

Theoretical and Experimental Verification of Using PZT Ceramic Plates as Strain Rate Transducers for Flexible Structures

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Abstract: This paper introduces the use of Piezoelectric Ceramic plates as strain rate transducers for light weight flexible structures. This has been applied and tested at different set points of a flexible oscillatory cantilever arm, where the introduced strain rate measurements used in turn to compute the flexible arm generalized velocities. The sensitivity of PZT Ceramic materials to strain rates has been theoretically formulated and experimentally tested. Stresses of oscillatory arm give rise to voltages being produced at the gauge electrodes due to piezoelectric effect. Transfer function amplitude and phase between experimental strain and strain rate measurements reflected excellent verification of PZT Ceramic plates as strain rate sensors.

Nomenclature:

ε	Strain
$\dot{\varepsilon}$	Strain rate (1/sec)
I	Current (Amp)
V_i	Strain rate gauge voltage (Volt)
S	Laplace operator
A	Surface area of PZT Ceramic plate (mm ²)
e_{31}	Piezoelectric constant (col/m ²)
V_o	Amplified strain rate output signal (Volt)
$S(\varepsilon)$	Strain signal (Volt)
$S(\dot{\varepsilon})$	Strain rate signal (Volt)

Keywords: Strain rate transducers, piezoelectric materials, PZT Ceramic plates, Flexible structures

1. Introduction

The dynamics and control of vibration of flexible structures require computation of the modeled structure generalized displacements and velocities [1]. While strain gauges Bridge circuits are normally used for the computation of generalized displacements, this Paper presents the Piezoelectric Zirconate Titanate (PZT) Ceramic plates as strain rate transducers from which the flexible structure generalized velocities to be computed.

Development of an active vibration damper by the use of piezoelectric polymer as an actuator was introduced by [2], where the material used as a distributed parameter actuator over the length of a beam. Piezoelectric materials are being used at an increasing rate by researchers in the fields of vibration and control [3, 4]. Due to their distinct features these materials can be employed in the distributed sensing and control of intelligent structures [5]. An experimental evaluation of piezoelectric actuator for controlling the vibration of a simple cantilever beam was demonstrated by [6] and the use of piezoelectric materials as sensors and actuators was recommended. A study on the optimal placement of collocated piezoelectric actuator/sensor pairs was carried out by [7] for different boundary conditions of beams. [8] Investigated the behavior of piezoelectric elements as strain sensors, the superior performance of piezoelectric sensors compared to conventional strain gauges in terms of sensitivity and signal to noise ratio was demonstrated. Further analytical and experimental verification of the strain rate self-sensing mechanism for a

cantilever piezoelectric beam was presented by [9]. The estimation of strain rate using adaptive filter was impacted by phase delay in piezoelectric material and structural shortcoming.

In this paper, the PZT plates were cemented to a flexible cantilever arm of 2.5m long of which its lowest natural frequency is 0.62Hz. The arm end is clamped to a rotating base which is served by a dc. Servo-motor. A sketch of the structure is shown in figure 1.1. The PZT Ceramic plates (19mm length X 3mm width), were positioned at the neutral axis of a the light weight flexible arm as shown in figure 1.2. The plates have been used as strain rate gauges, utilizing the piezoelectric effect exhibited by the modified forms of lead Zirconate titanate Ceramic. The gauges are polarized through the thickness and have fired Silver electrodes. This type of transducer has the advantage of high strain rate sensitivity. When strains occur on the surface of the arm, extensional or contractional stresses are induced within the gauges. This gives rise to voltages being produced at the gauge electrodes due to the piezoelectric effect.

2. Strain Rate Formulation

An amplifier has been considered to amplify the output signal of PZT plate. The circuit diagram is shown in figure 2.1. The governing equations are,

$$I = -C_o \dot{V}_i + K \dot{\varepsilon} \quad (2.1)$$

$$\frac{V_i}{R1} + S C_o V_i = K \dot{\varepsilon} \quad (2.2)$$

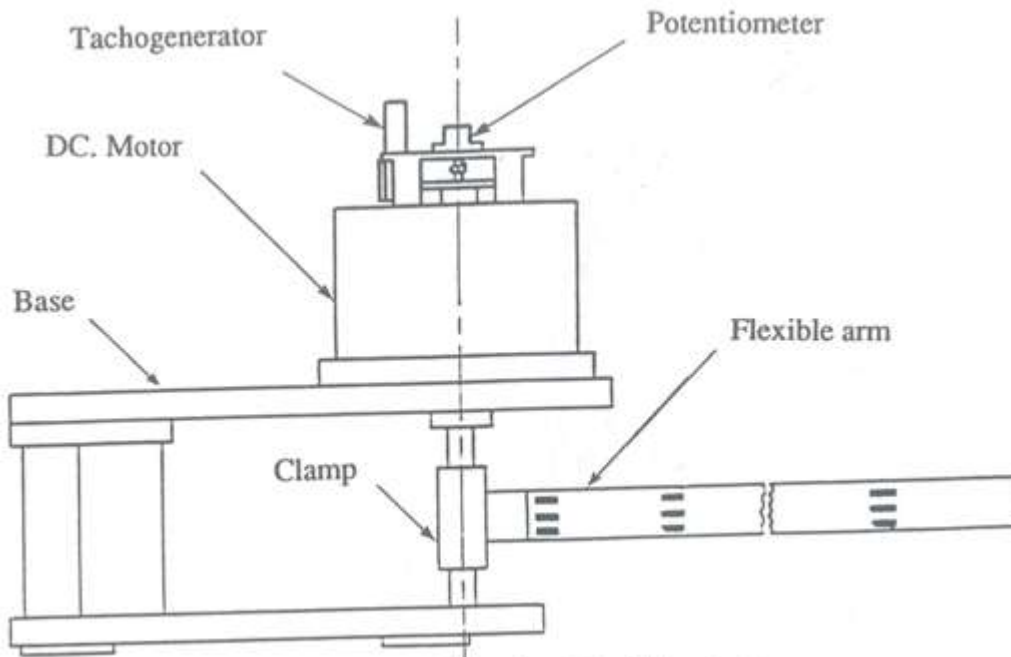
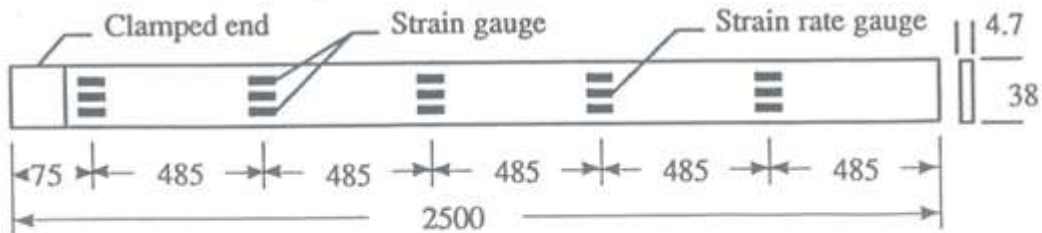


Figure 1.1 Sketch of the Structure



All dimensions in mm
 Not to scale

Figure 1.2 Schematic diagram of the flexible arm, showing locations of strain and strain rate sensors

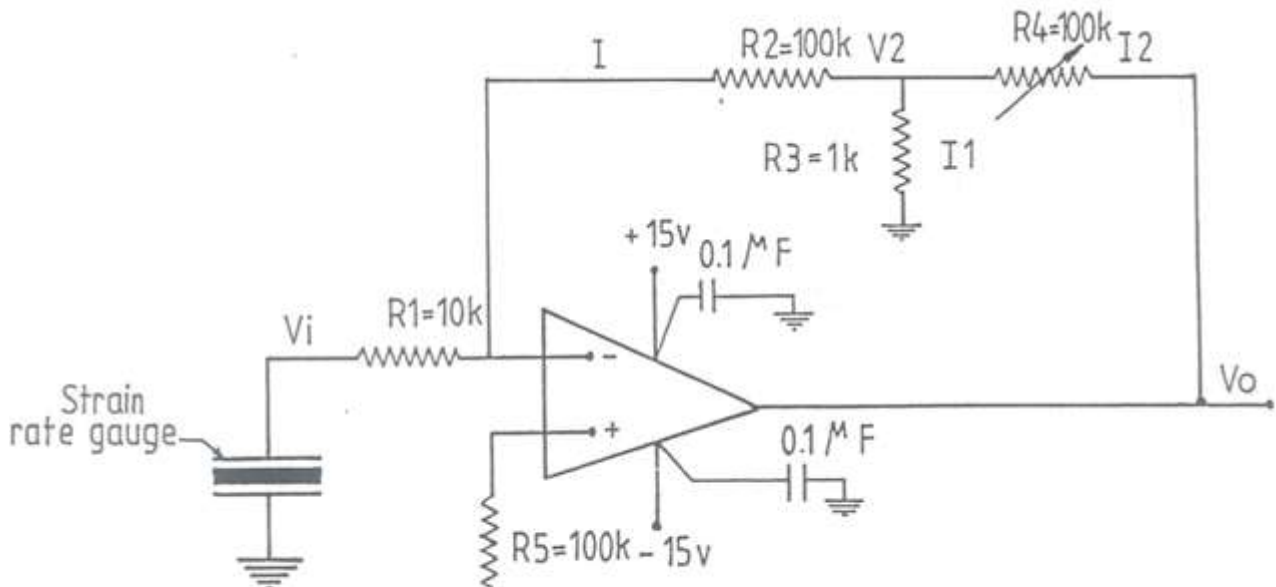


Figure 2.1 Circuit diagram of strain rate gauge amplifier

$$\text{Or } V_i = \frac{R1K\dot{\epsilon}}{(1+s.C_o.R1)} \quad (2.3)$$

Where $K = 2(Ae31)$
 $A = 60\text{mm}^2$, is the surface area of the Ceramic plate,
 $e31 = 9.176 \text{ Col/m}^2$, is the piezoelectric constant.

The term $(s.C_o.R1)$, is considered small 'at low frequencies' in comparison to the unit value one and has been neglected. Therefore equation (2.3) can be rewritten as

$$V_i = R1K\dot{\epsilon} \quad (2.4)$$

Now also from the amplifier circuit diagram (figure 2.1),

$$I = I_1 + I_2$$

$$I = \frac{V_i}{R1} = -\frac{V2}{R2}$$

i.e. $V2 = -\frac{V_i R2}{R1}$ (2.5)

But $I1 = \frac{V2}{R3}$

$$\text{And, } I2 = \frac{V2 - V_o}{R4}$$

Therefore, $\frac{R4}{R1} V_i = \frac{V2}{R3} + \frac{V2 - V_o}{R4}$ (2.6)

Substituting equation (2.5) into equation (2.6) gives,
 $V_o = \alpha V_i = s(\dot{\epsilon})$ (2.7)

Where V_o is the amplified output signal
 And

$$\alpha = -\left\{ \frac{R4}{R1} + \frac{R2(R4+R3)}{R1R3} \right\}$$

On substituting equation (2.4) into equation (2.7), equation (2.7) becomes,

$$\frac{V_o}{\dot{\epsilon}} = \alpha . R1 . K$$

The above defines the relationship between the strain rate ($\dot{\epsilon}$) and the output voltage (V_o).

3. Experimental Verification of Strain Rate Measurements

Two PZT Ceramic plates of opposite polarization were cemented at the centre line of the both sides of the flexible oscillatory cantilever arm at five equidistant points as shown

in figure 3.1. These points are properly coincident with the respective strain gauge locations, which are already fixed to measure the strain. To verify the functioning of PZT plates as strain rate transducers, the arm was actuated by an open loop swept sine signal applied to the motor which drives the arm. The signal was generated using a Hewlett-Packard (3652A) Dynamic Signal Analyser. At each of the five locations along the beam, signals from strain gauges and PZT Ceramic plates were connected to the Analyser as an input and output respectively. Frequency response tests were carried out over a frequency span of 50Hz, with a starting frequency of 0.1Hz. The frequency range covers the first five natural frequencies of the arm.

Let the strain gauge signal be represented by,
 $S(\epsilon) = A_o \sin \omega t$ (3.1)

Then its rate of change, i.e. the strain rate signal will be
 $S(\dot{\epsilon}) = A_o \omega \cos \omega t$ (3.2)

Transfer function amplitude and phase between ϵ and $\dot{\epsilon}$ were used as a measure to verify the sensors function. The results of tests for the five points are reflected by the plots of figures 3.2 to 3.6. Plot (a) of each figure represents the transfer function amplitude of the two signals, which increases linearly with frequency. Plot (b) shows a ninety degree phase shift between the two signals. The spikes in the plots (a) and (b) represent the anti-resonance behavior of the beam. The existence of these anti-resonances can be seen by considering plots (c) and (d) of each figure. For example consider the frequencies 9.58, 19.8 and 34.03 Hz, on figure 3.4. Plot (c) of figure 3.4 represents the transfer function amplitude of the strain gauge signal to the forcing source signal. Note that the output at these three points is almost zero. Similarly the transfer function amplitude of the strain rate signal to the forcing source signal at these points zero too, as shown in figure 3.4.d. Dividing strain rate signal by strain signal at such points will result in indeterminate values. This indeterminacy causes the spikes shown in plots (a) and (b). Also plots (e) of coherence function identify these nodes. The results of these tests prove the functionality of these strain rate transducers, and throughout this research it has been observed that they possess a high sensitivity to small changes in the strain rate.

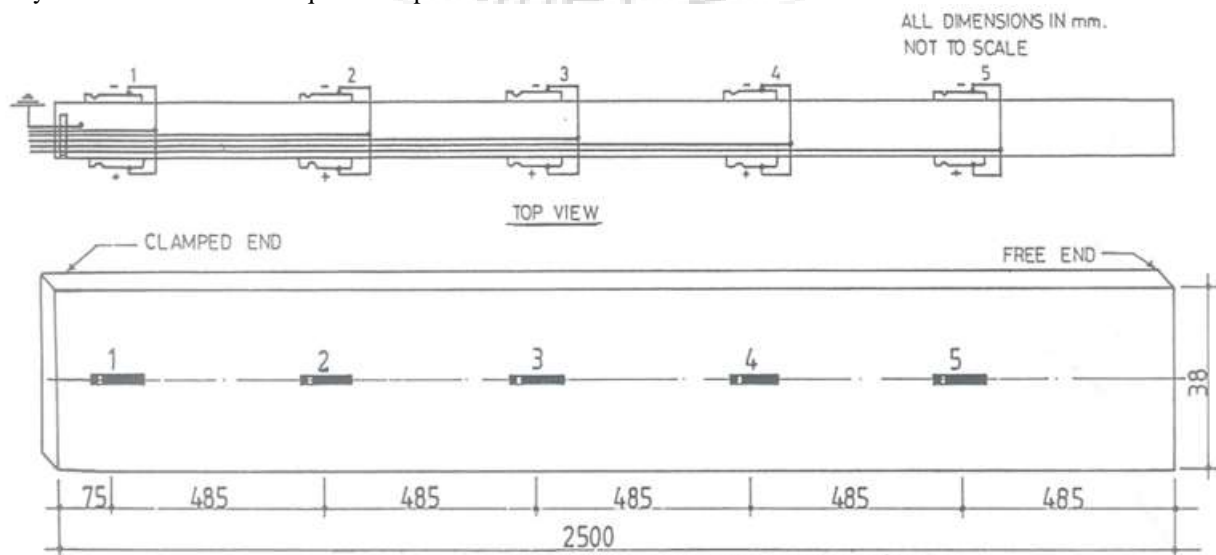


Figure 3.1 Strain rate gauge locations along the arm

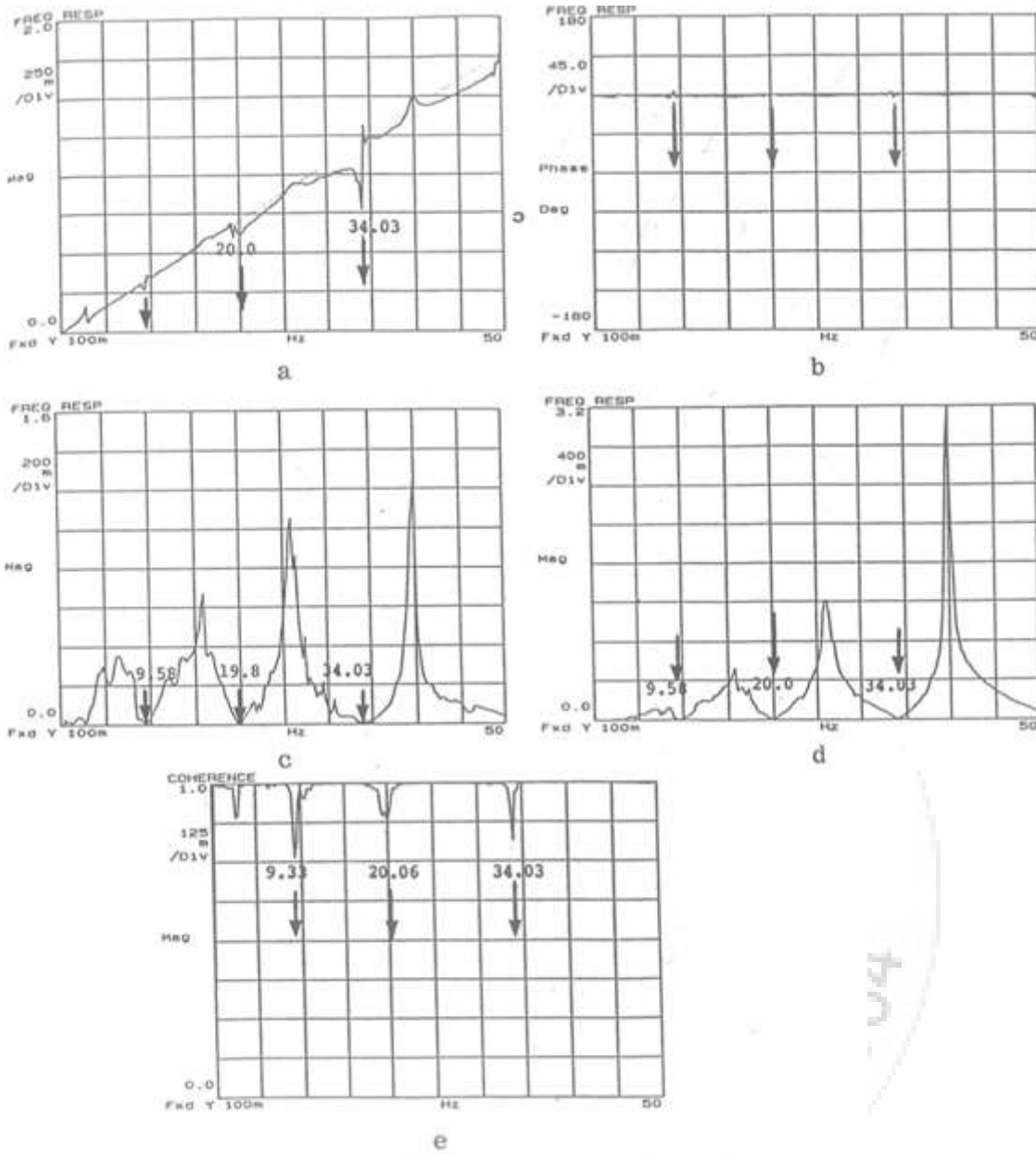
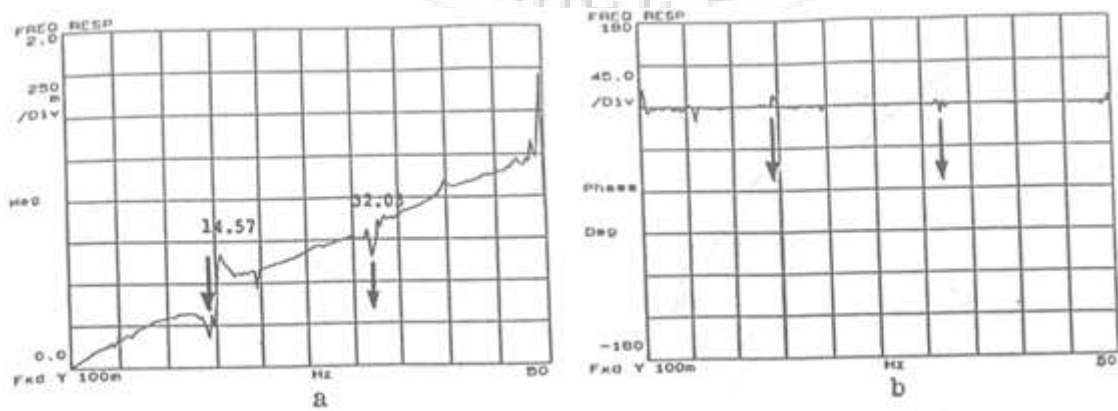


Figure 3.2 Experimental verification of strain rate gauges-point 1



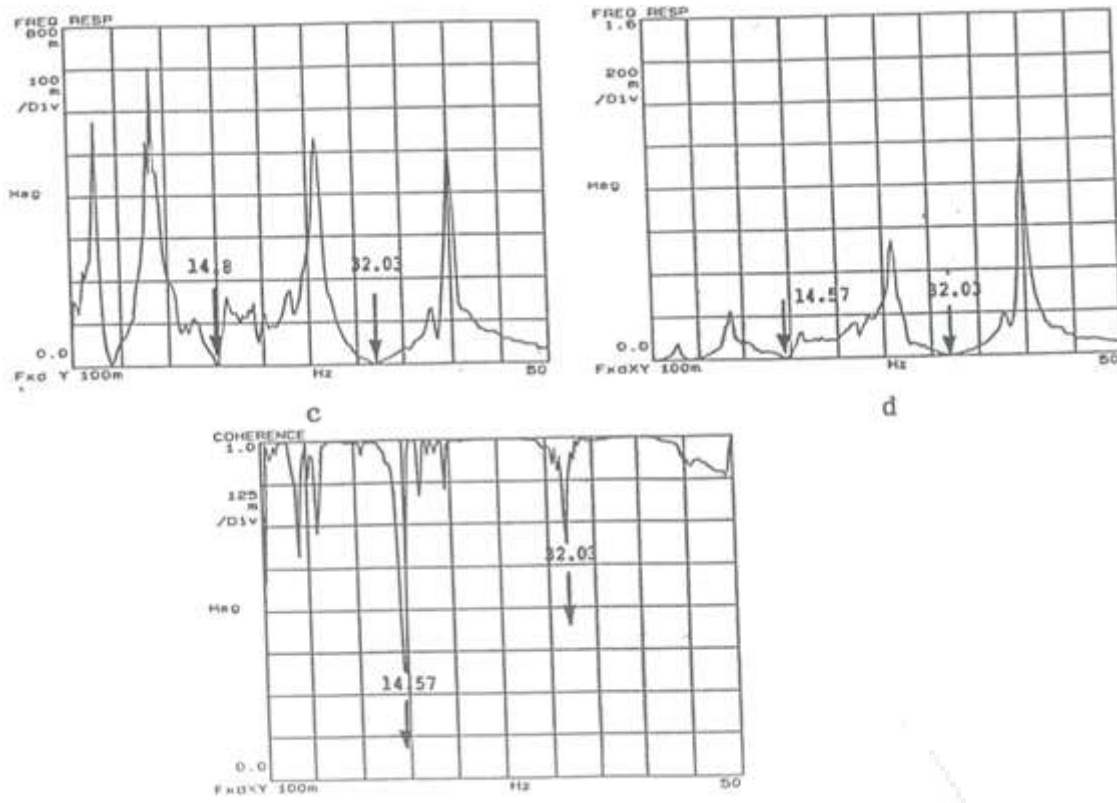
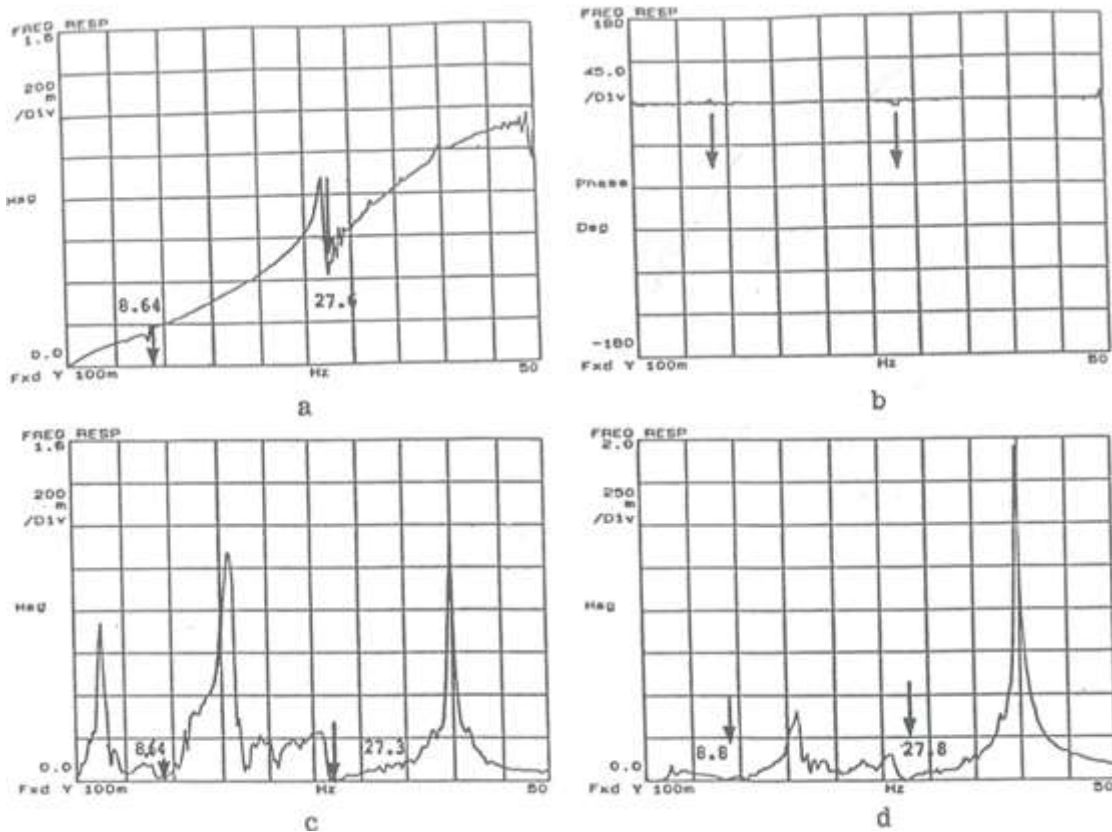
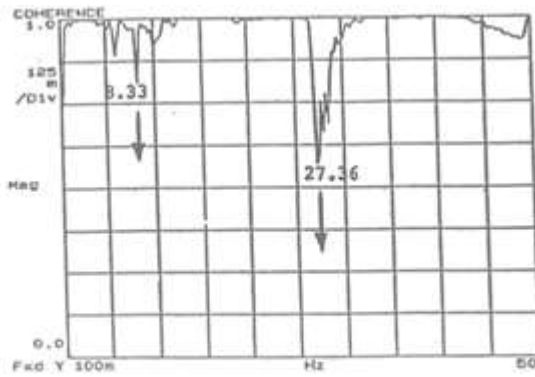


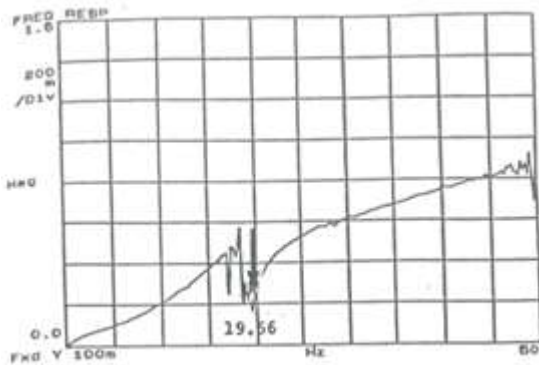
Figure 3.3 Experimental verification of strain rate gauge-point 2



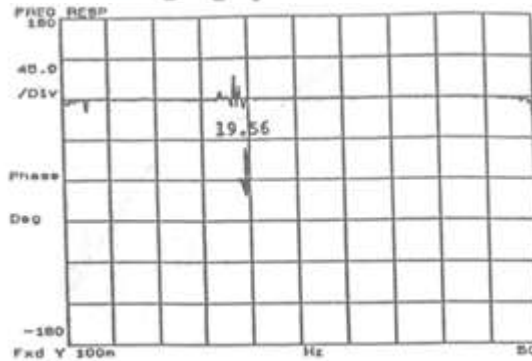


e

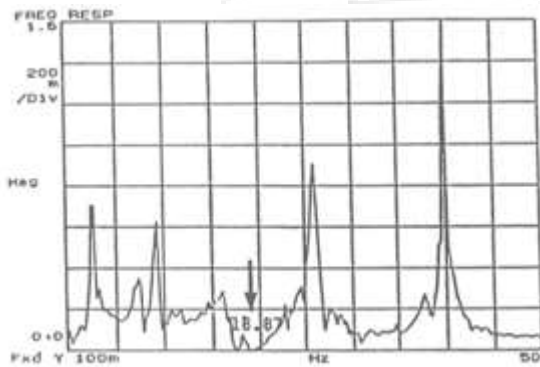
Figure 3.4 Experimental verification of strain rate gauge-point 3



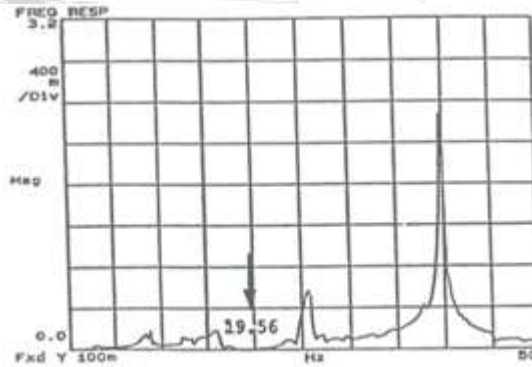
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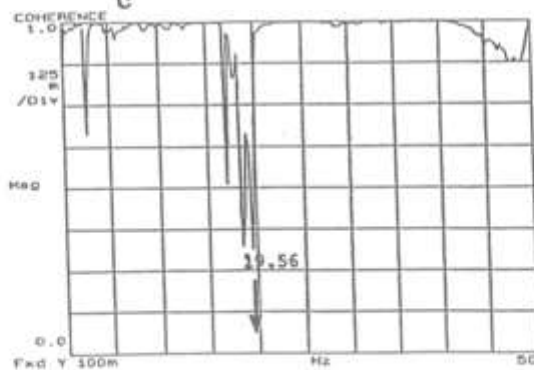
b



c



d



e

Figure 3.5 Experimental verification of strain rate gauge-point 4

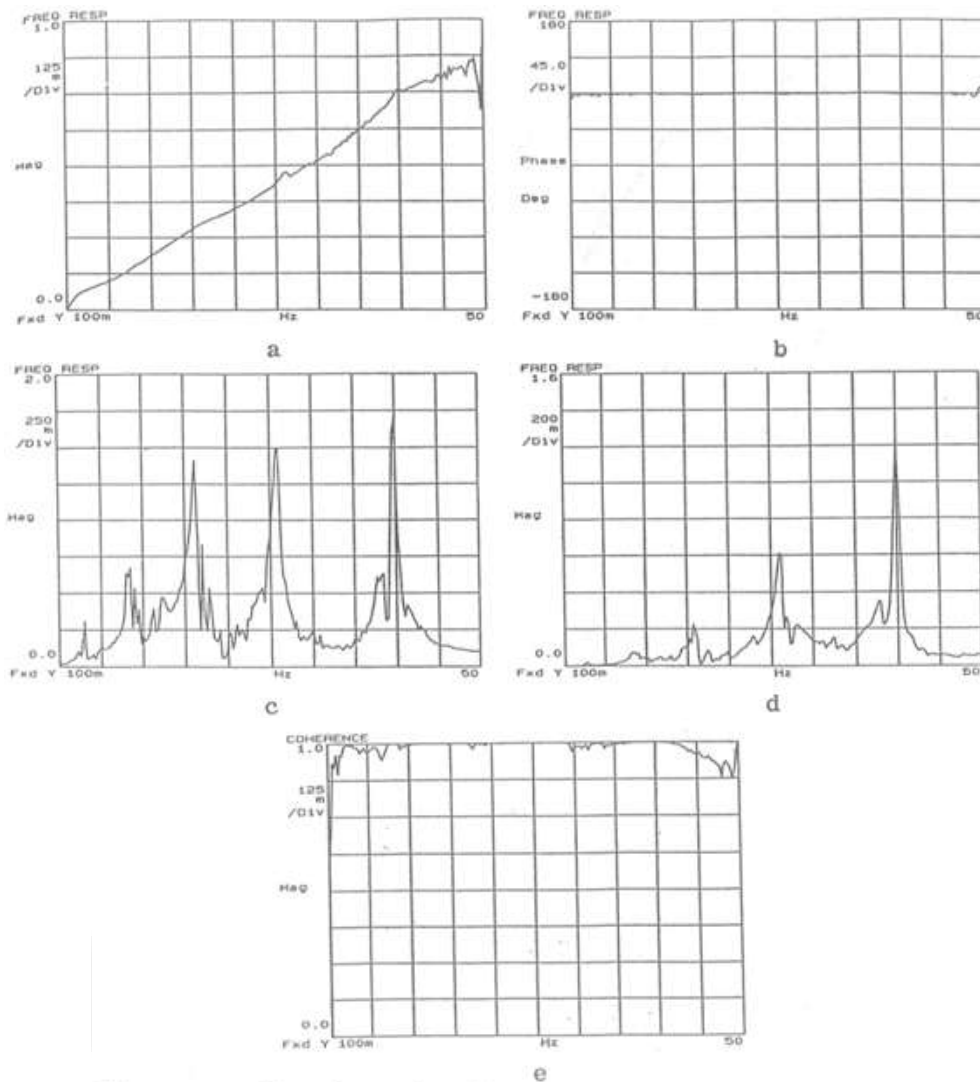


Figure 3.6 Experimental verification of strain rate gauge-point 5

4. Conclusions

PZT Ceramic plates were used and tested experimentally as strain rate sensors. Transfer function amplitude and phase between strain gauge and PZT plate sensor signals were determined as a measure of the sensors function. Results of swept sine frequency response tests, showed a linear increase of transfer function amplitude with frequency and a constant phase shift of ninety degree.

These results prove the functionality of PZT Ceramic plates as strain rate sensors. Throughout this work these devices were found to be very sensitive to small changes in strain rate, and with their negligible mass they are thus well recommended for measuring ‘velocities’ of oscillation of flexible structures.

Considering the obtained results, a further work is recommended through the application of PZT Ceramic plates at district points over the length of the arm as collocated sensors and actuators at the same time. These plates are light in weight and therefore their distribution will negligibly affect the dynamics of the structure. In concurrence with the recommendations of [1], at such a case the control of the structure will be decoupled into two parts.

The rigid body behavior of the arm might be controlled by the dc motor actuator, while the treatment of the arm flexural oscillation will be carried out by the collocated PZT plate sensors and actuators.

References

- [1] Al-Annaz, S. S., The dynamics and vibration control of a flexible arm, Ph. D. Thesis, University of Newcastle upon Tyne, UK, (1992).
- [2] Bailey, T. and Hubbard, J. E., Distributed piezoelectric – polymer active vibration control of a cantilever beam. *Journal of Guidance, Control and Dynamics*, Vol. 8, No. 5, (1985), pp. 605-611.
- [3] Fei, J. et al, The comparative study of vibration control of flexible structure using smart material. *Mathematical Problems in Engineering*, Vol. 2010, Article ID 768256 (2010)
- [4] Dafang, W., et al, Experimental study and numerical simulation of active vibration control of a highly flexible beam using piezoelectric intelligent material. *Aerospace of Science and Technology*, 37 (2014), pp. 10-19
- [5] Rao, S. S. and Sunar, M., Piezoelectricity and its use in disturbance sensing and control of flexible structures: A

- Survey, Applied Mechanics Reviews, Vol. 47, Issue 4 (1994), pp. 113-123.
- [6] Burdess, J. S. and Fawcett, J. N., Experimental evaluation of piezoelectric actuator for the control of vibration in a cantilever beam. Journal of Systems and Control Engineering, Vol. 206, (1992), pp. 99-106.
- [7] Ramesh, K. and Nerayanan, S., Active vibration control of beams with optimal placement of piezoelectric sensor/actuator pairs. Smart Materials and Structures, Vol. 17, No. 5, (2008)
- [8] Sirohi, J. and Chopra, I., Fundamental understanding of piezoelectric strain sensors. Journal of Intelligent Material Systems and Structures, Vol. 11, Issue 4 (2000), pp. 246-257.
- [9] Nam, Y. et al, Strain rate self-sensing of piezoelectric material with the phase delay compensation. JSME International Journal Series C, Mechanical Systems Machine Elements, Vol. 45, Issue 3 (2002), pp 722-729.

