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A New Image Restoration Technique Based on Fast Tensor Preconditioning and Iterative Filtering

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Abstract: ***Hyperspectral images (HSIs) are often corrupted by a mixture of several types of noise during the acquisition process, e.g., Gaussian noise, impulse noise, deadlines, stripes, and many others. Such complex noise could degrade the quality of the acquired HSIs. Image restoration is one of the main parts of image processing. Mathematically, this problem can be modeled as a large scale structured ill-posed linear system. Ill-posedness of this problem results in low convergence rate of iterative solvers. For speeding up the convergence, preconditioning usually is used. We show that the higher order singular value decomposition (HOSVD) of the blurring tensor is obtained very fast and so could be used as a preconditioner. Iterative median filtering for restoration of images corrupted by mixed noise is considered. A median filtering that can be applied iteratively is also proposed. The boundary condition for the iteration is based on minimum distance between any two successive iterations is less than a threshold value. Experimental results show that proposed system has higher convergence speed. The complexity of an image restoration process reduces highly further we measures Peak Signal Noise Ratio (PSNR) and Mean Square Error (MSE). The PSNR values appear to be high while the MSE values appear to be low.***

Keywords: Image restoration, Hyperspectral image (HSI), mixed noise, HOSVD, iterative median filter

1. Introduction

Image restoration is an emerging field of image processing in which the focus is on recovering an original image from a degraded image [1], [2]. Image restoration can be defined as the process of removal or reduction of degradation in an image through linear or non-linear filtering. Usually, iterative methods are used to solve this structured large scale and ill-posed linear system [1], [2], [5], [6], [7]. Therefore, at each step of iterative methods, the matrix-vector multiplication can be carried out very fast by FFT, without using the explicit form of the blurring matrix, which reduces the computational and storage costs [4]. Also, iterative methods like CGLS (Conjugate gradient for least square problems) have regularization property which is known as semi-convergency [8], [9].

Hyperspectral imaging employs an imaging spectrometer to collect hundreds of spectral bands ranging from ultraviolet to infrared wavelengths for the same area on the surface of the Earth. It has a wide range of applications including environmental monitoring, military surveillance, mineral exploration, among numerous others [3], [4]. Due to various factors, e.g., thermal electronics, dark current, and stochastic error of photocounting in imaging process, hyperspectral images (HSIs) are inevitably corrupted by severe noise during the acquisition process. This greatly degrades the visual quality of the HSIs. Hence, the task of removing the noise in hyperspectral imagery is a valuable research topic.

The discrete version of the blurring procedure can be modeled as the following large scale ill-posed linear system,

$$y = Ax, \; y = b + e$$

where A is a blurred matrix and its structure depends on the PSF array and boundary conditions. The concept of preconditioning for discrete ill-posed problems differs from the standard preconditioning. The standard preconditioning tries to speed up the convergence by clustering the entire singular values of the preconditioned system around 1, while in the context of the ill-posed problems, the preconditioner should provide regularization property and only the large singular values need special attention. Therefore, in ill-posed problems, a preconditioner should able to improve the distribution and location of the large singular values and leave the rest of them to prevent the propagation of the noise in solution [4]. So, the truncated version of singular or eigenvalue decomposition of the blurring matrix is an ideal preconditioner. But, its computation is very time-consuming and so is not reasonable. In the literature, different kinds of approximations have been proposed that implicitly approximate the blurring matrix in the space corresponding to large singular values (which called signal space). These preconditioning methods are dependent on the boundary conditions [10], [11].

To restore the lost samples of image infected with impulsive noise we should employ interpolation techniques. Basic well-known interpolation techniques for this purpose include low-pass filtering, bilinear interpolation and median filtering. The techniques are numerically efficient but they cannot restore the original images exactly even if the Nyquist Rate is satisfied [12], [13]. All of the mentioned methods are based on the approximation of the blurring matrix. In this paper, we propose a novel preconditioner with a different viewpoint based on tensor modelling. In recent years different tensor based methods has been used in image processing [14], [15], [16]. Here, we demonstrate that image restoration problem can be modeled as a contractive tensor-tensor equation, which its matricization is equal to matrix modeling. This modeling enables us to construct new preconditioners based on approximations of the blurring tensor which could not be obtained with the matrix framework. We show that due to the structure of the blurring tensor, the HOSVD of the blurring tensor can be obtained very fast.

The median filter is a simple nonlinear smoothing technique that takes the median value of the data inside a sliding window of finite length [17], [18]. Median Filtering is particularly efficient in suppressing impulse noise [19] that is usually caused by transmission errors, malfunctioning of pixel elements in the camera sensors, faulty memory locations, or timing errors in analog-to-digital conversion [20]. Median Filtering can preserve edges in restored images [19], but its falters when the probability of impulse noise occurrence becomes high [21].

We use this fast tensor-based preconditioner on some 2D and 3D image restoration problems. Experimental results confirm the high quality of this preconditioner. So, using HOSVD of the blurring operator as preconditioner is reasonable. The results show that applying the median filter iteratively results in well sighted resulting images. Moreover, despite the implementation simplicity, the proposed iterative median filtering scheme provides a considerably higher convergence speed which intends a lower numerical complexity.

This paper is organized as follows. Section II introduces some notations and preliminaries of tensors. In Section III, the proposed system and its motivations are introduced. We then develop algorithm for solving the proposed model. Section IV presents some experimental results. Finally, we conclude this paper with some discussions on future research in Section V.

2. Notation and Preliminaries

Tensors can be considered as a generalization of vectors and matrices of high dimensions. A real-valued tensor of order N is denoted by $X \in R^{I_1 \times I_2 \times \ldots \times I_N}$. It is known that a tensor can be seen as a multi-index numerical array, and its order is defined as the number of its modes or dimensions. Different "dimensions" of tensors are referred to as modes. It is represented as

$$A(i,j,k) = a_{ijk} \qquad \text{(II.1)}$$

A fiber is a sub-tensor, where all indices but one are fixed. For example mode-2 fibers of A, have following form

$$A(i,:,j) \in R^{I_2} \qquad \text{(II.2)}$$

All mode-n fibers of A are multiplied by the matrix X. A is the same as $A \times nX$ in that system. The Frobenius norm of the order-M tensor A can be defined as

$$||A|| = \left(\sum_{i1,\ldots\ldots iM} a^2_{i1\ldots\ldots\ldots iM}\right)^{\frac{1}{2}} \qquad \text{(II.3)}$$

The discrete N-dimensional true and blurred images, now by definition of contraction product in the image restoration model could be represented as the following tensor equation

$$Y = \langle A, X \rangle_{N+1:2N;1:N} \qquad \text{(II.4)}$$

The tensor equation can be reformulated as the following linear equation

$$y = Ax \qquad \text{(II.5)}$$

Although these two models are equal, using tensor framework allows us to use new tools to deal with this problem. For example in the following, we use an approximation of the blurring tensor A to derive a new preconditioner, which cannot be obtained with matrix framework. Since A is not guaranteed to be nonsingular, its least squares (LS) should be considered,

$$\min_x ||Ax - y|| \qquad \text{(II.6)}$$

Median filter replaces a pixel by the median, instead of average all pixels in a neighborhood *w*.

$$y[m,n] = median\{x[i,j], (i,j) \in w\} \qquad \text{(II.7)}$$

Where *w* represents a neighborhood defined by the user, centered around location [*m*, *n*] in the image. The important parameter in using median filter is size of the window. Choice of the window size depends on the context.

3. Proposed Method

The proposed system uses a tensor framework in the modeling of image restoration problem. This framework enables us to deal with two or three-dimensional images directly without folding them to the vectors and without dealing with complicated multilevel matrices. Also, based on the structure of the obtained tensor, for the first time, we introduce a new tensor-based preconditioner. The performance of the proposed strategy and demonstrate that the complexity of computing and applying this new preconditioner is comparable with well-known matrix preconditioners.

A. Motivation

Digital images could be contaminated by noise during image acquisition or transmission in a noisy environment. In such case, image restoration is an essential technique for noise suppression while preserving the detail of image. Hyperspectral imaging, collect and processes information from across the electromagnetic spectrum. The goal of hyperspectral imaging is to obtain the spectrum for each pixel in an image of a scene, with the purpose of finding objects, identifying materials, or detecting processes. In many real situations, the observed HSI data are contaminated by a mixture of several different kinds of noise. As a result, a noise HSI cube denoted by a three order tensor $X=\{X^1,X^2,\ldots..X^B\}$, where B denotes the number of bands, can be described as

$$Y = X + E \qquad \text{(III.1)}$$

where X and E are with the same size of Y, which represent the clean HSI cube and the mixed noise term, respectively. Now the objective of HSI restoration is to estimate X from the observed Y by exploiting the structures of the clean HIS X and the noise terms E. We divide the noise term E into two sub-terms as Gaussian noise term N and the sparse noise term S including stripes, impulse noise, and dead pixels, leading to the following degradation model:

$$Y = X + N + S \qquad \text{(III.2)}$$

As such, the Frobenius norm and the `L norm can be naturally used to model such two noise terms N and S respectively. The purpose of image restoration is to "compensate for" or "undo" defects which degrade an

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image. Degradation comes in many forms such as motion blur, noise, and camera misfocus. Gaussian noise is statistical noise having a probability density function (PDF) equal to that of the normal distribution, which is also known as Gaussian distribution. In other words, the values that the noise can take on are Gaussian-distributed. The probability density function *p* of a Gaussian random variable *z* is given by

$$P(z) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(z-\mu)^2}{2\sigma^2}} \qquad \text{(III.3)}$$

where *z* represents the grey level, μ the mean value and σ the standard deviation. Compared with Gaussian noise, the stripe has significantly structural characteristics. Moreover, these line structures exhibit directional characteristic.

Impulse noise is a category of (acoustic) noise, which includes unwanted, almost instantaneous sharp and sudden disturbances. It presents itself as sparsely occurring white and black pixels, also known as salt and pepper noise, noisy pixels take either maximum or minimum value. Thus, it could severely degrade the image quality and cause some loss of information details. Noises of the kind are usually caused by electromagnetic interference. An effective noise reduction method for this type of noise is a median filter or morphological filter.

B. Higher Order Singular Value Decomposition

Different kinds of preconditioning methods have been proposed for image restoration in two and three dimensions. All of these preconditioners have been computed based on the structure of blurring matrix and try to approximate this blurring matrix in the subspace corresponding to signal space. In this section, we demonstrate that based on tensor modeling framework used in, one can obtain a novel preconditioner with a new viewpoint based on an approximation of the blurring tensor operator, which cannot be achieved with the matrix modeling. Here, we show that HOSVD of the structured tensor operator can be used as a preconditioner. Now consider the HOSVD of the blurring tensor A as follows

$$A = (U^{(1)}, \dots, U^{(2N)})_{1:2N}.S \qquad \text{(III.4)}$$

This decomposition has the following intersecting properties that encourage us to use its truncated version as a regularized preconditioner. The first columns of singular matrices $U^{(i)}$ in every mode are smoother than the last ones. So, for an appropriate index

$$K_i, U_{ki}^{(i)} = [u_1^{(i)}, \dots, u_{ki}^{(i)}] \in R^{ni \times ki} v \qquad \text{(III.5)}$$

Which denotes the first *ki* columns of $U^{(i)}$ has an important role in the reconstruction of the exact image. For example, consider 2D satellite image restoration test problem which is presented as the first test in the experimental section. The most important parts of the core tensor S are located in the small indices. Also, these parts of the core tensor are corresponding to the smooth parts of the singular matrices. This means that if S_k denotes the first part of the core tensor, for appropriate small indices. By these properties, the truncated HOSVD of the blurring operator A defined as

$$M = (U_{k1}^{(1)}, \dots, U_{k2}^{(2N)})1:2N.^{S_k} \qquad \text{(III.6)}$$

It is a good approximation of A corresponding to the smooth singular vectors of each mode and has regularization property. Also, for small values of k_i in *k*,

$$||A - M||^2 = ||S||^2 - ||S_k||^2 \qquad \text{(III.7)}$$

This will be very small in ill-posed problems like image restoration problem. So this regularized approximation tensor M of the blurring tensor A can be used as a preconditioner. The matrix presentation of this preconditioner is

$$M = U_K S_K V_K^T \qquad \text{(III.8)}$$

Therefore, at each step of preconditioned iterative method finding the solution with minimum length of the following least squares problem

$$\min_x ||M_q - Z|| \qquad \text{(III.9)}$$

Since for ill-posed problems |k| is very small in comparison with the dimension of A, the complexity of this LS problem is very small and negligible. The tensor based Image restoration where the image is converted into tensor by the process of the tensorization. Similarly the tensor data is converted to matrix form by the process of matricization. The degraded image is first converted into tensor by using gradient function. The gradient is rate of change of a function. The term “gradient” is typically used for functions with several inputs and a single output. Image gradients can be used to extract information from images. Gradient images are created from the original image for this purpose. Each pixel of a gradient image measures the change in intensity of that same point in the original image, in a given direction. To get the full range of direction, gradient images in the x and y directions are computed. The Gaussian noise tensor, and sparse noise tensor is created. This noise tensor is added to create a mixture noise tensor. Using this noise tensor, the degraded image is restored to get back the restored image. Usually, iterative methods are used to solve this large structured and ill-posed linear equation. For this structured linear system, the matrix-vector multiplication can be done by FFT without a need to explicit form of the blurring matrix A. Unfortunately, despite these excellent properties, the convergence rate of iterative methods for image restoration is low and to speed up the convergence, preconditioning techniques should be used.

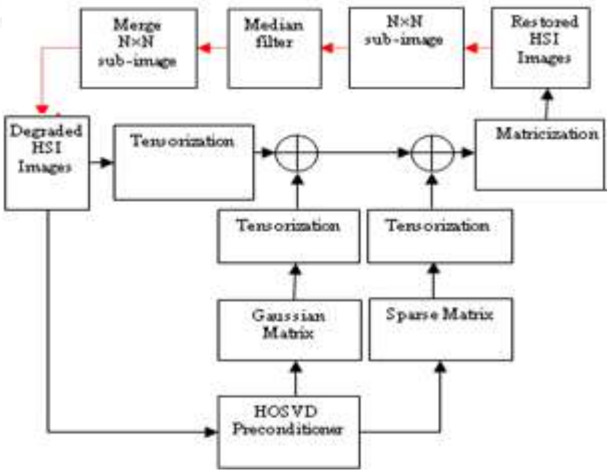

Figure 1: Block diagram of proposed system for HSI restoration.

The proposed system restores the degraded images that are degraded in remote sensing application. The degraded HSI are first tensorized to obtain the HSI tensor. Similarly a Gaussian tensor is created from the Gaussian matrix and sparse tensor is created from the sparse matrix. This Gaussian tensor and sparse tensor are added with the HSI tensor to obtain the restored image. Higher order singular value decomposition (HOSVD) is one common extension of singular value decomposition to the tensors. The HOSVD is most commonly applied to the extraction of relevant information from multi-way arrays. Singular values measure the 'energy' of the tensor. So, it is easy to see that the energy of core tensor S focused on the elements of S with small indices, especially in S(1,1,…,1). This property of HOSVD is very useful in the applications that encounter with denoising problem. The iteration stops if the mean square error between any two successive iterations is less than a threshold value.

C. Iterative Median Filter

Median Filtering is a type of nonlinear filtering technique, often used to remove noise from an image. Such noise reduction is a typical pre-processing step to improve the results of later processing. In this type of filtering all of the pixels in an *n* by *n* square mask of the image are selected, where *n* is an odd number usually 3 or 5. The center of the mask is a lost pixel that is to be restored. Median filter process which estimates by sorting all the pixel values from the surrounding neighborhood and replacing the pixel with center pixel value. This procedure is performed for all the lost pixels in the image. The main idea of the median filter is to run through the pixel entry by entry, replacing each entry with the median of neighbors is called "window", which slides, entry by entry over the entire pixels. Note that if the window has an odd numbers of entries, then the median is simple to define: it is just the middle value after all entries in the window are sorted numerically. If the neighborhood under consideration contains an even number of pixel, then average of the two pixel values is used. The detection of noisy and noise-free pixels is decided by checking whether the intensity of processed pixel lies between the maximum (max) and minimum (min) intensities values that occur inside the selected window. If the value of the processed pixel denoted by $p(x, y)$, is within the range $(0<p(x, y)<255)$, then it is an uncorrupted pixel and left unchanged. If the value does not lie within this range, then it is a noisy pixel and is replaced by median value of selected window. A median filtering that can be applied iteratively is proposed. If MSE between any two successive iteration is greater than 0.2, then subdivide an image into NxN sub-images to improve efficiency of median filter. The sub-image size selection is one of the important factors. In most of the applications, the sub-image size is selected as *n* x *n* such that *n* is an integer power of 2. The level of computation increases as the sub-image size increases. The experiment have been conducted by sub-image size *nxn* for *n*=16, or 32 and apply filtering to all sub-images. We propose to apply the median filter to image iteratively until MSE between any two successive iteration is less than 0.2. Median filtering does not need any arithmetic operation and requires a much smaller amount of memory. This results to a relatively low implementation complexity.

The reason for us to choose the median value is that uncorrupted pixels may be wrong classified as "noisy ones" at image flat areas with same gray value L_{min} or L_{max}. Using the median filter, the detection errors can be corrected, at the same time removing corrupted pixels as many as possible.

4. Experimental Results and Discussion

The proposed an image restoration technique is implemented in the working platform of MATLAB with machine configuration. In our proposed method, the degraded images are given to an image restoration process by using the techniques HOSVD and median filter. The original images captured by a remote sensing satellite (Remote sensing 1) are given Fig. 2. The image that was transmitted by the satellite was degraded because of the noise such as Gaussian noise, salt and pepper noise and strip noise. Therefore the received signal was degraded as shown in Fig. 3. The output restored images from HOSVD for 4 noisy inputs such as Gaussian noise, salt and pepper noise, strip noise and combination of all the three noises as shown in Fig. 4.

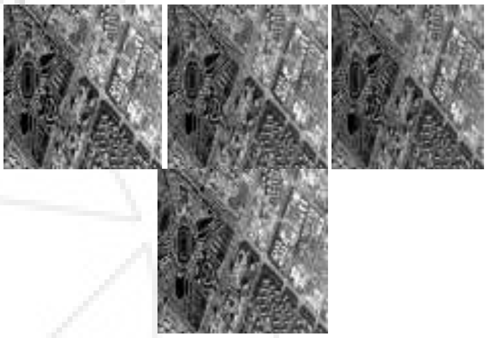

Figure 2: Original Image (Remote Sensing 1)

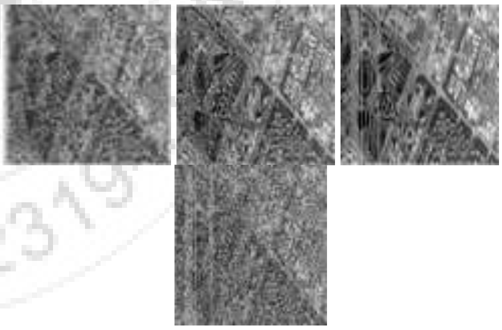

Figure 3: Received Degraded Image

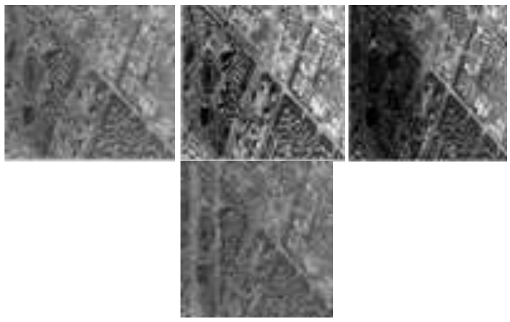

Figure 4: Restored Images from HOSVD

Similarly, Fig. 5. shows the original images captured by a remote sensing satellite (Remote sensing 2) and degraded images as shown in Fig. 6. Fig. 7. shows the restored images from HOSVD for an input images remote sensing 2. The restored image appears to be good for salt and pepper noise and strip noise, when compared to other two restorations.

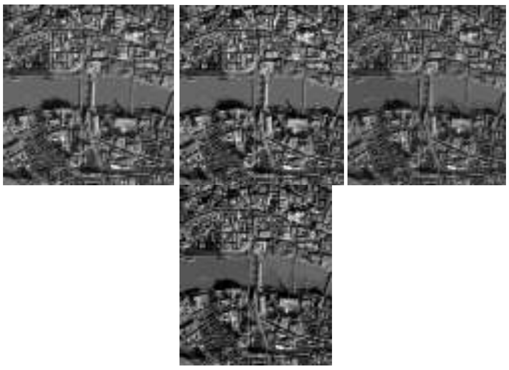

Figure 5: Original Image (Remote Sensing 2)

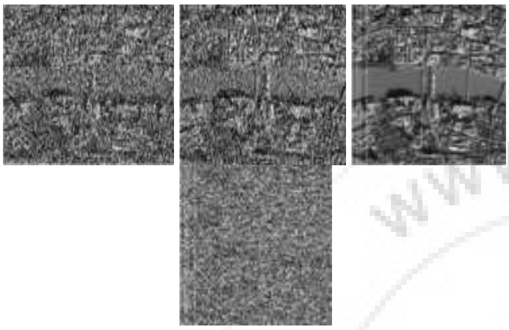

Figure 6: Received Degraded Image

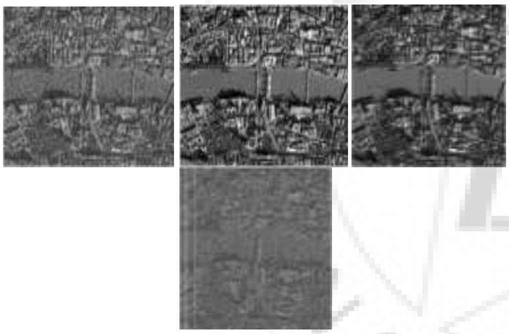

Figure 7: Restored Images from HOSVD

A. Performance Analysis

The performance evaluation of the proposed system is carried out by calculating the metrics such as Peak Signal to Noise Ratio (PSNR), Mean Square Error (MSE), Maximum Error, root Mean Square Error.

Let *f(x, y)* be the original image with size MxN and *f'(x, y)* be the restored image with the same size. The MSE measures the average of squares of errors that is, the difference between the restored and an original image is expressed as in

$$\text{MSE} = \frac{1}{MN}\sum_{xy}\left(f'(x,y) - f(x,y)\right)^2 \qquad \text{(IV.1)}$$

and the PSNR is the ratio of maximum possible power and the power of corrupting noise in decibel is expressed as in

$$\text{PSNR} = 10\log_{10}\left(\frac{255^2}{MSE}\right) \qquad \text{(IV.2)}$$

The Maximum Error is the difference between an original and restored value as expressed as

$$E = z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right) \qquad \text{(IV.3)}$$

Root Mean Square Error (RMSE) measures how much error there is between two data sets. In other words, it compares a predicted value and an observed value as expressed as in

$$\text{RMSE} = \sqrt{\frac{1}{MN}\sum_{xy}\left(f'(x,y) - f(x,y)\right)^2} \qquad \text{(IV.4)}$$

Table I shows the performance of proposed system for Remote sensing 1. From this table, the PSNR value is high for Image 3, that was degraded with strip noise and the PSNR value is low for image 4, which is the combination of all three noise. The number of iterations is less than 50, which is a less value. The performance of proposed system is best because it has high PSNR and less Mean square error

Table 1: Performance of Proposed System For Remote Sensing 1

Images	PSNR	MSE	Maximum Error	Root mean square Error	Iteration
Image 1	9.6088	7115	118	84.35	41
Image 2	10.2783	6098	127	78.09	
Image 3	12.5164	3642	127	60.35	
Image 4	7.6860	11079	127	105.25	

Table II shows the performance of the proposed system for the Remote sensing 2. It shows that the PSNR results for different noise models that have been implemented with different noise intensities of an image. The PSNR is high for Salt and pepper noise image restoration and strip noise restoration, the number of iterations is found to be 37, which is less than the conventions methods. It could be observed that the proposed method provide a far better performance when compared with other well known methods.

Table 2: Performance of Proposed System For Remote Sensing 2

Images	PSNR	MSE	Maximum Error	Root mean square Error	Iteration
Image 1	15.14	1987	98	44.57	37
Image 2	21.14	499	127	22.34	
Image 3	20.50	579	90	24.06	
Image 4	14.66	2220	106	47.12	

Fig. 8. shows the MSE between adjacent iterations. From the graph, black color resembles the MSE of Gaussian noise image restoration, red color resembles the MSE of salt and pepper noise image restoration, blue color resembles the MSE of strip noise image restoration and green color resembles the MSE of all noise image restoration. Fig. 8. graphically illustrate the MSE value of proposed system for an image corrupted by different noise.

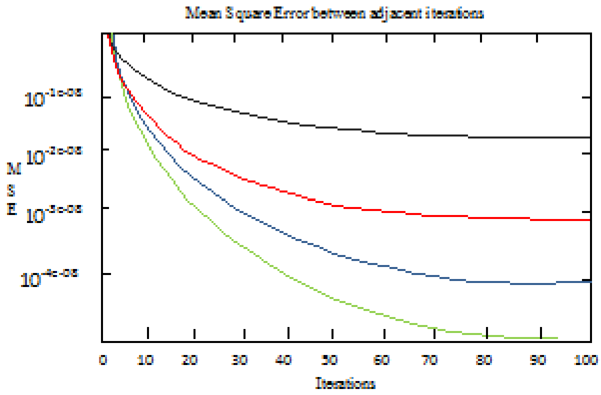

Figure 8: MSE between adjacent iterations

From this figure, it can be easily observe that proposed method yield more satisfying results and can be applied to different type of image.

5. Conclusion

In this paper, we demonstrated that image restoration could be modeled with the tensor framework. Based on this context, we proposed a tensor-based preconditioner and iterative filtering for removing mixed noise in HSIs based on using an approximation of HOSVD. We showed that due to the structure of the blurring tensor, this preconditioner could be constructed very fast. Instead of dealing as pixels the image is processed as tensor. This project reduces the noises that occur in remote sensing satellite images. The satellite images are degraded by the noise such as Gaussian noise, Impulse noise, Strip noise etc. The tensor of the degraded image is added with tensor of Gaussian, Impulse, Strip noise. The Singular values are extracted from the degraded image which controls intensity of Gaussian, Impulse and Strip noise tensor. This process gets repeated iteratively to get the restored image. The boundary condition for the iteration is based on minimum distance between any two successive iteration is less than a threshold value. Also, experimental results confirm the high quality of the proposed preconditioner in speeding up the convergence rate of iterative restoration methods. We suggested to apply median filtering on the images with mixed noise iteratively. The suggested method has a low implementation complexity. The Performance analysis of the median filter shows that an image with much lower pixel loss. The results shows that the complexity of the image restoration process reduces highly because the proposed system restores the image only on less number of iterations further we measures Peak Signal Noise Ratio (PSNR) and Mean Square Error (MSE). The PSNR values appear to be HIGH for the restored images while the MSE values appear to be LOW.

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