Three-Phase Mathematical Model of Dehydration and Granulation Process in the Fluidized Bed

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Abstract: The developed mathematical model takes into account the hydrodynamics of the fluidized bed, the contact of droplets with particles and their adhesion to the surface, as well as the kinetics of drying the solution on the surface of the particles. When creating a model, the process of dehydrating and granulating in a fluidized bed is considered as a heterogeneous three-phase process, during which interact with three separate phases: particles - granulation centers, the starting material - ammonium sulfate in the form of droplets and heat carrier - air. To get the dynamic characteristics of the developed system, the Simulink library of the Matlab application package is used. With the help of built-in library elements, a scheme of differential equations describing the model of the control object is obtained.

Keywords: mathematical modeling, fluidized bed, dehydration, granulation

1. Introduction

Currently, the granulation process is quite widespread, particularly in the food, chemical and pharmaceutical industries, because it makes possible to convert powdered raw materials into grain with a definite size and composition. Granulation is one of the leading operations in the mineral fertilizers production technology. It has been proved that the granular product has a number of advantages as compared with a powder. Granules are more convenient to use, because they are well absorbed and is not weather out. Therefore, since the mid 80’s, almost all solid fertilizers have been produced in granular form.

The development of the agrarian sector of Ukraine is one of the vectors of sustainable development, which is enshrined in the Strategy of sustainable development of Ukraine for the period up to 2030. Increasing the variety and number of crops grown is envisaged. It requires earth enrichment with fertilizers. Consequently, the modernization of mineral fertilizer production technologies becomes even more urgent.

The fluidized bed granulators have simple design, are well exposed to mechanization and automation. Therefore they have been widely used in industry. A granulate obtained in a fluidized bed has such advantages: balanced fractional composition, a rounded form, and a better looseness [1-4].

2. Three-Phase Mathematical Model

We chose a fluidized bed granulator as a research object because its characteristic feature is that dehydration of the granular material, heating and dehydration of the drops of solution, its crystallization on the particles and granulate’s drying occur in the granulator gradually. Combining these processes in one machine increases the intensity of moisture removal from the material due to a contact surface's increase between the particles and the drying agent, increases energy efficiency and reduces the resource intensiveness of production.

In the model of the process of dehydrating and granulating in a fluidized bed this process is considered as a heterogeneous three-phase process. Three separate phases interact with each: particles - granulation centers, solution - ammonium sulfate in the droplets form and heat carrier – air [5-9].

The following assumptions were made during the model creation:

- The parameters' change occurs by layer height and in time.
- There is convective heat exchange between air, particles and droplets.
- The particles are monodispersed, non-porous. The agglomeration is absent. The mixing intensity of particles between the layers is described by the coefficient of the axial dispersion. The coefficient depends on the velocity of the gas phase and the particles properties.
- The droplets have a narrow size distribution. It allows them to be considered as a mono-dispersed phase. There are no collisions and clinging between the drops, no sticking on the device’s walls. The droplets move in a co-ordinate way with the air flow through the fluidized bed. The heat transfer process in the granulator is described by the following system of equations.

The first equation describes the change of temperature in particles in the fluidized bed:

\[
M_p C_p \frac{d\Theta_p}{dt} = M_p C_p r_p \Theta'_p + R^{wh} M_p C_d (\Theta_d - \Theta_p) + \\
+ M_a v_{dry} Q_{cryst} - M_a v_{dry} Q_{resp} + a_p S_p (\Theta_a - \Theta_p),
\]

where

- \(M_p\) - mass of particles, kg;
- \(C_p\) - specific heat, J/kg·K;
- \(r_p\) - rate of drying of particles, kg/m²·s;
- \(R^{wh}\) - rate of evaporation of water, kg/m²·s;
- \(M_a\) - mass of air, kg;
- \(v_{dry}\) - rate of dry air flow, m/s;
- \(Q_{cryst}\) - rate of crystallization of water, kg/m²·s;
- \(Q_{resp}\) - rate of gasification of water, kg/m²·s;
- \(a_p\) - coefficient of axial dispersion, m²/s;
- \(S_p\) - heat capacity of particles, J/kg·K;
- \(\Theta'_p\) - initial temperature of particles;
- \(\Theta_d\) - temperature of droplets;
particles, K; \( r_p \) - coefficient of axiary dispersion of particles, \( \text{s}^{-1} \); \( \nu_{dry} \) - specific rate of moistures drying on particles or in drops, \( \text{s}^{-1} \); \( Q_{sup} \) – specific heat of evaporation of moisture, \( \text{J/kg} \); \( \Theta_{cryst} \) – specific heat of crystallization of drops, \( \text{J/kg} \); \( a_d \) – coefficient of heat transfer of particles, \( \text{W/(m}^2 \cdot \text{K}) \); \( S_d \) – surface area of particles, \( \text{m}^2 \);

\[
R_{ad} = \frac{G_d}{M_p^0 \cdot \chi_d} = \frac{G_d}{M_p^0 \left( \frac{\Sigma d_d}{\Sigma f + 0.35} \right)^2} - \text{ specific rate of contact of droplets with particles (adhesion), sec}^{-1} \;
\]

\( M_p^0 \) - weight of loadable particles, kg; \( \chi_d \) - the efficiency of contacting droplets with particles;

\[\text{St}_d = \frac{\rho_d \nu d_d^2}{\mu_d d_p^4} \] - Stokes number for drops.

The second equation describes the temperature change of the drops:

\[
M_d C_d \frac{d \Theta}{dt} = G_d C_d \Theta_{d}^0 - R_{ad} M_p C_d (\Theta_{d} - \Theta_{p}) + + M_{u} \nu_{dry} Q_{sup} + \alpha_d S_d (\Theta_{a} - \Theta_{d}), \tag{2}
\]

\( \Theta_{d}^0 \) - initial temperature of droplets, K; \( \alpha_d \) - coefficient of heat transfer drops, \( \text{W/(m}^2 \cdot \text{K}) \); \( S_d \) – area of heat transfer drops, \( \text{m}^2 \).

The growth of the granule is more possible when the adhesion forces between droplets and the particles increase. Adhesive properties of the droplet, depend on the stiffness of the surface of the granules and the spray substance’s properties.

The third equation describes changing the temperature of the heating air:

\[
M_a C_a \frac{d \Theta}{dt} = G_a (C_a^0 \Theta_{a}^0 - C_a \Theta_{a}) - \alpha_p S_p (\Theta_{a} - \Theta_{p}) - \alpha_d S_d (\Theta_{a} - \Theta_{d}), \tag{3}
\]

\( C_a^0 \) – initial air specific heat, \( \text{J/(kg} \cdot \text{K}) \); \( \Theta_{a}^0 \) – initial air temperature, K;

In the system of equations (1-3) are taken into account the empirical correlations for calculating the specific dry air pressure, the specific rate of droplet settling on the particles (as a result of adhesion), the coefficient of the axial dispersion of the particles, the coefficients of heat transfer, the ratios for calculating the material losses and the thickness of the coating layer, and also the initial conditions.

The amount of heat released during the removal of moisture from the surface of droplets can be rewritten as follows:

\[
M_u \nu_{dry} Q_{sup} = \beta M_{g,0} S_p \Delta P Q_{sup} \]

\( \beta \) - coefficient of mass return, \( \text{m/s} \); \( M_{g,0} \) – molecular weight of water, \( \text{kg/mol} \); \( R \) – universal gas became, \( \text{m}^3 \cdot \text{kg}/(\text{s}^2 \cdot \text{K} \cdot \text{mol}) \); \( \Delta P \) – difference of partial pressure, Pa;

The last expression can be rewritten, using the analytical connection between the partial pressure and temperature, as follows:

\[
\frac{M_{g,0} S_p \Delta P Q_{sup}}{R \Theta_d} = \beta \frac{M_{g,0} S_p Q_{sup} (P_d - P_{sg})}{R \Theta_d} = = \beta \frac{M_{g,0} S_p Q_{sup} (\xi_d \Theta_{d} - \xi_d \Theta_{sg})}{R \Theta_d}, \tag{4}
\]

\( \xi_1, \xi_2 \) - analytical coefficients of the partial pressures dependence on the temperature, Pa/K; \( P_d \) - pressure of drops, Pa; \( P_{sg} \) - pressure of the steam-gas mixture, Pa; \( \Theta_{sg} \) - temperature of the steam-gas mixture, K.

The dependence of the droplets’ density on their temperature can be described by the following formula:

\[
\rho_d(\Theta_a) = \frac{\rho_{d,0}^p \Theta_{p}^p}{\rho_{d,0}^p} P_d \frac{P_{p}^d}{P_{p}^d}, \tag{5}
\]

\( \rho_{d,0}^p \) – density of droplets under normal conditions, \( \text{kg/m}^3 \); \( \Theta_{p}^p \) – temperature of droplets at normal, K; \( P_d \) – current pressure of drops, Pa; \( P_{p}^d \) – the pressure of droplets under normal conditions, Pa;

The final system of equations describing the processes in the apparatus of dehydration and granulation has the following form:

Particle temperature change:

\[
M_p C_p \frac{d \Theta}{dt} = M_p C_p \Theta_{p}^0 + \frac{b_p}{\rho_{p} \Theta_{p} + 0.35} \cdot M_p C_p (\Theta_{d} - \Theta_{p}) + + \frac{G_x \nu_{dry} Q_{sup}}{\Theta_{p}} \cdot \frac{\beta_1 (\xi_d \Theta_{d} - \xi_d \Theta_{sg})}{\Theta_{p}} + \alpha_p S_p (\Theta_{a} - \Theta_{d}). \tag{6}
\]

Changes in temperature of drops:

\[
M_d C_d \frac{d \Theta}{dt} = G_d C_d \Theta_{d}^0 - G_d C_d \Theta_{d} + + \frac{\beta_1 (\xi_d \Theta_{d} - \xi_d \Theta_{sg})}{\Theta_{p}} + \alpha_p S_p (\Theta_{a} - \Theta_{d}). \tag{7}
\]

Air temperature change:

\[
M_a C_a \frac{d \Theta}{dt} = G_a (C_a^0 \Theta_{a}^0 - C_a \Theta_{a}) - - \alpha_p S_p (\Theta_{a} - \Theta_{p}) - \alpha_d S_d (\Theta_{a} - \Theta_{d}), \tag{8}
\]

where \( \Theta_{a0} = \frac{G_d}{M_p^0} \), \( \Theta_{a} = \frac{M_{p}^0}{\mu_d d_p^4} \).

\( \beta_d = \frac{M_{g,0} S_p Q_{sup}}{R} \) - auxiliary coefficients.

This mathematical model takes into account the fluidized bed’s hydromechanics, the contact of droplets with particles, their adhesion to the surface and the kinetics of drying the solution on the particles’ surface.

3. Experiment

To get the dynamic characteristics of the developed system, the Simulink library of the Matlab application package was used. We obtain the following scheme of the system of differential equations describing the model of the control object using the built-in library elements:
The mass flow of air $G_a$ was chosen as a controlling influence from the experience of previous analytical and practical studies. Using the simulation of the system reaction to change the controlling influence of $G_a$ by using the scheme, the system of differential equations of dynamics has been solved. As a result transient characteristics on the channels "air flow $G_a$ - particle temperature $\Theta_p$" (Fig. 2) and "air flow $G_a$ - temperature of droplets of solution $\Theta_d$" (Fig. 3) were obtained.

Fig. 4 shows the static characteristics of the granulator - cumulative dependence of the temperature of particles $\Theta_p$ and droplet flow rate $G_d$ on the air flow $G_a$.

**Figure 1:** Scheme of the system of differential equations of the control object

**Figure 2:** Transient characteristic of the channel "air flow $G_a$ - particle temperature $\Theta_p$"

**Figure 3:** Transient characteristic of the channel "air flow $G_a$ - temperature of drops of solution $\Theta_d$"

**Figure 4:** The static characteristics of depending the temperature of particles $\Theta_p$ on the air flow rate $G_a$ and the droplet flow rate $G_d$.

### 4. Conclusion

It is known that the maintenance of a stable operation of devices with a fluidized bed and the necessary hydrodynamic regime inside the apparatus requires the development of an efficient system for controlling the processes of dehydration and granulation. The quality of management of the developed control system depends on the accuracy of the object model. The results of the above studies allow to specify the model of the control object and become a reliable ground for developing an effective control system.

### References


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