International Journal of Science and Research (IJSR)
ISSN (Online): 2319-7064
Index Copernicus Value (2016): 79.57 | Impact Factor (2017): 7.296

Analysis of the Second Order Flow Equations Using Traffic Flow in Kisii Town

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Abstract: ***The analysis of second order traffic flow equations using Kisii town's traffic flow situations is crucial as the number of vehicles increases. The increase of traffic poses challenges ranging from but not limited to head to back collisions, lane change difficulties, congestion and pedestrian related accidents. There's no set safe inter-vehicle separation distance between traffic in motion, traffic at rest and traffic in a jam in Kisii town motorways. As a result the above stated hindrances are eminent. The study is seeking to determine the magnitudes of Kisii town's traffic flow variables which are limited to traffic flow rate, traffic density and traffic velocity, analyze the traffic flow equations using the different values of the variables. Besides, inter-vehicle spacing traffic flow equation is to be modeled using the Finite Difference Method. Analysis of the empirical data of traffic flow variables of Kisii town is done using MATLAB. The processed information is very useful to civil, mechanical engineers, town planners, computer programmers and upcoming Mathematicians.***

Keywords: Traffic flow, Macroscopic and Microscopic Modeling approaches and Finite difference method

1. Introduction

Macroscopic and microscopic modeling approaches or methods of traffic analysis are the most suitable in describing Kisii town's traffic flow situations. This is because macroscopic traffic flow analysis models are similar to models of fluid dynamics which employs a system of partial differential equations in expressing the quantities of interest such as traffic density, traffic flow rate and traffic velocity. For the case microscopic modeling, where at the most basic level, each vehicle is considered as an individual and as a result an ordinary differential equation can be written for each vehicle. In mesoscopic modeling, statistical mechanics techniques are employed where a probability function is generated which gives the possibility of finding a vehicle at a given time, position and with a given velocity travelling in a given direction in a traffic stream. In this research, vehicles are regarded as single whole entities, flowing in a traffic stream. This intertwines macroscopic and microscopic modeling techniques leaving mesoscopic method of traffic analysis slightly out. The consideration of the problem of modeling second order flow equations of vehicles within a traffic network in Kisii town is made.

1.1. Mathematical Background

Today there exist traffic flow models which not only aim at maximizing the rate of traffic flow but also controls traffic transfer. The models of vehicular flow are classified according to the order of differential equations, the number of independent variables and the number of dependent variables involved (Hoogendoom and Bovy, 2000).Macroscopic traffic flow models describe at a higher level of aggregation, traffic flow without distinguishing its constituent parts. For instance, the traffic stream is represented in an aggregate manner using characteristics such as traffic flow rate, traffic density and traffic velocity. Individual vehicle maneuvers such as lane change are usually not explicitly represented. Traffic network that is consist of avenues, highways, lanes, streets and other roadways provide a convenient and an economical conveyance of goods and passengers. The basic activity in transportation is a trip, defined by its origin, destination, departure time, arrival time and travel route. A myriad of trips interact on traffic network to produce a sophisticated pattern of traffic flows (Bovy, 2000).Nowadays traffic jam has become a major problem as far as transportation in developed and developing countries; where our country Kenya as a member is considered (Zhang, 2000).Traffic congestion on motorways of Kisii town has become an emerging challenge where almost in every weekday morning, weekday evening including weekends the capacity of many main roads is exceeded (Lighthill, 2000).In the last few decades much interest has been focused on traffic flow models as the amount of traffic more especially motorcycles continues to increase exponentially (Whitham, 2001).

2. Literature Review

2.1 Introduction

A number of different models have been proposed in the literature. Many of which share certain properties. Typically, these models are concerned with the flow rate of traffic, the traffic density and the average flow velocity. These models also involve the idea of an equilibrium flow rate, which prescribes a relationship between the traffic density and the flow rate.

2.2 Related Literature Review

Zhang (1998), in his investigation of congestion spillback and dynamics of shock waves in traffic flow, developed the first order traffic flow model. The model had a dynamically varying coefficient which ensured that no characteristics precede the traffic flow when overtaking possibilities are small. Useful information was borrowed from this model but with a slight contradiction showing that when roads are crowded overtaking may result to other traffic flow characteristics. Zhang (2000), in the study of his earlier traffic flow model, of the first order, developed a finite area difference for the model, which was macroscopic and relatively easier to compute. In his critical analysis he expressed traffic flow rate as a product of traffic density and traffic flow velocity. This research could enormously benefit from his discoveries but try to show whether traffic flow rate can be independently related to traffic density and traffic flow velocity. Lighthill and Whitham (2001) presented a model based on the analogy of vehicular flow and fluid particle flow in an open channel with a basement. This became a stepping stone for research and debate on a mathematical description of traffic flow by civil, mechanical engineers and Mathematicians. Nevertheless, this has yielded a broad scope of models describing different aspects of traffic operations. Payne and Whitham (2010) in their research work on traffic flow on short un-crowded roads; investigated how the velocity of a car is related to its road position between two designated points x_0 and x_1 of a traffic stream on an highway, in the long run they developed a first order traffic flow model of the form

$$\frac{dx_1}{dt} = \beta(x_0 - x_1) \tag{2.1}$$

This model could not explain how the velocity of the car is affected if the road is congested. In this regard our model borrowed from this model but considered crowded short roads where there are pedestrians, motorists passing or crossing randomly. Payne and Whitham model was improved later in the same year by Petrosyan and Balabanyan (2010). Petrosyan and Balabanyan (2010) in their research work on crowded roads; investigated how the driver's behavior can be influenced by the behavior of the leading car or vehicle a head. In so doing they came up with a traffic flow model of the second order of the form:

$$\frac{d^2x_1}{dt^2} = \beta(x_0 - x_1) \tag{2.2}$$

Since vehicles are incompressible; this model never accounted for the separation distance between vehicles in motion, at high speed, low speed or at rest and in dense traffic jam. This model borrowed a leaf from this model but concentrated on separation distance between vehicles in a traffic jam; in minimizing head to back collisions and pedestrian crossing related accidents. Our model enhanced the ability of drivers to look ahead and adjust to changes in traffic conditions, such as shocks, before they arrive at the vehicle itself. Nagatani (2004) developed a stochastic traffic flow model in order to investigate a wide range of traffic configurations. However, under great scrutiny, it was realized later on that if the population is sufficiently large, these stochastic processes tend towards a continuum approach as opposed to discretization which is main method of study. Consequently, this approach is most useful when considering long, densely-populated roads, but becomes invalid as the traffic on the road becomes sparse. Contrary to this, in this study of the second order flow equations using Kisii town's traffic flow situations involved division of the road sections and taking measurements of the traffic parameters at the entry and exit point of the REA regions. Additionary the model pays more emphasis to short, densely-populated roads.Michalopoulos et al (2003) in their investigation of a two-way traffic flow road section developed a traffic flow model of the general form:

$$\upsilon_e(\rho) = u_f[1-(\frac{\rho}{\rho_j})^{\alpha}]^{\beta} \tag{2.3}$$

Where u_f represents the unimpeded traffic speed, ρ_j represents the density at which traffic can no longer flow, ρ represents the traffic flow density, υ_e represents the equilibrium traffic flow velocity, α and β are positive constants that depend upon the characteristics of the road section in question. For example, setting $\alpha = \beta = 1$ returns the relationship between average traffic speed and density posed by Greenshields (1999). Other relationships have been considered, with Greenberg (2000) proposing the form

$$q_e(\rho) = \alpha\rho \log\left[\frac{\rho_j}{\rho}\right] \tag{2.4}$$

Where q_e is the equilibrium traffic flow rate. This form seems to be problematic, as the flow does not approach zero as the density approaches zero, but as continuum models are no longer valid in this limit, useful results may still be obtained. This study borrowed from this model, the two lane aspect but would feature a traffic condition where the traffic flow will tend to zero as traffic velocity tends to zero.The most elementary continuum traffic flow model was the first order model developed by Lighthill & Whitham (2001) and Richards (2002), based around the assumption that the number of vehicles is conserved between any two points if there are no entrances (sources) or exits (sinks). This produces a continuum model known as the Lighthill-Whitham-Richards (LWR) model, given as

$$\frac{\partial \rho}{\partial t} + \frac{\partial q}{\partial x} = 0 \tag{2.5}$$

This will be briefly demonstrated in a subsequent section. This model has been used to analyze a number of traffic flow problems. Notably, both Lighthill and Whitham and Richards used the model to demonstrate the existence of shockwaves in traffic systems. This particular model does suffer from several limitations, as noted by Lighthill and Whitham. The model does not contain any inertial effects, which implies that the vehicles adjust their speeds instantaneously, nor does it contain any diffusive terms, which would model the ability of drivers to look ahead and adjust to changes in traffic conditions, such as shocks, before they arrive at the vehicle itself. In order to address these limitations, Lighthill and Whitham propose a second-order model of the form

$$\frac{\partial \rho}{\partial t} + c\frac{\partial \rho}{\partial x} + T\frac{\partial^2 \rho}{\partial t^2} - D\frac{\partial^2 \rho}{\partial x^2} = 0 \tag{2.6}$$

Where T is the inertial time constant for speed variation, c is the wave speed obtained from the relationship between traffic flow rate and traffic density, and D is a diffusion coefficient representing how vehicles respond to nonlocal changes in traffic conditions. Second-order models were not explored again for some time, Payne (2001) and Whitham (2002) developed a second-order continuum model governing traffic flow, given by Payne as

$$\frac{\partial \upsilon}{\partial t} + \upsilon\frac{\partial \upsilon}{\partial x} = \frac{-\upsilon - \upsilon_e(\rho)}{T} - \frac{\mu}{T}\frac{1}{\rho}\frac{\partial \rho}{\partial x}; \tag{2.7}$$

Where $\mu = -\upsilon_e(\rho)/2$, υ_e is the equilibrium traffic flow velocity, υ is the flow velocity, T is the inertial time for speed variation, μ is half negative equilibrium velocity. Daganzo (2000) demonstrated that the Payne model, as well as several other second-order models available in the literature, produced flawed behavior for some traffic conditions. Specifically, it was noted that traffic arriving at the end of a densely-packed queue would result in vehicles travelling backwards in space, which is physically unreasonable. This is due to the isotropic nature of the models, as the behavior of vehicles is influenced by vehicles behind them due to diffusive effects Aw and Rascle (2000) were able to produce an anisotropic second-order model that averted the flaws noted by Daganzo, obtained by considering the convective derivative of some pressure-type function of the density, given as p (ρ). This model took the form

$$\frac{\partial\left(\upsilon+p(\rho)\right)}{\partial t}+\upsilon\frac{\partial(\upsilon+p(\rho))}{\partial x}=0 \qquad (2.8)$$

Where $p\approx\rho^{\gamma}$ with γ > 0 near (ρ) = 0, and ρp (ρ) is strictly convex.While there are different traffic models in the literature depending on the number of traffic variables analyzed and the nature of differential equations they underlie.This study would concentrate on the mathematical investigation of the model proposed by Petrosyan and Balabanyan (2010) with particular emphasis on considering the kinematic varying vehicle separation term, d which presumably depends on the speed of the inflowing vehicle

3. Materials and Methods

3.1 Equations Governing Traffic Flow

Gupta (2001), in his investigation of traffic flow, established that the following equations are basic:

3.1.1 Equation of Continuity

$$\frac{\partial\rho}{\partial t}+\vec{\nabla}.(\rho\vec{v})=0 \qquad (3.1)$$

This is a mathematical representation of the principle of conservation of mass, that is to say, the amount of flux entering a given channel of flow or control area is constant.

3.1.2 Conservation of Momentum Equation

The principle of conservation of momentum states that; 'the rate of change of momentum is equal to the net or resultant forces'

$$\rho\left(\frac{\partial\vec{v}}{\partial t}+(\vec{v}.\vec{\nabla})\vec{v}\right)=-\vec{\nabla}.\vec{p}+\mu\nabla^{2}\vec{v}+\rho\vec{F} \qquad (3.2)$$

Equation two is Navier-Stokes equation of motion, where the term on the LHS represents rate of change of momentum, on RHS we have the terms pressure forces, viscous forces and body forces.

3.2 Finite Difference Method

It is a numerical, discretization method for solving differential equations where the derivatives or the gradients of the differential equations are replaced by finite difference quotients. The solution of finite difference equations approximates the solutions of the differential equations based upon the application of the local Taylor series expansion. The finite difference method uses a topographically square or a rectangular network of lines to construct the discretization of the demarcated region. The finite difference method is suitable in the analysis of the second order flow equations in Kisii town since the discretized road sections, that is to say, the control areas (representative elementary area), have a rectangular outlines; with a regular polygonal network of the discretization lines unlike the Finite Element Method and the Finite Volume Method which deals with complicated geometrical regions enclosing irregular areas and volumes.

3.3 Discretized Area of Study

In this work the Finite Difference Method was used where the flow field is divided into rectangular cells (**Fig. 3.1**). The magnitude of the flow variables that is to say traffic flow rate q, traffic density, ρ and traffic velocity v, with respect to their independent variables road position x, road width y, and time taken t, are measured at the edge of the nodes set for each cell. For an elementary control area (REA) measuring 50m by 3m on the left-lane of Kisii town's two way traffic roads; where the length of a control area is 50m and 3.0m is the width of a lane the following tabulated figures were realized,(it is worth noting that the road sections of concern are sub-divided in a number of REAs where the flow rate into and out, velocity and traffic density can be more than zero).Considering a small portion of the road between $x=x_1$ and $x=x_2$, where x_1 and x_2 are the node positions at the entry and exit points respectively. Let the width of the road be y, and hence the lane width is $\frac{y}{2}$. It's worth noting that most roads in Kisii town are two way traffic with 2 lanes, the left and the right lane. In Kenya the basic law of traffic is "**keep left**". The designated portion of the left lane between X_1 and X_2 is called the control area or Representative Elementally Area (REA). We assume that the number of vehicles in the control area is conserved; such that at any point in time t, the change in the number of vehicles within the control area is given by the difference between the number of traffic entering at x_1 and the number of vehicles leaving the control area at x_2

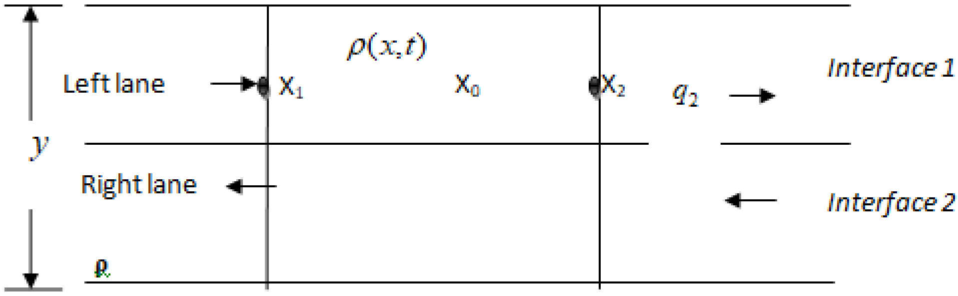

Figure 3.1: Elements and Nodes in the Control Area

3.4 The Model Equation

The flow rate into the control area at x_1 is given by $q(x_1,t)=q_1$ the flow rate out the control area at x_2 is given by $q(x_2,t)=q_2$. Where-as ρ(x, t) is the vehicle density in the control area. The change in the number of vehicles in the control area is given as

$$\frac{\partial}{dt}\int \rho(x,t)dx = q_1 - q_2 \tag{3.5}$$

This is the integral-differential form of the conservation equation; as the interval reduces, implying that $x_2 \to x_1$, then

$$\Delta x = (x_2 - x_1) \to 0 \tag{3.6}$$

the integral becomes a partial differential equation that governs traffic flow system, given by the following equations

$$\rho = \rho(x,t) \tag{3.7}$$

Implying that the full derivative of traffic density with respect to the space interval, x and time interval, t is given as

$$d\rho = \frac{\partial\rho}{\partial t}dt + \frac{\partial\rho}{\partial x}dx = 0 \tag{3.7a}$$

On dividing throughout by dt, the full derivative with respect to time becomes

$$\frac{d\rho}{dt} = \frac{\partial\rho}{\partial t} + \frac{\partial\rho}{\partial x}\frac{dx}{dt} = 0 \tag{3.7b}$$

$$\frac{dx}{dt} = \text{velocity of the incoming vehicle} = \frac{dq}{d\rho} = q'(\rho) \tag{3.7c}$$

In order to formulate the equation completely, we require some relationship between flow rate, q and traffic density, ρ

$$q = q(\rho) \tag{3.8}$$

Thus, using equation (3.7c) equation (3.7b) can be re-written as

$$\frac{\partial\rho}{\partial t} + q'(\rho)\frac{\partial\rho}{\partial x} = 0 \tag{3.9}$$

This relation is used in configuration of traffic lights in order to ensure that the traffic does not ''bank up" or worse still "crowd up" indefinitely; more especially at junctions and round about. In equation (3.9) the first part of the second term also gives the velocity of the incoming vehicle, in justifying the equality we use dimensional analysis

$$q'(\rho) = \frac{dq}{d\rho} = \frac{dx}{dt} \tag{3.10}$$

This is change of the number of vehicles flowing per unit time divided by the number of vehicles flowing per unit length, by traffic definition, which is equivalent to

$$\frac{dq}{d\rho} = \frac{\frac{1}{s}}{\frac{1}{m}} = \frac{m}{s} = ms^{-1} \tag{3.11}$$

ms^{-1} is the unit for velocity of the inflowing vehicle. From the traffic flow model equation (2.2) of Petrosyan and Balabanyan (2010), the equation is void of the inter-vehicle spacing, d which controls head to back collisions. In introducing s'd' this consideration is made. The acceleration of the incoming vehicle is proportional to the distance between the leading vehicle and the coming vehicle. Petrosyan and Balabanyan (2010) express this as

$$\frac{d^2x_1}{dt^2} \propto (x_0 - x_1) \tag{3.12}$$

Introducing the constant of proportionality

$$\frac{d^2x_1}{dt^2} = \beta(x_0 - x_1) \tag{3.13}$$

where β is the constant of proportionality and is the sensitivity coefficient, let x_0 be the position of the leading vehicle between x_1 and x_2, x_1 is the position of the incoming vehicle approaching x_0 . The incoming vehicle at x_1 approaching x_0 which is right in the control area can't just move covering the entire distance $(x_0 - x_1)$, it will have head to back collision. To avoid this collision it should only cover the distance $(x_0 - x_1) - d$, where d is the inter-vehicle separation distance required. From equation (3.13), inter-vehicle separation distance is the product of velocity and temporal separation time, thus

$$d = \tau\frac{dq}{d\rho} \tag{3.14}$$

Where τ is temporal separation time and $\frac{dq}{d\rho}$ is the velocity of the incoming vehicle.

The velocity of the incoming vehicle is also given as

$$\text{Velocity} = \frac{dx_1}{dt} \tag{3.15}$$

Implying that

$$\frac{dq}{d\rho} = \frac{dx_1}{dt} \tag{3.16}$$

The inter-vehicle separation distance, d as in equation (3.14) is also expressed as

$$d = \tau\frac{dx_1}{dt} \tag{3.17}$$

Once again the inter-vehicle spacing model becomes

$$\frac{d^2x_1}{dt^2} \propto \{(x_0 - x_1) - d\} \qquad (3.18)$$

Substituting for d and introducing the equal signs results to

$$\frac{d^2x_1}{dt^2} = \beta(x_0 - x_1 - \tau\frac{dq}{d\rho}) \qquad (3.19)$$

β is the constant of proportionality which is the sensitivity coefficient. Transposing gives

$$\frac{d^2x_1}{dt^2} + \tau\beta\frac{dq}{d\rho} + \beta x_1 = \beta x_0 \qquad (3.20)$$

Due to dimensional homogenization and using equation (3.17); equation (3.20) can be re-written as

$$\frac{d^2x_1}{dt^2} + \beta\tau\frac{dx_1}{dt} + \beta x_1 = \beta x_0 \qquad (3.21)$$

Equation (3.21) is the required inter-vehicle spacing traffic flow model equation that is used in investigating the stopping distances of inflowing vehicles. The preferred separation is 'Short for slow moving vehicles and long for first moving vehicles'

4. Data Analysis

4.1 Actual Findings

There are five main types of roads in Kisii County Roadways. In the analysis of second order flow equations using traffic flow in Kisii town, the empirical data was obtained from two categories of the following roads

4.2. KCGU Roads. Category 1

These are Kisii county urban roads which are built and maintained by the Kisii county government. These are the Kisii town roads. They are the concerned roads in this study.For an elementary control area (REA) measuring 50m by 3m on the left-lane of Kisii town's two way traffic roads; where the length of a control area is 50m and 3.0m is the width of a lane the following tabulated figures were realized,(it is worth noting that the road sections of concern are sub-divided in a number of REAs where the flow rate into and out, velocity and traffic density within can be more than zero)

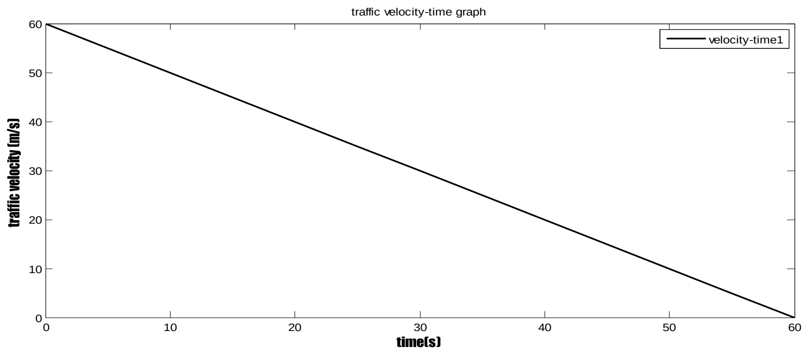

Figure 4.1: Traffic Flow Velocity against Traffic Density

For a vehicle moving from a less congested road section to an overcrowded lane its velocity is a function of time. The velocity reduces from its maximum urban value of 60m/s to its minimum in 60 seconds.

$v = v(t)$; To be very particular;

$$v \alpha \frac{1}{t}$$

$$v = \frac{k}{t} \qquad (4.1)$$

Velocity is inversely proportional to time, where k is a constant of proportionality and is equal to the distance covered by the incoming vehicle.

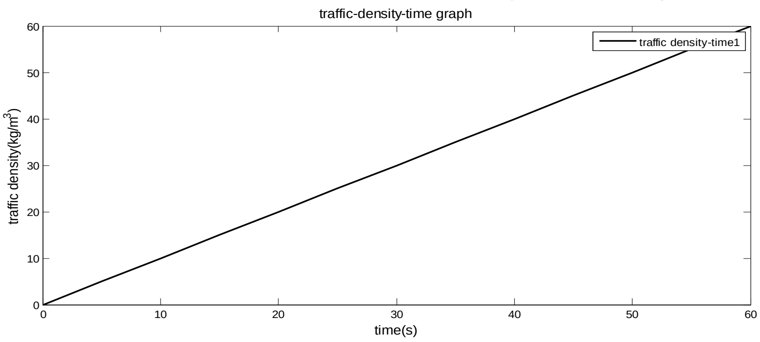

Figure 4.2: Traffic Density against Time Graph

It is quite evident that in the Kisii town roads during the rush hours of the week days and weekends as the capacity of the roads is exceeded; traffic density and time are proportionate. The traffic density increases from zero to its maximum value 60kg/m^3 in a period of 60 seconds $\rho \alpha t$ and $\rho = kt$ Where k is the constant of proportionality, which equal to the rate at which traffic density increases with time.

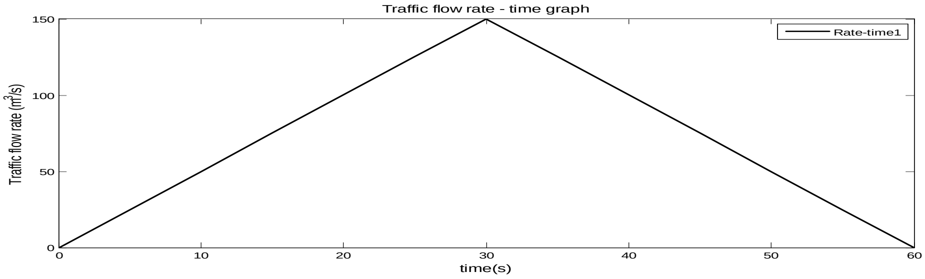

Figure 4.3: Traffic Flow Rate against Time Graph

Traffic flow rate increases to maximum value of 150m^3/s, at the critical time, t = 30s; beyond which it's zeroed when the traffic situation is bumper to bumper.

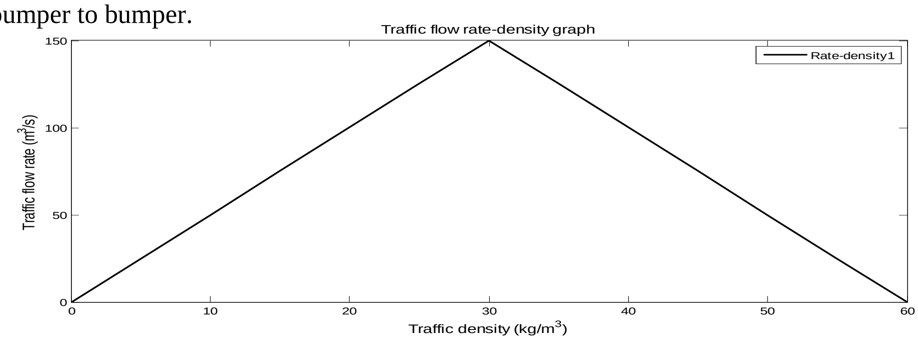

Figure 4.4: Traffic Flow Rate against Traffic Density Graph

Traffic flow rate and traffic density are linearly related and in fact; traffic flow rate is a function of traffic density. At the critical density of 30kg/m^3 the flow rate attains its maximum value of 150m^3/s, beyond which it uniformly reduces to zero at jam density of value 60kg/m^3.

$$q=q(\rho) \tag{4.2}$$

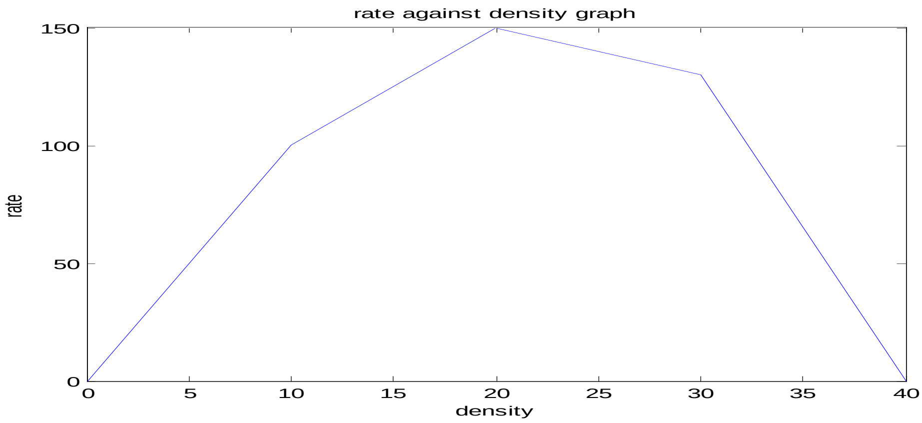

Figure 4.5: Traffic Flow Rate against Traffic Density Partially Smoothened Graph

At the critical density of 20kg/m^3, the flow rate is at its peak value of 150m^3/s; beyond the critical density the road becomes so overcrowded till no vehicle flows. As the traffic flow rate reduces to zero.

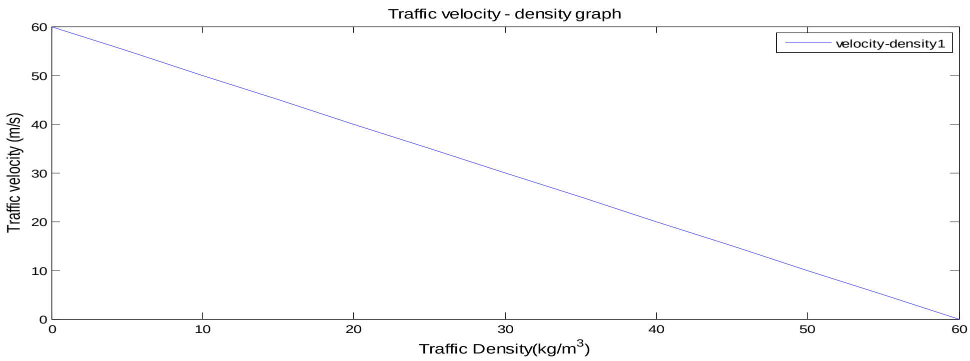

Figure 4.6: Traffic Velocity against Traffic Density Graph

It's quite evident that as the traffic density increases to its maximum value 60kg/m^3 the traffic velocity decreases to zero.

$$v = v(\rho) \qquad (4.3)$$

$$v \, \alpha \, \frac{1}{\rho} \qquad (4.4)$$

$$v = \frac{q}{\rho} \qquad (4.5)$$

$q = v\rho$ =constant=traffic flow rate.

5. Conclusions

This study was seeking to determine the magnitudes of the flow variables, analyze the flow equations and develop the inter-vehicle separation traffic flow model equation for Kisii town roads. The magnitudes of the traffic flow variables were determined for 11 road sections; which were categorized as Kisii County government Urban Roads (KCgUR), which were basically Kisii town roads and Kenya National Highway Authority roads (KeNHA), these are Kisii town's main feeder roads. The road sections investigated, indicated that the maximum traffic flow velocity for vehicles was 60km/h. This value was more than the traffic recommended optimum urban speed of 50km/h, where roads were less crowded. Obviously, the speed was 0km/h (zero) at jam density, the jam density varied from one road section to another. The critical traffic density, which is the density at maximum flow rate for each road was equal to the average traffic density for each road section. The development of the inter-vehicle spacing traffic flow model equation (3.21) was based on the analysis of the traffic flow equations. The mathematical, second order traffic flow model, established for Kisii town would only be physically meaningful if we restrict the traffic flow parameters vis-à-vis flow rate, flow velocity, traffic density, vehicle width, lane width, departure and arrival time. The driver may be quick thinking but the vehicle will have head to back collision or worse still will infinitely collide if the separation distance between vehicles is not observed.

The stopping distances are proportional to the initial velocity of the vehicle. Hence, short inter-vehicle distance of about 3.0m for slow moving traffic and long inter-vehicle distance of separation for faster moving traffic of approximately 5.0m. The separation relaxation time ranges between 1.05s to 0.65s.

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