (32)$$

At the interface between the two middles, a variation of angular momentum occurs while L = 0, for $n_1 = n_2$. The variation of angular momentum:

$$L_{n1 \neq n2} - L_{n1 = n2} = 2\hbar (1 - \frac{n_1^2}{n_2^2}) \quad (33)$$

Similarly, we find this variation in re-entry to the original middles:

$$L_{n1 = n2} - L_{n1 \neq n2} = -2\hbar (1 - \frac{n_1^2}{n_2^2}) \quad (34)$$

In this perspective, the photon experiences a torque whenever it crosses the interface between the two middles, which act in the direction of the new trajectory, such that:

$$\tau = \frac{\Delta L}{\Delta t} \quad (35)$$

The effects associated with the torque at the interface are determinant for the new trajectory of the photon, since after being immersed in the middle 2, it is noticed that the angular momentum is constant and the torque is zero:

$$\tau = \frac{d}{dt} \left[2\hbar (1 - \frac{n_1^2}{n_2^2}) \right] = 0 \quad (36)$$

Considering equation (36) and the law of Snell-Descartes, we can write:

$$\frac{sen^2\,\theta_2 - sen^2\,\theta_1}{sen^2\,\theta_1} = \frac{L}{2\hbar} \quad (37)$$

For small angles, we can identify that the gradient of trajectory deviation along the thickness is zero, such that the difference is constant:

$$\frac{d}{dx}(\theta_2^2 - \theta_1^2) = \frac{d}{dx}\left(\frac{L}{2\hbar}\right) = 0 \quad (38)$$

4.2.2 Conservation of the angular moment in at the fonto-elétron collision

In the perspective of the photon presenting conservation of energy in the form of rotational kinetic energy, it indicates that the causes of a torque are associated with the action of forces at a distance or collisions with entities of the electronic structure, be they the electrons. Assuming a process by collisions, satisfying the conservation of the angular momentum while the external torque is zero, we have:

$$\vec{L}_{e_0} = \vec{L}_e + \vec{L}_f \quad (39)$$

The magnitude of the angular momentum of the electron after the collision:

$$L_e^2 = L_{e_0}^2 + L_f^2 - 2L_{e_0} L_f \cos \emptyset \quad (40)$$

Considering the result presented in equation (40) and using the quantum model for the total angular momentum of the electron, according to equation (7), we can write:

$$L_e^2 = j(j+1)\hbar^2 + 4\hbar^2 \left(1 - \frac{n_1^2}{n_2^2}\right)^2 - 4\sqrt{j(j+1)}\hbar^2 (1 - \frac{n_1^2}{n_2^2}) \cos \emptyset \quad (41)$$

From equation (9), we can predict the angular momentum of the electron by:

$$\Delta L_e \geq \frac{\hbar}{4\pi} \quad (42)$$

It results in an expression for the angular momentum of the electron:

$$L_e \geq \frac{\hbar}{4\pi} + \sqrt{j(j+1)}\hbar \quad (43)$$

Substituting equation (43) into (41):

$$\frac{1}{16\pi^2} + \frac{1}{2\pi}\sqrt{j(j+1)} \geq$$
$$4(1-\frac{n_1^2}{n_2^2})^2 - 4\sqrt{j(j+1)}(1-\frac{n_1^2}{n_2^2}) \cos\emptyset \quad (44)$$

In this relation it is possible to define the quantum numbers allowed to the electrons involved, in the transition of light in different middles, according to equation (45):

$$\sqrt{j(j+1)} \leq \frac{8\pi(1-\frac{n_1^2}{n_2^2})^2 - \frac{1}{8\pi}}{\left[1+8\pi\left(1-\frac{n_1^2}{n_2^2}\right)\cos\emptyset\right]} \quad (45)$$

Being $\emptyset$ the angle between the directions of the angular moments of the electron and the photon. From equation (44), one can write:

$$\lambda_2 = \eta\lambda_1 \quad (46)$$

Such that:

$$\eta = \sqrt{1 + \frac{\sqrt{j(j+1)}\cos\emptyset \pm 4\sqrt{[j(j+1)\cos^2\emptyset] + \left[\frac{1}{16\pi^2} + \frac{1}{2\pi}\sqrt{j(j+1)}\right]}}{2}} \quad (47)$$

Equations (45) and (46) are revealing. However, the dependence of the ângulo angle generates the need for its own determination. In this sense, it is suggested that the analysis of the Linear moment, as the one done by Compton, is more appropriate to the step that involves the angle of trajectory deviation, which can be obtained experimentally with some ease.

4.2.3 Conservation of the linear moment in the transition between two middles

In Compton's description, for X-ray scattering, it was considered that the linear momentum of the electron before the collision would be zero. In the present work, the fact of introducing here the conservation of energy in rotational kinetic energy, is assumed in the analysis of the conservation of the angular momentum that the electron has angular momentum before the collision, $\sqrt{j(j+1)}\hbar$, in which it implies that it also presents initial linear momentum, since $L = \vec{p} \times \vec{r}$. In this sense we can satisfy:

$$\vec{p}_{f_0} + \vec{p}_{e_0} = \vec{p}_f + \vec{p}_e \quad (48)$$

Such as:

$$p_f^2 = p_{f_0}^2 + \Delta p_e^2 - 2p_{f_0}\Delta p_e \cos\theta \quad (49)$$

Expressing the variation of the linear moment in terms of the kinetic energy of translation of the electron, according to equation (8):

$$\frac{h^2}{\lambda_f^2} = \frac{h^2}{\lambda_{f0}^2} + 2m_e\Delta E_{transl.} - \frac{h}{\lambda_{f0}}\sqrt{2m_e\Delta E_{transl.}}\cos\theta \quad (50)$$

Considering the conservation of energy, it is assumed that the variation of the kinetic energy of translation of the electron is equivalent to the variation of the kinetic energy of translation of the photon, whose difference is given by equation (30), where we can write:

$$\frac{h^2}{\lambda_f^2} - \frac{h^2}{\lambda_{f0}^2} = 2m_e h\nu_f(1-\frac{n_1^2}{n_2^2}) - \frac{h}{\lambda_{f0}}\sqrt{2m_e h\nu_f(1-\frac{n_1^2}{n_2^2})}\cos\theta \quad (51)$$

Comparing both sides of equality, it is possible to infer that:

$$\frac{h}{\lambda_{f0}} = \sqrt{2m_e h\nu_f(1-\frac{n_1^2}{n_2^2})\cos\theta} \quad (52)$$

$$\frac{h}{\lambda_f} = \sqrt{2m_e h\nu_f(1-\frac{n_1^2}{n_2^2})} \quad (53)$$

In the ratio of equations (51) and (52), considering equation (3), we obtain:

$$\cos\theta = \frac{\sin\theta_{ref}}{\sin\theta_{inc}} \quad (54)$$

The wavelength shift is given by:

$$\lambda_f - \lambda_{f0} = \frac{h}{\sqrt{2m_e h\nu_f(1-\frac{n_1^2}{n_2^2})}}(1-\sec\theta) \quad (55)$$

5. Final Considerations

Considering the hypotheses presented for the modeling of the corpuscular behavior of the photon when moving between two middles, it was found that the predominance of the Compton displacement cannot be extended to low energy photons such as those found in the visible range, in the processes of refraction because the displacement of the wavelength would occur at each infinitesimal thickness of the material generating a gradual increase of the angle of refraction along the thickness, reserving to the photon a curved trajectory.

In the proposition of this work, to satisfy the conservation of the mechanical energy of the photon in rotational kinetic energy, we obtained a description in which the variation of the trajectory in the interface of two middles, is due to the appearance of a torque and that from this , the photon conserves part of its mechanical energy in rotational kinetic

energy in a constant angular motion along the new trajectory and that reentering the original middle, again at the interface, would experience a torque capable of moving it to a new trajectory.
In this study, the photon was assumed to be a free particle along its trajectories. However, the actions of torques indicated that at the interface the potential energy is not null, where it was assumed that the process occurs by the action of forces involved in the photon-electron collision. The assumed description of a rotation motion for the photon after the collision allowed predicting, unlike Compton's theory that the electron involved was not at rest before the collision, having linear and angular moments. From this it was possible to relate the possible quantum numbers allowed to the electron with the refractive indices, so that the collision occurs as theorized. The displacement of the wavelength, in the light of this work, is directly proportional to the relative constant between the two middles, which is assumed to be $(1-\frac{n_1^2}{n_2^2})^{-\frac{1}{2}}$.

6. Other Recommendations

Although it was a matter of known literature and experimental effects of refraction are widely diffused, new studies are indicated, such as the ratio between the intrinsic angular momentum and the orbital, as well as to classify for different transitions between two middles it the quantum numbers n, l, s and j, for the occurrence of collisions.

References

[1] BALLANTINE, Kyle E.; DONEGAN, John F.; EASTHAM, Paul R. There are many ways to spin a photon: Half-quantization of a total optical angular momentum. Sci. Adv. 2016; 2 : e1501748
[2] EINSBERG, Robert; RESNICK, Robert. Física Quântica. Rio de Janeiro: Elsevier, ISBN 85-700-1309-4, 24ª reimpressão, p. 51, 1979.
[3] HALLIDAY, D.; RESNICK, R.; KRANE, K. S. Física 1, LTC - Rio de Janeiro, 5 ed., vol. 4, 384 p., 2004.
[4] NUSSENZVEIG, H. Moysés. Física Básica 4, São Paulo: Ed. Blücher, ISBN 85-212-0163-X, vol. 4, 4 ed., 437 p., 2002.
[5] MORGON, Nelson H. O comportamento do elétron, uma análise do efeito Compton e da relação de Broglie. Quim. Nova, Vol. 31, No. 7, 1869-1874, 2008.
[6] SA, José Roberto et al . Interação da Física das Radiações com o Cotidiano: uma prática multidisciplinar para o Ensino de Física. Rev. Bras. Ensino Fís., São Paulo , v. 39, n. 1, e1503, 2017.
[7] SERWAY, Raymond A.; JR., John W. Jewett. Princípios de Física. São Paulo: CENGAGE Learning, ISBN 85-221-0437-9, 4 vol., 3 ed., p. 992, 2009.
[8] Thornton, Stephen T.; Marion, Jerry B. Dinâmica clássica de partículas e sistemas. São Paulo: CENGAGE Learning, ISBN 978-85-221-0906-7, 5 ed., p. 376-505, 2011.

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