Evaluation of Smart Growth of a City Based on Entropy Weight and Gray Relational Analysis - Using Suzhou and Saint Louis as Examples

Zhibing Liu[1], Xuelian Zhu[1], Yidan Zhu[1], Shanchao Cui[2]

[1]College of Mathematics and Physics, Huanggang Normal University, Hubei 438000, China

lzb8401552[at]hgnu.edu.cn

[2]College of Electronics and Information, Huanggang Normal University, Hubei 438000, China

2205190953[at]qq.com

Abstract: ***Smart growth is about helping every town and city become a more economically prosperous, socially equitable, and environmentally sustainable place to live. How to evaluate the success of smart growth of a city plays an important role in construction and development of a city. By construing the evaluation system of smart growth city, this paper measures the success of smart growth of two cities—Suzhou and Saint Louis—from 2010 to 2015 based on entropy weight and gray relational analysis methods, and using smart growth principles develops growth plans for both cities over the next few decades. Moreover, we find that the indexes of economy and community of Suzhou continue to increase while its environment begins to become worse and worse from 2012. For Saint Louis, the indexes of community and environment remain growth and the economy is ups and downs. Nevertheless, the current growth plans of both cities are successful as a whole. Furthermore, the plan of smart growth of Suzhou is far more successful than that of Saint Louis.***

Keywords: evaluation; city smart growth; entropy weight method; gray relational analysis

1. Introduction

The discussion around the cities and their developments has never stopped. Moving from the “industrial city” to the “city of services” and, finally, to contemporary urban conurbations, the city has indeed experimented issues of a different nature related to its growth. In these years of “urban reconcentration” social problems have emerged, related to the provision of essential services such as housing and education at an affordable cost, a social environment without crime, safe and inclusive, but also innovative services, in addition to the problems of congestion, pollution and physical degradation. In response to these challenges, after the models of urban sustainability represented by the “green city” and the “creative city”, taking shape new paradigm for the modern city, the “smart growth city”[1].

Many communities are implementing smart growth initiatives in an effort to consider long range and sustainable planning goals. Smart growth covers a range of development and conservation strategies that help protect our health and natural environment and make our communities more attractive, economically stronger, and more socially diverse [2]. Smart growth focuses on building cities that embrace the E’s of sustainability--Economically prosperous, socially Equitable, and Environmentally Sustainable. This task is more important than ever because the world is rapidly urbanizing. It is projected that by 2050, 66 percent of the world’s population will be urban--this will result in a projected 2.5 billion people being added to the urban population [3].

Smart growth of a city has 10 principles [4], which must be tailored to a community’s unique needs to be effective. In general, any measure of success must incorporate the demographics, growth needs, and geographical conditions of a city as well as the goal to adhere to the three E’s. Therefore, this paper investigates how to use smart growth theory to measure the smart growth degree of two cities--Suzhou and Saint Louis. Particularly, we want to answer the following questions.

1) How to measure the success of smart growth of a city, which considers the three E’s of sustainability and/or the 10 principles of smart growth.
2) Research the current growth plan of the selected cities. Measure and discuss how the current growth plan of each city meets the smart growth principles. How successful are the current plans according to our metric?
3) Using smart growth principles develops a growth plan for both cities over the next few decades. Support why we chose the components and initiatives of our plans based on the geography, expected growth rates, and economic opportunities of our cities. Use our metric to evaluate the success of our smart growth plans.
4) Also using our metric, ranks the individual initiatives within our redesigned smart growth plan as the most potential to the least potential. Compare and contrast the initiatives and their ranking between the two cities.

2. Assumptions

1) The selected cities are representative. The developments of the cities are relatively stable and smooth, which can be used to represent the majority of the developments of the cities.
2) Assume that the selected cities have no natural disasters,

and the city governments are stable and have no help from any other city. The chosen cities can be used to do research for the sustainable development of city without any regime-threatening instability or conflict, and they only depend on the developments of their governments without external force.

3) Assume that the data found in this paper is reliable and persuasive. This is because the data information searched by us comes from the governments' official websites.

3. Notations

For convenience, the main notations used in this paper are shown in Table 3-1.

Table 3.1: Notation description

symbol	Dedinition
A1	GDP
A2	Total amount of import and export
A3	Total export
B1	City total population
B2	Basic old – age insurance number
B3	Number of beds in health care institutions of per one thousand people
B4	Professional and technical personnel
B5	Urban built – up area
B6	Total length of bus operating line
B7	Taxi operation vehicle
C1	Green area of built – up area
C2	Intensity of sulfur dioxide emissions per unit of GDP
C3	Industrial waste water processing
x_{ij}	Original data of sample i and index j
w_j	Weight of index j
ξ_{ij}	Correlation coefficient of sample i and index j

4. Evaluation System of Smart Growth City

"Smart growth aims to help each town and city into a more prosperity, social justice, environmental and economic sustainable development of the city". According to the three E principles--economic prosperity, social equity and environmental sustainable development--of city sustainable development, the city smart growth composite index is determined.

4.1 Establishment of city smart growth evaluation system

The science evaluation system for a city is the basis of economic prosperity, social justice and sustainable development of environment. The establishment of integrated city smart growth evaluation system must follow the following principles: economic prosperity, social justice and environmental sustainable development.

According to these principles, we take the success rate of city smart growth as the primary. It includes three secondary indicators: intelligent economy, intelligent community and intelligent environment, and thirteen third level indicators: GDP, total amount of import and export, total export, city total population, basic old-age insurance number, number of beds in health care institutions of per one thousand people, professional and technical personnel, city built-up area, total length of bus operating line, taxi operating vehicles, green area of built-up area, intensity of sulfur dioxide emissions per unit of GDP and industrial waste water processing (Table 4-1).

4.2 Reason of indicator selection

- All levels of indicators for city smart growth evaluation are chosen due to the following reasons.
- A city economic level is mainly reflected in the GDP, the total import and export, export volume.
- The intelligent level of socialization of a city is mainly reflected in the city total population, the number of primary endowment insurance, the basic old-age insurance number, the number of beds in health care institutions of per one thousand people, the professional and technical personnel, the city built-up area, the total length of bus operating line and the taxi operation vehicles.
- A city economic environment optimization degree is mainly reflected in the green area of built-up area, the intensity of sulfur dioxide emissions per unit of GDP, the industrial waste water processing.

Table 4.1: Indicator system of smart growth of the city

City smart growth evaluation	Intelligent economy	GDP (A1)
		Total amount of import and export *(A2)*
		Total export *(A3)*
	Intelligent community	City total population *(B1)*
		Basic old-age insurance number *(B2)*
		Number of beds in health are institutions of per one thousand people *(B3)*
		Professional and technical personnel *(B4)*
		City built-up area *(B5)*

		Total length of bus operating line *(B6)*
		Taxi operation vehicles *(B7)*
	Intelligent environment	Green area of built-up area *(C1)*
		Intensity of sulfur dioxide emissions per unit of GDP *(C2)*
		Industrial waste water processing *(C3)*

5. Current development plan evaluation

In this section, we use the method combined with entropy weight and gray correlation analysis to determine the comprehensive smart growth index of a city and analyze the corresponding results.

5.1 Determination of comprehensive city smart growth index

Entropy weight method [5] is a method for objective determining weights. It uses information entropy to calculate the weights of each evaluation index according to the variation degree of all the evaluation indexes in specific applications. Then the weights of all the indexes are modified through entropy weight and the more objective weights of indexes are obtained. Entropy weight is not the important coefficient of representing the practical meaning of individual index in decision or evaluation problems. It represents the volume of useful information provided in these problems, that is, it represents the discriminating degree of evaluation object under individual index.

Grey system theory proposes the concept of doing the grey correlation analysis for all the subsystems, which purpose is, by a certain method, to seek the numerical relationship between all the subsystems or factors in the total system. Grey correlation analysis [6] provides the quantified measure for the trend of development and change of a system, which is very suitable for dynamic process analysis.

In the following, we use the entropy weight method to determine the weight of each index and apply grey correlation analysis method to calculate the correlation coefficients of all the secondary indicators. According to these weights and coefficients, the evaluation value of the comprehensive city smart growth index is obtained and the corresponding results are analyzed.

5.1.1 Using entropy weight method to establish index weights

There are *m* samples and *n* indexes which can compose an original date matrix $X = \left(x_{ij}\right)_{m\times n}$. In this paper, we select the corresponding data of each index from 2010 to 2015 in Suzhou and Saint Louis (see the Appendix), and thus $m=6$, $n=13$, where m represents the total years. The process of using the entropy weight method to establish index weights is as follows.

- The standardization of original data is provided according to the formula

$$y_{ij} = \frac{x_{ij} - (x_{ij})_{\min}}{(x_{ij})_{\max} - (x_{ij})_{\min}},$$

where y_{ij} is the standardization result for evaluation index j in the *ith* year. $(x_{ij})_{\max}$ and $(x_{ij})_{\min}$ represent the maximum and minimum values of each index in the original data, respectively.

- The ratio of index value of index j and sample i is given by the equation

$$p_{ij} = \frac{y_{ij}}{\sum_{i=1}^{m} y_{ij}}.$$

- The information entropy of index j is calculated according to the formula

$$e_j = -k\sum_{i=1}^{m} p_{ij} \ln p_{ij},$$

where $k = \dfrac{1}{\ln m}$.

- The weight of evaluation index j is obtained by the formula

$$w_j = \frac{d_j}{\sum_{j=1}^{n} d_j},$$

where $d_j = 1 - e_j$.

The weights of all the evaluation indexes in Suzhou are A1: 0.0264, A2: 0.0021, A3: 0.0039, B1: 0.0003, B2: 0.024, B3: 0.8539, B4: 0.047, B5: 0.0019, B6: 0.0235, B7: 0.0118, C1: 0.0032, C2: 0, C3: 0.002.

The weights of all the evaluation indexes in Saint Louis are A1: 0.0069, A2: 0.0254, A3: 0.8795, B1: 0.0107, B2: 0.0002, B4: 0.0003, B5: 0.0042, B6: 0.0474, C1: 0.013, C2: 0.0112, C3: 0.0011. Note that: the weights of B3 and B7 are omitted due to the data lack of these two indexes.

5.1.2 Using a composite method to determine comprehensive city smart growth index

In this subsection, we first apply grey correlation analysis method to calculate the correlation coefficients of intelligent economic, intelligent community and intelligent environment indexes, respectively. Second, we determine the comprehensive smart growth indexes of Suzhou and Saint Louis according to the calculated correlation coefficients and weights. The process is as follows.

- The original data is treated with dimensionless method to be transformed into the corresponding dimensionless matrixes, denoted by

$$y_{ij} = \frac{x_{ij} - (x_{ij})_{\min}}{(x_{ij})_{\max} - (x_{ij})_{\min}},$$

where $X=(x_{ij})_{kl}$ represents the original data matrix. For example, for intelligent economic index, $k=6$ and $l=3$ due to the data with six years and three sub-indexes.

- The absolute difference sequence Δ_{ij}, the maximum difference $\Delta_{\max}$ and the minimum difference $\Delta_{\min}$ are denoted as

$$\Delta_{ij} = \left| y_{ij} - y_{0j} \right|, \ \Delta_{\max} = \max_i \cdot \max_j \Delta_{ij},$$

$$\Delta_{\min} = \min_i \cdot \min_j \Delta_{ij},$$

where $y_{0j} = \max_i y_{ij}$ represents the reference sequence.

- The correlation coefficients are calculated according to the formula

$$\xi_{ij} = \frac{\Delta_{\max} + k\Delta_{\max}}{k\Delta_{\max}},$$

where k is identification coefficient and its value usually is 0.5 in real computation.

- The indexes of comprehensive smart growth of a city are calculated according to

$$f_i = \sum_{j=1}^{l} w_j \xi_{ij}.$$

By calculating them, the city smart growth indexes from 2010 to 2015 are shown in Tables 5-1 and 5-2.

From Table 5-1, we know that the intelligent economic and intelligent community indexes of Suzhou have an increasing trend over time, while its intelligent environment index firstly increases and then decreases. Its comprehensive smart growth index drops slightly in 2015 compared with 2014 because the intelligent environment index decreases more rapidly than the other two indexes in this year. But it increases as a whole. Therefore, the environment of Suzhou is suggested to be improved.

Similarly, from Table 5-2, we find that the intelligent community and intelligent environment of Saint Louis have an increasing trend over time, while its intelligent economic index is ups and downs. However, its comprehensive smart growth index firstly decreases and then increases. Especially, it drops significantly in 2011 and returns to the level of 2011 until 2015. The main reason is that the intelligent economic index of Saint Louis drops significantly in 2011, compared with 2010. As a consequence, the economy development of Saint Louis should be attached importance to.

Table 5.1: Suzhou smart growth index from 2010 to 2015

year	Intelligent Economic index	Intelligent community index	Intelligent environment index	Comprehensive smart growth index
2010	0.1077	0.0484	0.0492	0.2053
2011	0.1613	0.0569	0.0658	0.2840
2012	0.2188	0.0800	0.1292	0.4280
2013	0.2299	0.0873	0.0906	0.4078
2014	0.3170	0.1412	0.0577	0.5159
2015	0.3232	0.1451	0.0431	0.5114

Table 5.2: Saint Louis smart growth index from 2010 to 2015

year	Intelligent economic index	Intelligent community index	Intelligent environment index	Comprehensive smart growth index
2010	0.4263	0.0704	0.0704	0.5671
2011	0.1758	0.0802	0.0802	0.3362
2012	0.1584	0.0929	0.0929	0.3442
2013	0.1426	0.1122	0.1122	0.3670
2014	0.1531	0.1447	0.1447	0.4425
2015	0.1421	0.2113	0.2113	0.5647

5.2 Analysis for change rate of comprehensive smart growth index

The year-on-year increase rate of the comprehensive smart growth index of Suzhou and Saint Louis from 2011 to 2015 can be calculated according to Tables 5-1 and 5-2 (see Figures 5-1 and 5-2). We use the calculated rate to measure the success or failure of smart growth of a city in the current year. If it is greater than zero, it is successful, otherwise it is unsuccessful. The average value of all the rates is represented as the measurement of the success degree of smart growth of a city.

Figure 5-1 says that the year-on-year increase rate of comprehensive smart growth index of Suzhou goes up or down with time. The increase rates of 2013 and 2015 are negative, meaning that the plans of smart growth in these two years are unsuccessful. The positive increase rates of 2011, 2012 and 2014 imply that the plans of smart growth in these three years are successful. Since the average value of all the rates of Suzhou is 0.2199, the plan of smart growth of this city is successful as a whole. Similarly, the year-on-year increase rate of comprehensive smart growth index of Saint Louis increases with time. The negative increase rate of 2011 indicates that the plan of smart growth in this year is unsuccessful. The plans of smart growth in the other years are successful due to their positive increase rates. However, the plan of smart growth of this city is successful as a whole since the success degree of smart growth of this city is 0.0328.

In comparison, the plan of smart growth of Suzhou is far more successful than that of Saint Louis because 0.2199 is far bigger than 0.0328.

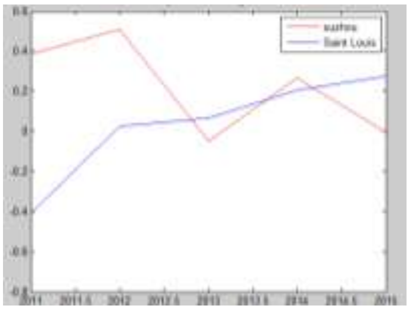

Figure 5.1: Year-on-year increase rate

6. City Plans for Future Growth

In this section, the plans of growth by 2050 of Suzhou and Saint Louis are made according to the comprehensive analysis for the geographic location, economic opportunities and expected growth rates of these two cities.

6.1 Growth plan in Suzhou

6.1.1 Location

Suzhou construction land area is 2432 square kilometers, accounting for 28.1% of the city area, 49.8% of the land area and 80.7% of the construction land area. Rapid consumption of land resources has broken through the scale for the planning and control. Land index, population scale, facilities, etc. are difficult to support the current development. It is worth noting that the area traffic location advantage of Suzhou is being weakened under the networked pattern of Yangtze River Delta and the intra plates of Suzhou city region develop separately and independently. The support points of rapid transport links are insufficient and the urban centricity is not strong. Urban traffic organization still keeps ancient city as the core, which is close to the limit capacity and contradicts with ancient city protection. Aiming at these problems, we should take control of urban area, utilize the compact area maximally, and build “bus + go slow” traffic mode. Leading in and out of the old city is dominated by rail transit. Limit the use of cars, lengthen the operation distance of buses and increase operation quantity of taxes.

6.1.2 Economic opportunities

In recent years, manufacturing industry is the core of the Suzhou industry and is the most important brand resources. However, Suzhou’s manufacturing processing is given priority to, resulting in low added value and low output benefits with high land input. Therefore, we should make a selection for Suzhou industry. And we build the industry layout structure of “transverse promotions, vertical axis agglomeration; science and technology production, cultural innovation; civilized innovation service and advanced manufacturing technology”, to improve the GDP, the total import and export and the total exports.

6.1.3 Expected rate of growth

According to the data in Table 5-1, the variation trends of intelligent community, intelligent economy and intelligent environment of Suzhou from 2010 to 2015 are shown in Figure 6-1.

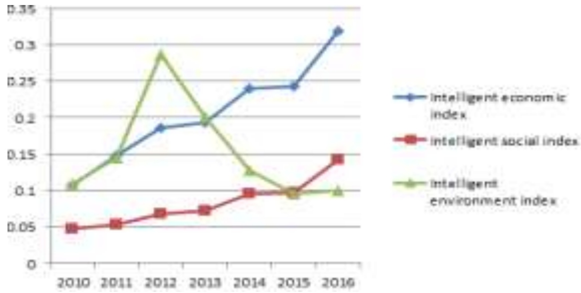

Figure 6.1: Variation trend of individual index of Suzhou

From Figure 6-1, the intelligent environment index has been dropping since 2012, so we should increase the green area of built-up area and the quantity of industrial waste water processing, and reduce the sulfur dioxide emissions per unit of GDP to strengthen the construction of environment.

6.1.4 Growth plan in the future

Due to the contradiction between intelligent environment and intelligent economy (or intelligent community), the current environment construction of Suzhou represses its overall development. Therefore, in the next few decades, the environment construction should be given priority and at the same time the government should formulate the corresponding political measures to promote public participation to promote the urban development of Suzhou. According to the trend of average values of thirteen third indexes, we propose a bold assumption: in 2050, the growth rates of the thirteen third level indexes are 21%, 13%, 8%, 10%, 12%, 7%, 15%, 6%, 7%, 3%, 8%, 12% and 20%, respectively.

6.1.5 Growth plan success degree evaluation

We calculate all the index values in 2050 based on the index values in 2015 and the expectations hypothesis in Section 6.1.4, and then compare the corresponding values in the two years to evaluate the degree of success of this plan (Table 6-1).

Table 6.1: Comprehensive smart growth index of Suzhou in 2015 and 2050

year	Intelligent economic index	Intelligent social index	Intelligent environment index	Comprehensive smart growth index
2015	0.1905	0.1139	0.1288	0.4332
2050	0.3194	0.2026	0.3863	0.9083

From Table 6-1, compared with the indexes in 2015, the comprehensive index of smart growth of Suzhou in 2050 increases significantly and the increase rate is 109.67%. Therefore, we believe that the plan we make is very successful.

6.2 Saint Louis growth plans

6.2.1 Location and economic opportunities

Saint Louis is the only independent city and also is the largest city of Missouri. It is located in the confluence of the Missouri river and the Mississippi River. Its urban area is 170 square kilometers, where the water area is accounting for

6.2%.

Saint Louis has long been a major industrial and transportation hub. It is a leading rail and trucking center, and its airport and river port are among the country's busiest. Its industries produce a variety of manufactures, including chemicals, consumer goods, motor vehicles and parts, electronic components, foods and beverages, textiles, shoes, paper, plastic, and metal products, paints, soap and detergents, hardware, and pharmaceuticals. Saint Louis is also a wholesale, banking, and financial center. Based on these features, the development of Saint Louis focuses on the three indexes--total amount of import and export, total export and industrial waste water processing--in order to improve the intelligent economic comprehensive index.

6.2.2 Expected rate of growth

According to the data in Table 5-2, the variation trends of intelligent community, intelligent economy and intelligent environment indexes of Saint Louis from 2010 to 2015 are shown in Figure 6-2.

Figure 6-2 says that the intelligent economy and intelligent community indexes continue to drop since 2010, so we should increase their third level indicators, such as GDP, total export, city total population, etc., to strengthen the constructions of economy and community.

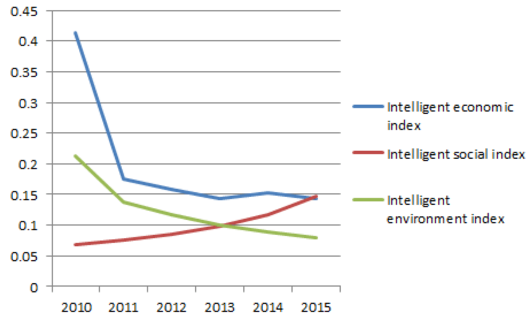

Figure 6.2: Variation trend of individual index of Saint Louis

6.2.3 Growth plan in the future

Due to the contradiction between intelligent economy and intelligent environment (or intelligent community), the current economy construction of Saint Louis represses its overall development. Therefore, in the next few decades, the economy construction should be given priority and at the same time the government should formulate the corresponding political measures to promote public participation to promote the urban development of Saint Louis.

6.2.4 Growth plan success degree evaluation

We calculate all the index values in 2050 based on the index values in 2015 and the expectations hypothesis, and then compare the corresponding values in the two years to evaluate the degree of success of this plan (Table 6-2).

Table 6.2: Comprehensive smart growth index of Saint Louis in 2015 and 2050

year	Intelligent Economic index	Intelligent Social index	Intelligent environment index	Comprehensive smart growth index
2015	0.2365	0.1492	0.1324	0.5181
2050	0.2397	0.1948	0.3810	0.8155

From Table 6-2, compared with the indexes in 2015, the comprehensive index of smart growth of Saint Louis in 2050 increases significantly and the increase rate is 57.4%. Therefore, we believe that the plan we make is very successful. Comparing the plans of Suzhou and Saint Louis we make, we find that the plan of smart growth of Suzhou is far more successful than that of Saint Louis because 109.67% is far bigger than 57.4%.

7. Sensitivity Analysis

In order to do a sensitivity analysis only for Suzhou, we increase 10% of each third level index and other indexes remain the same. We use entropy weight method and grey correlation analysis method to redetermine the comprehensive smart growth indexes (Table 7-1). In Table 7-1, the first line represents the corresponding values of comprehensive smart growth index from 2010 to 2015 when the growth of the first third level index, A1, is 10%, and the meanings of the other lines are similar.

Table 7-1: Impact of each third level index on comprehensive index of Suzhou

	2010	2011	2012	2013	2014	2015	Average
A1	0.3215	0.3428	0.346	0.35425	0.3621	0.3608	0.34791
A2	0.3191	0.3315	0.343	0.35088	0.3585	0.3643	0.34455
A3	0.3191	0.3315	0.343	0.35088	0.3582	0.3643	0.3445
B1	0.382	0.3507	0.362	0.37043	0.3783	0.3843	0.37129
B2	0.3916	0.3315	0.343	0.35093	0.3586	0.3644	0.35667
B3	0.3191	0.3315	0.343	0.35085	0.3585	0.3643	0.34454
B4	0.3191	0.3315	0.343	0.35087	0.3585	0.3643	0.34455
B5	0.3191	0.3315	0.343	0.35083	0.3585	0.3643	0.34454
B6	0.3215	0.3362	0.348	0.3564	0.3637	0.3699	0.34928
B7	0.3186	0.3323	0.344	0.3518	0.3595	0.3653	0.34525
C1	0.3251	0.3375	0.349	0.35727	0.6516	0.3711	0.3986
C2	0.3191	0.3315	0.343	0.35081	0.3585	0.3643	0.34454
C3	0.3204	0.3329	0.344	0.35225	0.3598	0.3656	0.34583

Table 7-1 indicates that third level indexes to influence degree of comprehensive index of Suzhou press from high to the low row preface is C1, B1, B2, B6, A1, C3, B7, A2, B4, B3, B5, C2 and A3. Because the size sort of the average value of comprehensive index from 2010 to 2015 is in accordance with this size sort.

8. Improved Model

According to the change trends of third level indexes of Suzhou and Saint Louis, the developments of both cities reach their peaks at some point. Therefore, urban sprawl is necessary. Next, the remote sensing technology is used to research it which is benefic to the development of growth plans for both cities over the next few decades.

Remote sensing is a very useful tool that detects and analyzes the urban expansion. Using remote sensing data, we can get the latest information like highway traffic, residential areas, farmland, etc., and thus analyze the spatial relations between urban sprawl and farmland loss, which can't be matched by conventional methods.

Using the spatial analysis function of geographic information system (GIS), we can quantitatively analyze the spatial distribution of urban sprawl and farmland erosion and its impact factor. Urban sprawl causes a loss of farmland, which is necessary for us to dynamically detect and analyze the diffusion process. As many other geographical phenomena, the process of city expansion and occupying farmland can be explained by using diffusion theory. Entropy can be used as describing the quantitative indicators of the spatial distribution and its variation. On the remote sensing map, the change of entropy caused by urban expansion can be easily calculated to provide an objective evaluation index.

Attenuation property with distance

Since the land location of urban development is great related with distance factor, the decreasing relationship between the number of land development and distance can be described by the following exponential function,

$$y = ae^{-bx},$$

where y is the density of land development, x is the distance from the city or highway, and a and b are parameters. The function is used by Clark to reflect the function relationship of the population density with the distance from the city center [7].

Using the buffer analysis function of GIS, the relationship between the land development density and the distance from the town center can easily be established. First of all, the different buffer areas can be obtained by the figure about the present situation of highway and the urban dot figure. Highway network and the latest information of residential areas can be obtained by remote sensing images. Further using the lamination analysis function of GIS, the classification results of remote sensing and the buffer areas are overlapped to count the frequency of land development fallen into each buffer area. And thus, the regression analysis method can be used to establish the gradient relationship of land development with these factors.

- Entropy and urban development decentralization -- space features

Shannon formula reflects the amount of information in information theory. Shannon's entropy can be expressed as [8]

$$H_n = \sum p_i \log(1/p_i).$$

Entropy can be used to describe the decentralization and centralization of the geographical phenomena. First, observations x_i can be converted to percentages p_i,

$$p_i = x_i / \sum x_i.$$

In order to make the calculated value belonging to zero and one, the decentralization can be calculated by the following relative entropy equation

$$H_n = \sum p_i \log(1/p_i)/\log n.$$

The centralization relative to the decentralization can be expressed as

$$c_n = 1 - H_n.$$

The bigger value of decentralization implies the disorganized expansion of urban or the lack of a reasonable standard layout.

- **Differential entropy and urban expansion -- time features**

As many other geographical phenomena, the development of a city is a diffusion process. Due to the needs of economic development and population growth, urban sprawl causes the farmlands near the city periphery and highway to be occupied and used as new city farmlands. We can calculate the entropy variation of city farmlands on the multidate remote sensing images to quantitatively monitor the process of urban expansion, which can be reflected by the differential entropy $\triangle H$

$$\Delta H = H(t+1) - H(t) = \sum p_i(t+1)\log(1/p_i(t+1))/\log n - \sum p_i(t)\log(1/p_i(t))/\log n.$$

The increase of entropy means that urban land spreads outward. The bigger the increased value is, the faster the diffusion is. The decrease of entropy implies that urban land tends to be more compact.

9. Conclusion

This paper uses the entropy weight and gray correlation analysis methods to measure the success of smart growth of a city and analyze the smart growth index changes from 2010 to 2015 to evaluate the success of city smart growth. Moreover, we find that the indexes of economy and community of Suzhou continue to increase while its environment begins to become worse from 2012. For Saint Louis, the indexes of community and environment remain growth and the economy is ups and downs. Nevertheless, the current growth plans of both cities are successful as a whole. Furthermore, the plan of smart growth of Suzhou is far more successful than that of Saint Louis. Finally, the suggestions of development for both cities are proposed.

In this paper we only use thirteen third level indexes to characterize the smart growth of a city. It remains for future research to consider more detailed information to describe the smart growth of a city more accurately. On the other hand, in the paper we use entropy weight and gray correlation analysis methods to measure the success of smart growth of a city. Instead, the other evaluation methodologies may be applied to better deal with this problem. In a word, the problem of evaluation of smart growth of a city implies fruitful research topics that would expand the scope of the current city smart growth evaluation literature.

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Appendix

Table 5.3: Values of third level indexes of Suzhou from 2010 to 2015

	2010	2011	2012	2013	2014	2015
A1 (billion dollars)	9228.91	10716.99	12011.65	13015.7	13760.89	14504.07
A2 (billion dollars)	2740.76	3008.63	3056.92	3093.48	3113.06	3053.5
A3 (billion dollars)	1531.08	1672.33	1746.89	1757.06	1811.78	1814.59
B1 (billion dollars)	6376558	6423336	6478054	6538372	6610766	6670124
B2 (unit)	346.78	388.44	472.88	492.44	506.06	511.57
B3 (piece)	3.92	4.09	4.37	4.88	5.21	5.29
B4 (ten thousand people)	84.66	96.45	109.99	123.48	136.61	149.64
B5 (square kilometers)	660.4	698.9	719.94	727.16	735.15	747.84
B6 (kilometers)	8077	10345	11624	11847	12187	12561
B7 (ten thousand person-time)	6838	7327	8132	8297	8988	9018
C1 (square kilometers)	187829	188750	192206	201926	209479	214816
C2 (kilogram/ten thousand yuan)	42.84	42.46	42.58	42.79	42.93	43.08
C3 (ten thousand tons)	66968	69746	73790	72006	68602	65401

Table 5.4: Values of third level indexes of Saint Louis from 2010 to 2015

	2010	2011	2012	2013	2014	2015
A1 (billion dollars)	0.126	0.129	0.135	0.139	0.143	0.147
A2 (billion dollars)	188.4	135.7	151.9	150.5	149.1	147.8
A3 (billion dollars)	144.9	62.5	47.1	29.9	41.8	29.3
B1 (billion dollars)	15.9	16.6	17.6	18.3	18.6	19.4
B2 (unit)	1.29	1.281	1.293	1.294	1.303	1.319
B4 (ten thousand people)	83.3	83.9	84.5	85.1	85.7	86.2
B5 (square kilometers)	1.065	1.092	1.12	1.148	1.176	1.206
B6 (kilometers)	1.962	2.15	2.334	2.534	2.751	2.987
C1 (square kilometers)	16.7	18	18.7	19.5	20.3	20.9
C2 (kilogram/ten thousand yuan)	0.638	0.69	0.714	0.738	0.764	0.79
C3 (ten thousand tons)	141.9	139	137.5	135.9	134.5	132.9