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Mathematical Analysis of Static and Dynamic Forces of Spur Gear Drive with High Contact Ratio

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Abstract: ***In this work, the procedure used for determination of the load acting on the various teeth in mesh of spur gear drive with high contact ratio is mathematical using mathematical expressions. In this procedure, it is assumed that in all circumstances the tooth deflections are equal at every pairs of contact; either it is two pairs or three pairs. Also it is assumed that summation of normal loads shared by each of two or three pairs of contact is equal to peak normal load. In this, a mathematical method based on torsional vibration of gears is used to determine the dynamic load of spur gear drive having high contact ratio. In the duration of meshing of gear, the dynamic load variations are determined using this. For calculations of dynamic load, it is required to determine variable stiffness, which is determined by using mathematical expressions. In the study of tooth stiffness calculations, various factors like bending compression, shear compression, axial compression and Hertzian contact deflections are considered. Friction amongst various teeth in contact and viscous damping are also taken in consideration in this analysis. In the analysis two elastic bodies is used which is held in contact by forces normal to the area of contact together with tangential frictional force is used for calculation of contact stresses. The variations in contact stresses during meshing are also determined. It is also found that the dynamic load which is acting on the gear tooth is greatly influenced by the operating speed. Due to change in operating speed, the value and location of peak dynamic load also changes.***

Keywords: Mesh stiffness, Dynamic load, High contact ratio, Spur gears

1. Introduction

1.1 History of Gearing

There are many modes of power transfer available in the industry. Gears are one amongst the modes of power transfer wide utilized in nearly every kind of machineries within the industry. Together with bolts, nuts and screws; gears are often used part in machines and it's required often by machine designers to finish their designs in most fields of mechanical applications. The first gear was designed over 3000 years past and they became an important part in all manner of tools and machineries. In early days, gear drives were unprocessed associated used rods inserted in one wheel meshing with identical rods mounted axially in an another wheel and it is presented in Figure 1.1. These toothed wheels were used to transmit circular motion or rotational force from one part of a machine to a different. It should be noted that every gear is typically connected to a rod and gears are utilized in pairs. The study of the velocity ratio of the gear system was not so easy although it is not showing any problem at that time. The velocity ratio was not constant due to the unprocessed design of the system. As a result of this, there was regular acceleration and deceleration of each tooth of the other gear when one gear ran at constant speed. Due to the loads generated by the acceleration, it affects the steady drive load which produces vibration and it leads to failure of the gear system. The gear drives designed mainly focusing on constant contact stress since the 19th century. It is required to keep the contact stresses below materials elastic limits. In order to modify the smoothness of the drive, by keeping velocity ratios as constant as possible. When the velocity ratio is kept constant, the major outcome is the reduction of dynamic effects due to which; it will give rise to increase in stress, vibration and noise. Design of gear requires a high skilled man. The constant pressure to create cheaper, quieter running, lighter and additional powerful machinery has increased steady and advance changes geared styles over the past.

1.2 Contact Ratio

Contact ratio is defined as ratio of length of arc of contact to circular pitch. When two gear teeth mesh, the meshing zone is usually limited between the intersecting radii of addendum of the respective gears as shown in Figure 1.2. From the figure it can be seen that the initial tooth contact occurs at point a and final tooth contact occurs at b. If the tooth profiles are drawn through points a and b, they will intersect the pitch circle at points A and B respectively. The radial distance AP is called the arc of approach q_a, and the radial distance PB is called the arc of recesses q_r and the sum of these being the arc of action q_t.

$$q_t = q_a + q_r$$

When the circular pitch p of a mating gear pair is equal to the arc of action q_t, there is always only one pair of teeth in contact, one gear tooth and one pinion tooth in contact and their clearance occupies the space between the arc AB.

$$q_t = p$$

During this case, once the contact ends at b, another tooth at the same time makes contact with a. In alternative things, once the action line is larger than the spherical pitch, over one tooth of the cogwheel is often to bear with over one tooth within the pinion, which suggests that if a tooth ends the contact at b, another tooth has already been to bear for a brief amount beginning at a. For a brief time there'll be 2 teeth to bear, one within the close to A and also the alternative close to B. once the gear try rotates through their meshing cycle, the tooth stops close to B to be to bear and solely one contact can stay teeth, and this method repeats itself throughout the amount of operation. The contact ratio provides the common variety of tooth pairs that square measure to bear. Most gears are generally designed with a contact ratio of more than 1.2, as the contact ratio is generally reduced due to errors in mounting and assembly of the gear pairs. Gear pairs operating with low contact ratio are susceptible to interference and damage as a result of impacts between teeth and there by leading to an increased level of noise and vibration. Gears are generally designed with contact ratios of 1.2 to 1.6. A contact ratio of 1.6, for example, means that 40 per cent of the time one pair of teeth will be in contact and 60 per cent of the time two pairs of teeth will be in contact. A contact ratio of 1.2 means that 80 per cent of the time one pair of teeth will be in contact and 20 per cent of the time two pairs of teeth will be in contact. Gears with contact ratio greater than 2 are referred to as "high-contact-ratio gears." For these gears there are never less than two pairs of teeth in contact. A contact ratio of 2.2 means that 80 per cent of the time two pairs of teeth will be in contact and 20 per cent of the time three pairs of teeth will be in contact. High contact ratio gears are generally used in selected applications where long life is required. Analysis should be performed when using high contact ratio gearing because higher bending stresses may occur in the tooth addendum region. Also higher sliding in the tooth contact can contribute to distress of the tooth surfaces. In addition higher dynamic loading may occur with high contact ratio gearing.

1.3 High Contact Ratio Gears

The contact ratio is explained as the average number of tooth pairs that are in contact during static conditions, and there is no error or variation in the tooth profile. The term high contact ratio (HCR) refers to a gear having at least two pairs of constantly contacting teeth, ie a contact ratio of 2 or more. The fraction change in the mesh stiffness of the HCR meshing is less than that of the low contact ratio (LCR) meshing, so the high-quality HCR meshes has lower vibration and noise than the LCR meshes. In the advanced gear drive design, the main objectives are increased life and reliability and weight reduction of a gear drive. High contact ratio gears (HCRG) are an efficient way to attain these objectives. In HCRG, at least two pairs of teeth are in contact at any time, while in the standard reports of low contact ratio (LCRG), alternating between one and two pairs are in contact. This allows a higher weight to power ratio, increased life and more stable because the acting load is divided between two or more pairs of teeth. The loading of the teeth and the individual stress are lower for HCRG as compared to LCRG designs. HCRG must be dynamically more sensitive to the profile of the teeth. In the performance of gears, loads and dynamic constraints are important. High dynamic loads increase noise of gears and the probability of surface fracture, and a high dynamic stress value at the root of the tooth can result in premature teeth fatigue and fracture. A dynamic analysis is needed to find the load distribution between the two and three pairs in contact.

1.4 Objectives of Present Wok

The objectives of present work are listed below;

- Better study of the working of high contact ratio gears.
- To determine the load sharing between gear teeth.
- Calculation of stiffness variation with pinion roll angle.
- To find out the dynamic load variation with respect to operational speed.
- To calculate the principal stresses variation with respect to operational speed.

2. Mathematical Modelling

A typical gear system with a rotor is shown in Figure 3.1. It composed of a motor that is connected to at least one of the shaft by suggests that of a coupling, load on the opposite finish of the second shaft and a combine of gears that connect the shafts. Each shafts area unit supported in several places by bearings. That is why the system consists of the subsequent components. 1) Shafts 2) Rigid disks 3) Flexible bearings 4)Gears

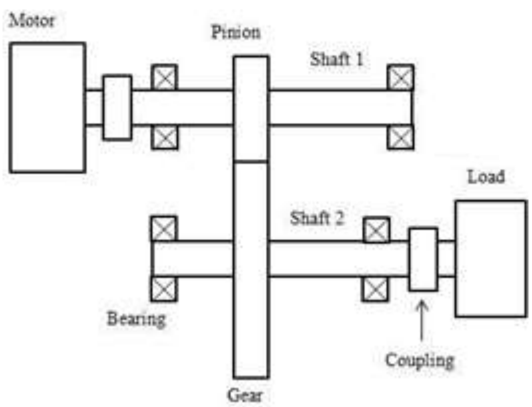

Figure 3.1: Typical gear rotor system

When two shafts are not coupled, each gear can be modelled as a rigid disk. However when they are in mesh, these rigid disks are connected by a spring damper element representing the mesh stiffness and damping. A typical gear mesh represented by a pair of rigid disks connected by a spring and a damper along the pressure line which is tangent to the base circles of the gears.

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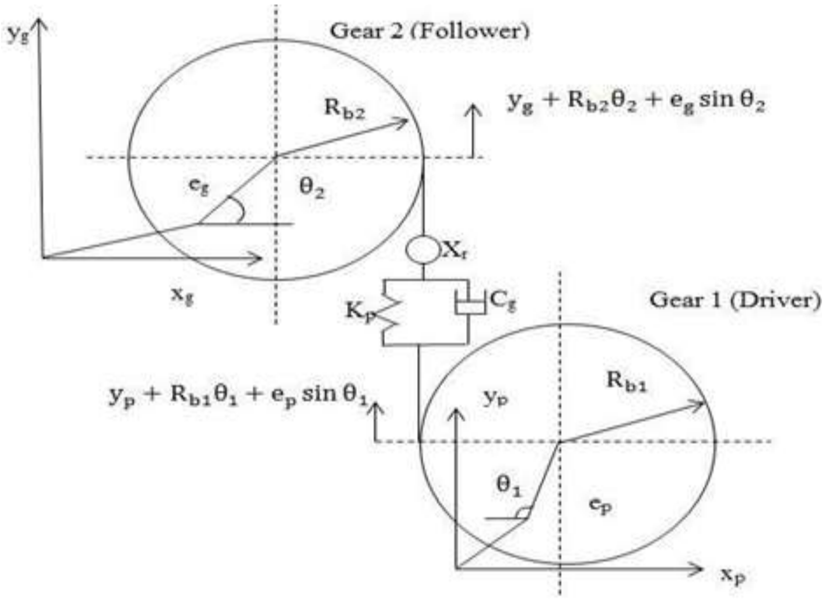

Figure 3.2: Modelling of a gear mesh

X_{gi} is displacement of tooth profile of gear i along the line of action.

$$X_{g1} = y_p + R_{b1}\,\theta_1 + e_p \sin\theta 1 \tag{1}$$

$$X_{g2} = y_p + R_{b2}\,\theta_2 + e_g \sin\theta 2 \tag{2}$$

. The angles θ_1 and θ_2 are the total angular rotations of the driving and driven gears, respectively, and are equal to

$$\theta_1 = \theta_p + \omega_p t \tag{3}$$

$$\theta_2 = \theta_g + \omega_g t \tag{4}$$

In this work, associate mathematical model supported torsional vibration of gears is employed to determine the dynamic load on spur gears with high contact ratio. Within the gear analysis, the gear and pinion teeth square measure assumed to be the sole energy storing components. Snap of the opposite elements of the drive has not been taken into consideration. Equation of motion governing the angular displacements of the kit and pinion together with the friction forces between the contacting teeth and a damping force with viscosity is derived by victimization the free body diagrams of every gear that square measure given.

$$J_1\ddot{\theta}_1 = [T_1 - \{F_{p1} + G_{p1}\}R_{b1} - F_{f1}X_1 - \{F_{p2} + G_{p2}\} R_{b1} - F_{f2}X_3 - \{F_{p3} + G_{p3}\}R_{b1} - F_{f3}X_5] \tag{5}$$

$$J_2\ddot{\theta}_2 = [-T_2 + \{F_{p1} + G_{p1}\}R_{b2} + F_{f1}\{L_p - X_1\} + \{F_{p2} + G_{p2}\}R_{b2} + F_{f2}\{L_p - X_3\} + \{F_{p3} + G_{p3}\}R_{b2} + F_{f3}\{L_p - X_5\}] \tag{6}$$

Using the relations given below, the equation of motion in terms of transmission error X_r is obtained

$$m_1 = \frac{J_1}{R_{b1}^2} \tag{7}$$

$$m_2 = \frac{J_2}{R_{b2}^2} \tag{8}$$

$$F_{st} = \frac{T_1}{R_{b1}} = \frac{T_2}{R_{b2}} \tag{9}$$

$$F_{pj} = K_{pj} * X_r \tag{10}$$

$$G_{pj} = C_{gj} * \dot{X}_r \tag{11}$$

$$F_{fj} = \mu * F_{pj} \tag{12}$$

The equation of motion in terms of X_r is given below

$$\ddot{X}_r + [K_{p1}(s_{p1}m_2 + s_{g1}m_1) + K_{p2}(s_{p2}m_2 + s_{g2}m_1) + K_{p3}(s_{p3}m_2 + s_{g3}m_1)] * \frac{X_r}{m_1 m_2} + [C_{g1}(s_{p1}m_2 + s_{g1}m_1) + C_{g2}(s_{p2}m_2 + s_{g2}m_1) + C_{g3}(s_{p3}m_2 + s_{g3}m_1)] * \frac{\dot{X}_r}{m_1 m_2} = F_{st}\left(\frac{m_1 + m_2}{m_1 m_2}\right) \tag{13}$$

$$X_r = X_{g1} - X_{g2} \tag{14}$$

$$\dot{X}_r = \dot{X}_{g1} - \dot{X}_{g2} \tag{15}$$

$$\ddot{X}_r = \ddot{X}_{g1} - \ddot{X}_{g2} \tag{16}$$

$$C_{gj} = 2 * \xi * \sqrt{K_{pj} * m_e} \tag{17}$$

$$m_e = \frac{m_1 m_2}{m_1 + m_2} \tag{18}$$

$$S_{pj} = 1 \pm \frac{X_{2j-1} * \mu}{R_{b1}} \tag{19}$$

3. Conclusions

The present work gives an analytical method for calculation of the tooth load sharing in high contact ratio spur gear pairs at any point in mesh cycle. It also gives the stiffness variation of driving gear, driven gear and mesh with respect to pinion roll angle. For dynamic load analysis of high contact ratio spur gear system, a torsional vibration based model is used. This gives the effect of operational speed on magnitude of dynamic load acting on pinion tooth also on the principal stresses acting on pinion tooth. Effect of operational speed on the location of maximum dynamic load is predicted here. From the foregoing discussion it is concluded that

- Stiffness of the driving gear tooth decreases along the path of contact whereas the stiffness of the driven gear tooth increases along the path of contact. In general tooth is more flexible at the outside than it is near the root.
- Normal load acting on pinion tooth has maximum value in the two pairs contact region.
- At low speed, the instantaneous load acting on pinion tooth is same as that of static transmitted load.

- Due to change in effective stiffness in second change of three pairs contact zone to two pair of contact zone, maximum dynamic load and principal stresses are observed in this region.
- Variation of operational speed has an influence on the magnitude of dynamic load acting on pinion tooth.

4. Future Scope

- This study can be carried out further including the flexibility of disks, shafts on which gears are mounted and bearings.
- This method of analysis can be applied to other forms of tooth profile like cycloid, non-involute, non-standard etc. with the proper modification.
- This analysis can be extended for other type of gears with composite materials.
- Two dimensional and Three Dimensional contact analysis can be done using ANSYS and then compared with the mathematical results.

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