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Finite Element Based Vibration and Stability Analysis of Functionally Graded Rotating Shaft System under Thermal Environment

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Abstract: ***The present work deals with the study of vibration and stability analyses of functionally graded (FG) spinning shaft system under thermal environment using three nodded beam element based on Timoshenko beam theory (TBT). Temperature field is assumed to be a uniform distribution over the shaft surface and varied in radial direction only. Material properties are assumed to be temperature dependent and graded in radial direction according to power law gradation and exponential law gradation respectively. In the present analysis, the mixture of Aluminum Oxide (Al_2O_3) and Stainless Steel (SUS304) is considered as FG material where metal contain (SUS304) is decreasing towards the outer diameter of shaft. The FG shafts are modeled as a Timoshenko beam by mounting discrete isotropic rigid disks on it and supported by flexible bearings that are modeled with viscous dampers and springs. Based on first order shear deformation (FOSD) beam theory with transverse shear deformation, rotary inertia, gyroscopic effect, strain and kinetic energy of shafts are derived by adopting three-dimensional constitutive relations of material. The derivation of governing equation of motion is obtained using Hamilton's principle and solutions are obtained by three-node finite element (FE) with four degrees of freedom (DOF) per node. . In this work the effects of both internal viscous and hysteretic damping have also been incorporated in the finite element model. A complete code has been developed using MATLAB program and validated with the existing results available in literatures. The analysis of numerical results reveals that temperature field and power law gradient index have a significance role on the materials properties (such as Young modulus, Poisson ratio, modulus of rigidity, coefficient of thermal expansion etc.) of FG shaft. Various results have also been obtained such as Campbell diagram, stability speed limit (SLS), damping ratio and time responses for FG shaft due unbalance masses and also compared with conventional steel shaft. It has been found that the responses of the FG spinning shaft are significantly influenced by radial thickness, power law gradient index and internal (viscous and hysteretic) damping and temperature dependent material properties. The obtained results also show that the advantages of FG shaft over conventional steel shaft.***

Keywords: Power law gradient index; Functionally graded shaft; Temperature dependent material properties; Viscous and hysteretic damping; Rotor-Bearing-shaft system; Finite element method; Campbell diagram; Damping ratio; stability speed limit (SLS)

1. Introduction

Composite materials and structures are more and more frequently used in advanced engineering fields mainly because of their high stiffness-to-weight ratio that is particularly favorable. However the main downside of composite materials is represented by the weakness of interfaces between adjacent layers known as delimitation phenomena that may lead to structural failure. To partially overcome these problems, a new class of materials named Functionally Graded Materials (FGMs) has recently been proposed whose various material properties vary through the radial and thickness direction in a continuous manner and thus free from interface weakness. The gradation of material properties reduces thermal stresses, residual stresses, and stress concentrations. A functionally graded structure is defined as, those in which the volume fractions of two or more materials are varied continuously as a function of position along certain dimension (typically the radius and thickness) of the structure to achieve a require function. FGMs can provide designers with tailored material response and exceptional performance in thermal environments. For example, the Space Shuttle utilizes ceramic tiles as thermal protection from heat generated during re-entry into the Earth's atmosphere. An FGM composed of ceramic on the outside surface and metal on the inside surface.

1.1 Background and Importance of Rotor Dynamic

Rotor dynamics has a remarkable history, largely due to the interplay between its theory and its practice. **Rotor dynamics** is a specialized branch of applied mechanics concerned with the behavior and diagnosis of rotating structures. It is commonly used to analyze the behavior of structures ranging from jet engines and steam turbines to auto engines and computer disk storage. Basic level of Rotor Dynamic is concerned with rotor and stator. Rotor is a rotating part of a mechanical device or structures supported by bearings and influenced by internal phenomena that rotate freely about an axis fixed in space. Engineering components concerned with the subject of rotor dynamics are rotors in machines, especially of turbines, generators, motors, compressors, blowers, alternators, pumps, brakes, distributors and the like. In Rotor Dynamics field William John Macquorn Rankine (1869) performed the first analysis of a spinning shaft. He chose a two-degrees-of-freedom model consisted of a rigid mass whirling in an orbit, with elastic spring acting in the radial direction. He defined the whirling speed of the shaft but he can be shown that beyond this whirling speed the radial deflection of Rankine's model increases without limit and this speed is called threshold speed for the divergent instability.

In 1883 Swedish engineer Carl Gustaf Patrik de Laval developed a single-stage steam impulse turbine for marine applications and succeeded in its operation at 42,000 rpm. He first used a rigid rotor, but latter used a flexible rotor and shown that it was possible to operate above critical speed by operating at a rotational speed about seven times the critical speed. In 1895, Stanley Dunkerley published a study of the vibration of shafts loaded by pulleys. The first sentence of his paper reads, "It is well known that every shaft, however nearly balanced, when driven at a particular speed, bends, and, unless the amount of deflection is limited, might even break, although at higher speeds the shaft again runs true. This particular speed or 'critical speed' depends on the manner in which the shaft is supported, its size and modulus of elasticity, and the sizes, weights, and positions of any pulleys it carries." In 1895 German civil engineer August Foppl who showed that an alternate rotor model exhibited a stable solution above Rankine's whirling speed. In England W. Kerr (1916) published experimental evidence that a second "critical speed" existed and it was obvious to all that a second critical speed could only be attained by the safe traversal of the first critical speed. In 1918 Ludwig Prandtl was the first to study a Jeffcott rotor with a non-circular cross-section.

1.2 Composite Materials

Composite materials are formed by combining two or more material on a micro scale form and their constituents do not dissolve or merge into each other, to achieve superior enhanced properties. These are widely used in a variety of structures, including army and aerospace vehicles, nuclear reactor vessels, turbines parts, buildings and smart highways (i.e. civil infrastructure applications) as well in sports equipment and medical prosthetics. Laminated composite structures consist of several layers of different fiber-reinforced laminate bonded together to obtain desired structural properties (e.g. stiffness, strength, wear resistance, CTE, Thermal conductivity, damping, and so on). Varying the lamina thickness, lamina material properties, and stacking sequence desired structural properties can be achieved. The increased use of laminated composites in various types of structures led to considerable interests in their analysis. Composite materials exhibit high strength-to weight and stiffness-to-weight ratios, which make them ideally suited for use in weight sensitive

1.3 Drawback of Composite Materials

Though laminated composites give numerous advantages over conventional materials, their major downside is however represented by the repeated cyclic stress, impact load and so on can causes to separate layers and weakness of interfaces between adjacent layers, known as delamination phenomena (i.e. Mode of failure or failure mechanisms of composite materials). It may lead to failure of the structure. Additional problems include the presence of residual stresses due to the difference in coefficient of thermal expansion and coefficient of moisture expansion of the fiber and matrix. For anisotropic constitution of laminated composite structures often results in stress concentrations near material and geometric interface that can lead to damage in the form of de-lamination, matrix cracking and adhesive bond separation. These problems can be reduced if the sudden change of material properties is somehow prevented.

1.4 Conceptual Idea about FGMs

First FGM concepts have come from Japan in 1984 during a space plane project. There a combination of materials used would serve the purpose of a thermal barrier capable of withstanding a surface temperature of 2000 K and a temperature gradient of 1000 k across a 10 mm section. Recently FGMs concept has become more popular in Europe (Germany). A collaborative research center Transregio (SFB Transregio) is funded since 2006 in order to exploit the potential of grading mono-materials, such as steel, aluminum and polypropylene, by using thermo mechanically coupled manufacturing processes**.** Functionally Graded Materials (FGMs) are those composite materials where the composition or the microstructure is locally varied so that a certain variation of the local material properties is achieved. FGM is also defined as, those in which the volume fraction of two or more materials are achieved continuously as a function of position along certain directions of the structure to achieve a required function (e.g. mixture of ceramic and metal). It is materially heterogeneous which is defined for those objects with and/or multiple material objects with clear material domain. By grading of material properties in a continuous manner, the effect of inter-laminar stresses developed at the interfaces of the laminated composite due to abrupt change of material properties between neighboring laminas is mitigated. As many thin walled members, i.e., plates and shells used in reactor vessels, turbines and other machine parts are susceptible to failure from buckling, large amplitude deflections, or excessive stresses induced by thermal or combined thermo mechanical loading. Thus, FGMs are primarily used in structures subjected to extreme temperature environment or where high temperature gradients are encountered. Mainly they are manufactured from isotropic components such as metals and ceramics, since role of metal portion is acts as structure support while ceramics provides thermal protection in environments with severe.

1.5 Applications of FGMs

Due to progressing of technology it is need for advanced capability of materials to become a priority in engineering field for higher performance systems. FGMs are relatively new materials and are being studied for the use in high temperature applications. FGM is an extensive variety of applications in engineering practice which requires materials performance to vary with locations within the component. The following applications are noticeable such as

1) Aerospace field (space planes, space structures, nuclear reactors, insulations for cooling structures, Aerospace skins, Rocket engine components, Vibration control, Adaptive structures etc.).
2) Engineering field (Turbine blade, shaft, cutting tool etc.)
3) Optical field (optical fiber, lens etc.)
4) Electronics field (sensor, graded band semiconductor, substrate etc.)
5) Chemical field (Heat Exchanger, Reactor Vessel, Heat Pipe etc.)
6) Biomaterial field (artificial skin, drug delivery system, prosthetics etc.)

7) Commodities (Building materials, Sports good, Car body etc.)
8) Energy conversion (Thermoelectric generator, Thermo ionic converter, Fuel cells, Solar cells etc.)
9) Optoelectronics
10) Piezoelectricity

1.6 Objectives of Present Work

The specific objectives of the present thesis have been laid down as

- Development of material modeling for FG shaft based on the different laws of gradation
- Modeling of FG shaft with temperature dependent material properties
- Effects of different temperatures and power law gradient indexes on variation of mechanical properties through the radial direction of the FG shaft.
- Finite modeling of FG spinning shaft system (i.e. rotor-bearing-shaft system) in order to study the vibration behavior of this shaft system
- To study the vibration and stability analysis of FG shaft system by incorporating internal viscous and hysteretic damping
- To comparative study of the various responses of FG shaft over steel shaft
- To study the effects of different temperatures and power law gradient indexes on the various responses of the FG shaft.
- To study the dynamic behaviors (i.e. critical speed, fundamental frequencies, Campbell diagram, Damping ratio, Time response and Stability Limit Speed) of rotating FG shaft system under thermal and mechanical loadings by incorporating internal viscous and hysteretic damping

2. Modeling for Effective Materials Properties of Fg Shaft

This chapter modeling of FG material to obtain the effective material properties of FG shaft by considering power law gradation and exponential law gradation.

2.1 Effective Materials Properties of FGM

As FGMs are heterogeneous materials so there is need for the determination of effective material properties. To achieve best performance, accurate material property estimation is essential for analysis and design of FG structures/system. There are various models developed to determine the material properties of FGM such as

1) Rules of mixtures: Linear rule of mixtures and Harmonic rule of mixtures
2) Variational approach
3) Micromechanical approaches

2.2 Modeling for Material Properties of FG Rectangular Cross-section

A FGM beam is considered having with finite length L and Thickness t and also made of a mixture of ceramics (aluminum oxide) and metal (stainless steel).The effective material properties P can be written as,

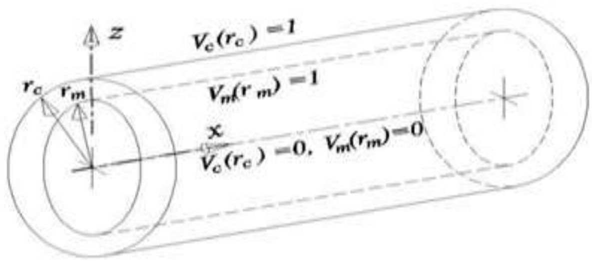
$$P = P_c V_c + P_m V_m \qquad (1)$$

Where, P_c and P_m are the material properties of the ceramic and metal respectively. Now V_c and V_m are the volume fractions of ceramic and metal respectively and they are related by

$$V_c + V_m = 1 \qquad (2)$$

2.3 Modeling for Material Properties of FG Circular Cross-section

A FGM shaft is considered with finite length L, inner radius (r_i) and outer radius (r_o) . Material of the shaft is considered in top surface (z □ r_i / 2) as ceramic and in bottom surface (z □ r_o / 2) as metal.



2.4. Formulation for Governing Equations of Rotor Shaft System

Based on the FOSD theory, shaft is modeled as a Timoshenko beam with considering rotary inertia and gyroscopic effect. The shaft is considered uniform circular cross-section and it is rotate at constant speed about its longitudinal axis. The displacements variables and schematic diagram of rotor-bearing system are shown with coordinate systems.

3. Conclusions

The present work enables to arrive at the following important conclusions:

- A three nodded beam finite element has been implemented for modeling and vibration analysis of the FG shaft system by incorporating both the internal viscous and hysteretic damping in the thermal environment.
- The temperature distribution is nonlinear along the radial direction of the cross-section of FG shaft.
- The material property distribution of the FG shaft has been performed very smoothly along the radial direction by accounting different temperatures and power law gradient indexes.
- From the comparison of various responses between steel and FG shaft, it has been found that FG shaft is more stable than the steel shaft.
- From the comparison of various responses of the FG shaft without and with temperatures consideration, it has been noticed that the FG shaft is more stable in case of without temperature consideration than that of with temperature consideration.
- The power law gradient index plays an important role in the responses (viz. Campbell diagram, damping ratio,

critical speed, stability limit speed and time responses) of the FG shaft system.

- It is observed that the less value of temperature and the power law gradient index promotes more stable system than that of higher values of temperature and power law gradient index
- Finally, it can be concluded that the present work can be used for modeling and vibration analysis of the FG shaft system considering with or without temperature dependent material properties according to power law gradation by incorporating both internal viscous and hysteretic damping.

4. Scope of Future Works

1) Study of vibration and stability analysis for FG rotor shaft system under electro-thermo-mechanical environment.
2) Study of vibration and stability analysis by using a fluid film journal bearing for this present model
3) Active vibration control of FG rotor shaft system
4) Nonlinear modeling of FG shaft and
5) Multi-objective optimization of FG shaft system

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