International Journal of Science and Research (IJSR)
ISSN (Online): 2319-7064
Index Copernicus Value (2016): 79.57 | Impact Factor (2015): 6.391

Three Coupler Position Synthesis and Kinematic Analysis for a Couple of Four-Bar Mechanism

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Abstract: ***The four-bar mechanism is a class of mechanical linkage in which four links are pinned together to form a closed loop in order to perform some useful motion. In this paper the four bar mechanism is designed by forming mathematical model for mechanism position incorporating three coupler position. The three coupler position synthesis is done for four possible alternatives. Kinematic analysis is performed for the mechanism designed in the first alternative. The algebraic position equation for the mechanism is derived. The first and the second derivative of the position equation provide the velocity and acceleration of the mechanism. For the known position, velocity and acceleration of the input link, the position, velocity and acceleration of the other links are determined. MATLAB programs are written to solve the mathematical model and equations developed in the kinematic analysis. The result of four-bar mechanism design and its kinematic analysis is included.***

Keywords: Four-bar mechanism, Three coupler position synthesis, Kinematic analysis, MATLAB program

1. Introduction

A four-bar linkage is the most fundamental of the plane kinematic chains. It is a much preferred mechanical device for the mechanization and control of motion due to its simplicity and versatility. A four-bar linkage is a versatile mechanism that is widely used in machines to transmit motion or to provide mechanical advantage. Their low friction, higher capacity to carry load, ease of manufacturing, and reliability of performance in spite of manufacturing tolerances make them preferable over other mechanisms in certain applications. It is also the most fundamental linkage mechanism, and many more complex mechanisms contain the four-bar linkage as elements [3]. A four-bar linkage has four binary links and four revolute joints; hence from Gruebler's Equation there are3*(4 - 1) - 2*4 = 1 degree of freedom. This means that only one input is required to make the linkage move. Because of its simplicity, and perhaps also because of the rapid increase in design complexity suffered by linkages with more than four bars, the four-bar linkage is one of the most commonly used linkages . Of course, the link that is unable to move is referred to as the frame. Typically, the pivoted link that is connected to the driver or power source is called the input link. The other pivoted link that is attached to the frame is designated the output link or follower. The coupler or connecting arm "couples" the motion of the input link to the output link. Given that all the dimensions of a linkage and the input angle of the crank, we can easily determine the position of the coupler. The problem of determining the position of a linkage's elements given their dimensions and constraints, either relative to each other or to the positions of the actuators, is called the forward (or direct) kinematics problem. Kinematic analysis is used to solve the forward kinematics problem. Given that desired positions of a coupler, we can find the link parameters that would enable the linkage to move the coupler through the desired positions. This is called the inverse kinematics problem. Linkage synthesis is when the lengths and positions of the links themselves must also be determined[1][2].

2. Three Position Kinematic Synthesis

The design problem for known three coupler position is called three position synthesis. The problem here is to find the appropriate length of all four links and to determine the location of the pivot points, so that the coupler achieve the desired displacement.

Consider two points B and C that is attached to the end of the coupler and moves from B_1C_1 to B_2C_2 to B_3C_3 as shown in figure 1. The following methodologies are used to determine the appropriate length of the links and the position of the pivot points.

- Draw four line B_1 to B_2, B_2 to B_3, C_1 to C_2 and C_2 to C_3
- Draw a perpendicular bisector of B_1B_2, B_2B_3, C_1C_2 and C_2C_3.
- Pivot point A is the intersection of perpendicular bisector of B_1B_2 and perpendicular bisector of B_2B_3
- Pivot point D is the intersection of perpendicular bisector of C_1C_2 and perpendicular bisector of C_2C_3.
- Write the mathematical model to do the analytical synthesis.
- Write the MATLAB program to get the length of each link and to get the four bar linkage in three location which is similar with the coupler positions.

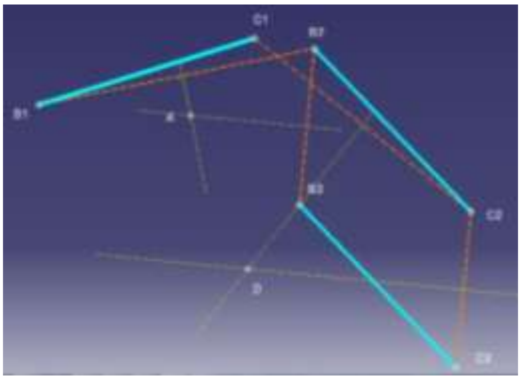


Figure 1: Three coupler position

A. Mathematical model for analytical synthesis

The following variables are used to obtain the model equations that can be used for analytical synthesis.

xb_1 is B_1 position in x-direction
yb_1 is B_1 position in y-direction
xb_2 is B_2 position in x-direction
yb_2 is B_2 position in y-direction
xb_3 is B_3 position in x-direction
yb_3 is B_1 position in y-direction
xc_1 is C_1 position in x-direction
yc_1 is C_1 position in y-direction
xc_2 is C_2 position in x-direction
yc_2 is C_2 position in y-direction
xc_3 is C_3 position in x-direction
yc_3 is C_1 position in y-direction
xa is the position of pivot A in x=direction
ya is the position of pivot A in y=direction
xd is the position of pivot D in x=direction
yd is the position of pivot D in y=direction
xb_{12} is the midpoint of **B_1**and B_2 in x- direction
yb_{12} is the midpoint of B_1 and B_2 in y -direction
xb_{23} is the midpoint of **B_2**and B_3 in x- direction
yb_{12} is the midpoint of B_2 and B_3 in y -direction
xc_{12} is the midpoint of C_1 and C_2 in x direction
yc_{12} is the midpoint of C_1 and C_2 in x direction
xc_{23} is the midpoint of C_2 and 3 in x direction
yc_{23} is the midpoint of C_2 and C_3 in x direction
Sb_1b_2 is slope of the line B_1B_2
Sb_2b_3 is slope of the line B_2B_3
Sc_1c_2 is slope of the line C_1C_2
Sc_2c_3 is slope of the line C_2C_3
$Sb_{12}$is slope of the line perpendicular bisector of B_1B_2
$Sb_{23}$is slope of the line perpendicular bisector of B_2B_3
$Sc_{12}$is slope of the line perpendicular bisector of C_1C_2
$Sc_{23}$is slope of the line perpendicular bisector of C_2C_3

once we have the variables we can derive the mathematical equations to get the pivot points A and D and the lengths of the four bar.

$$xb_{12} = \frac{xb_1+xb_2}{2} \quad (1)$$
$$yb_{12} = \frac{yb_1+yb_2}{2} \quad (2)$$
$$xb_{23} = \frac{xb_2+xb_3}{2} \quad (3)$$
$$yb_{23} = \frac{yb_2+yb_3}{2} \quad (4)$$
$$xc_{12} = \frac{xc_1+xc_2}{2} \quad (5)$$
$$yc_{12} = \frac{yc_1+yc_2}{2} \quad (6)$$
$$xc_{23} = \frac{xc_2+xc_3}{2} \quad (7)$$
$$yc_{23} = \frac{yc_2+yc_3}{2} \quad (8)$$
$$Sb_1b_2 = \frac{y\,b_2-yb_1}{xb_2-xb_1} \quad (9)$$
$$Sb_2b_3 = \frac{y\,b_3-yb_2}{xb_3-xb_2} \quad (10)$$
$$Sc_1c_2 = \frac{y\,c_2-yc_1}{xc_2-xc_1} \quad (11)$$
$$Sc_2c_3 = \frac{y\,c_3-yc_2}{xc_3-xc_2} \quad (12)$$
$$Sb_{12} = -\frac{1}{Sb_1b_2} \quad (13)$$
$$Sb_{23} = -\frac{1}{Sb_2b_3} \quad (14)$$
$$Sc_{12} = -\frac{1}{Sc_1c_2} \quad (15)$$
$$Sc_{23} = -\frac{1}{Sc_2c_3} \quad (16)$$
$$x_a = \frac{((Sb_{23}*xb_{23})-(Sb_{12}*xb_{12})-(yb_{23}+yb_{12}))}{Sb_{23}-Sb_{12}} \quad (17)$$
$$x_d = \frac{((Sc_{23}*xc_{23})-(Sc_{12}*xc_{12})-(yc_{23}+yc_{12}))}{Sc_{23}-Sc_{12}} \quad (18)$$
$$y_a = Sb_{12} * (x_a - xb_{12}) + yb_{12} \quad (19)$$
$$y_d = Sc_{12} * (x_d - xc_{12}) + yc_{12} \quad (20)$$

The pivot points are

$$A = (x_a, y_a)$$
$$D = (x_d, y_d)$$

length of the links are

$$L_1 = len(DA) = sqrt((x_d - x_a)^2 + (y_d - y_a)^2) \quad (21)$$
$$L_2 = len(AB_1) = sqrt((x_a - xb_1)^2 + (y_a - yb_1)^2) \quad (22)$$
$$L_3 = len(B_1C_1) = sqrt((xb_1 - xc_1)^2 + (yb_1 - yc_1)^2) \quad (23)$$
$$L_4 = len(C_1D_1) = sqrt((xc_1 - x_d)^2 + (yc_1 - y_d)^2) \quad (24)$$

B. Result and discussion

To solve this equations and to plot the graph of the four link in the specified three positions, the mat lab program is written. the values of xb_1, yb_1,xb_2,yb_2,xb_3,yb_3,xc_1, yc_1,xc_2, yc_2, xc_3, yc_3 , for five alternative coupler positions is given below

Alternative 1
xb_1=100, yb_1=100
xb_2=200, yb_2=150
xb_3=210, yb_3=40
xc_1=180, yc_1=140
xc_2=280, yc_2=110
xc_3=290, yc_3=0

Alternative 2
xb_1=0, yb_1=100
xb_2=100, yb_2=200
xb_3=300, yb_3=210
xc_1=0, yc_1=180
xc_2=180, yc_2=200
xc_3=300, yc_3=130

Alternative3
xb_1=50, yb_1=100
xb_2=150, yb_2=150
xb_3=280, yb_3=120
xc_1=100, yc_1=150
xc_2=200, yc_2=200
xc_3=330, yc_3=170

Alternative4
xb_1=100, yb_1=105
xb_2=190, yb_2=140
xb_3=205, yb_3=95
xc_1=180, yc_1=145
xc_2=270, yc_2=100
xc_3=285, yc_3=55

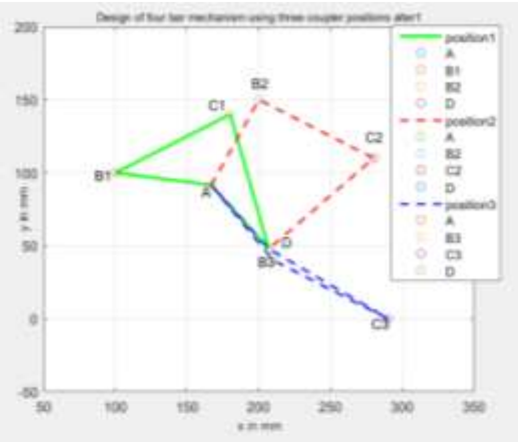
a. MATLAB results

The pivot points and the length of the bar results are shown in table below

Table 1: MATLAB result for different alternative of coupler positions

Coupler positions	*Pivot Points in mm* A xa	ya	D xd	yd	*Legth of the links in mm* L1	L2	L3	L4
Alternative1	159	108	211	61	59	67	89	96
Alternative	211	-11	116	-47	101	238	80	255
Alternative3	177	29	227	21	71	181	71	181
Alternative4	151	103	188	48	65	53	90	98

The graph of the four bar-linkage in three coupler position for the four alternative is shown as in figure 2.



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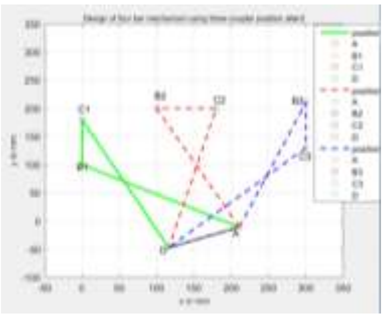

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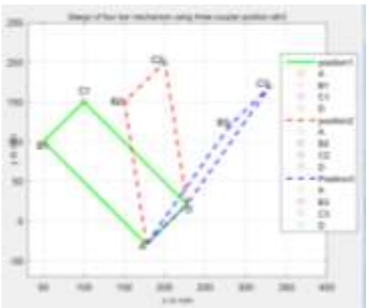

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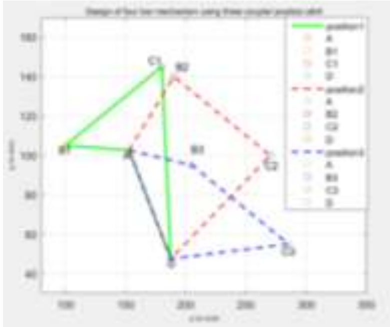

d)

Figure 2: four-bar linkage design using three coupler position a) for alternative 1 b) for alternative 2 c) for alternative 3 d) for alternative 4

b. Discussion on the result

In this design the link corresponding to L_1 is the frame, the link corresponding to L_2 is input link, the link corresponding to L_3 the coupler and the link corresponding to L_4 is the output link.

According to Grashof's criterion

s= length of the shortest link
L=length of the longest link
p=length of one end of the intermediate length link
p=length of one end of the intermediate length link
Based on Grashof's criterion
For firs alternative
s+L<p+q=59+96<67+89=155<156 and the frame is the shortest link
For second alternative
s+L<p+q=80+255<238+101=335<339 and the coupler is the shortest link
For third alternative
s+L=p+q=71+181=71+181=252=252 and the input link and output is equal in length.
For fourth alternative
s+L<p+q=53+98<90+65=151<155 and the input link is the shortest link

For the first alternative, the mechanism is double crank mechanism. The input link and output link can rotate through a full revolution. For the second alternative, the mechanism is double rocker mechanism. The coupler can complete a full revolution. Both the input and output links are constrained to oscillate between limits For the third alternative, the mechanism is change point mechanism. This is a parallelogram linkage because the frame and coupler have the same length and are the two pivoting links. For this type of mechanism the motion becomes indeterminate. For the fourth alternative the mechanism is crank-rocker mechanism. If the input link is continuously rotated the output link oscillates between a limit.

3. Kinematic Analysis of Four-Bar Linkage

For kinematic analysis of mechanisms, either analytical or graphical methods can be performed. Analytical kinematics is systematic process that can be easily developed in to computer program. Even though, very simple analytical kinematics can be done by hand calculations, it is not easy to do simple analytical kinematics without computer program. In a kinematic analysis, the position, velocity, and acceleration of the input link must be given or assumed. The task is to compute the other link coordinates, velocity and acceleration[4]. In current work kinematic analysis for four-bar mechanism is performed analytically. For our analysis we chooses four-bar mechanism designed by the first alternative. The MATLAB software is used for the analysis of analytical kinematics.

A. Position, velocity and acceleration Analysis

Consider the for bar linkage designed in alternative1.

For our analysis the following nomenclature are used.

Nomenclatures

θ_1= frame angular position

θ_2= Input link angular position

θ_3= coupler angular position

θ_4= Output link angular position

ω_2= angular velocity of the input link

ω_3= angular velocity of the coupler

ω_4= angular velocity of the output link

α_2= angular acceleration of the input link

α_3= angular acceleration of the coupler

α_4= angular acceleration of the output link

$R_{AD} = L_1$ is the length of the frame

$R_{BA} = L_2$ is the length of the input link

$R_{CB} = L_3$ is the length of the coupler

$R_{DC} = L_4$ is the length of the output link

Vector loop equation

$$R_{BA} + R_{CB} - R_{DC} - R_{DC} = 0$$

$$L_2 + L_3 - L_4 - L_1 = 0$$

The projection of vector loop equation on to x-and y-axis gives us two algebraic equations. Here θ_1 is zero

$$L_2 cos\theta_2 + L_3 cos\theta_3 - L_4 cos\theta_4 - L_1 = 0$$

$$L_2 sin\theta_2 + L_3 sin\theta_3 - L_4 sin\theta_4 = 0 \quad (25)$$

equation (25) is position equation for the mechanism

The time derivative of the position equation gives us velocity equation.

$$-L_2 sin\theta_2\omega_2 - L_3 sin\theta_3\omega_3 + L_4 sin\theta_4\omega_4 = 0$$

$$L_2 cos\theta_2\omega_2 + L_3 cos\theta_3\omega_3 - L_4 cos\theta_4\omega_4 = 0$$

Expressing the equation in matrix form

$$\begin{bmatrix} -L_3 sin\theta_3 & L_4 sin\theta_4 \\ L_3 cos\theta_3 & -L_4 cos\theta_4 \end{bmatrix} \begin{bmatrix} \omega_3 \\ \omega_4 \end{bmatrix} = \begin{bmatrix} L_2 sin\theta_2\omega_2 \\ -L_2 cos\theta_2\omega_2 \end{bmatrix}$$

solving for ω_3 and ω_4

$$\omega_3 = \frac{L_2\omega_2 sin(\theta_4-\theta_2)}{L_3 sin(\theta_3-\theta_4)}$$

$$\omega_4 = \frac{L_2\omega_2 sin(\theta_2-\theta_4)}{L_3 sin(\theta_4-\theta_3)} \quad (26)$$

Equation (26) is velocity equation

Time derivative of velocity equation gives us acceleration equation

$$-L_2 cos\theta_2\omega_2{}^2 - L_2 sin\theta_2\alpha_2 - L_3 cos\theta_3\omega_3{}^2 - L_3 sin\theta_3\alpha_3 + L_4 cos\theta_4\omega_4{}^2 + L_4 sin\theta_4\alpha_4 = 0$$

$$-L_2 sin\theta_2\omega_2{}^2 + L_2 cos\theta_2\alpha_2 - L_3 sin\theta_3\omega_3{}^2 + L_3 sin\theta_3\alpha_3 + L_4 sin\theta_4\omega_4{}^2 - L_4 cos\theta_4\alpha_4 = 0$$

Expressing the equation in matrix form

$$\begin{bmatrix} -L_3 sin\theta_3 & L_4 sin\theta_4 \\ L_3 cos\theta_3 & -L_4 cos\theta_4 \end{bmatrix} \begin{bmatrix} \alpha_3 \\ \alpha_4 \end{bmatrix} = \begin{bmatrix} L_2(sin\theta_2\alpha_2 + cos\theta_2\omega_2{}^2) + L_3 cos\theta_3\omega_3{}^2 - L_4 cos\theta_4\alpha_4 \\ -L_2(cos\theta_2\alpha_2 - sin\theta_2\omega_2{}^2 + L_3 cos\theta_3\omega_3{}^2 - L_4 cos\theta_4\alpha_4) \end{bmatrix}$$

solving for α_3 and α_4

$$\alpha_3 = \frac{-L_2\omega_2 cos(\theta_2-\theta_4) + L_3\omega_3{}^2 cos(\theta_3-\theta_4) + L_4\omega_4{}^2}{L_4 sin(\theta_3-\theta_4)}$$

$$\alpha_4 = \frac{L_2\omega_2 cos(\theta_2-\theta_3) - L_4 4^2 cos 4-3) + L_{3\omega 3}{}^2}{L_4 sin(\theta_4-\theta_3)} \quad (27)$$

Equation (27) is acceleration equation

We have already the lengths of the links. We can assume the angular velocity of the link ω_2 and analysis the mechanism for the input link position θ_2 from zero to 2π (complete rotation). For our analysis the input link is rotating with constant angular velocity and α_2 is assumed to be zero.

The MATLAB program m.files are done. The first m.file is fun.mfile which is a function written to solve the position of coupler(θ_3) and position of output link(θ_4) for the given values of θ_2 . The second m.file is fsolve@fun.mfile. This m.file provide us the values of θ_3 and θ_4. Other m.files are also written to solve the values of velocity and acceleration for other links as well to plot the results in relation with position of input link.

B. MATLAB Results

From table 1. Link lengths corresponding to the first alternative are:- L_1=59mm L_2= 67mm L_3=89mm L_4=96mm. Assuming that $\omega_2 = 40 rad/sec$, for value of θ_2 changing from zero to 2π, in the interval of $\pi/9$ values of θ_3 , θ_4, ω_3, ω_4, α_3, α_4 are shown in table.2.

The relationship between the input link and the other two links can be plotted. The graph for position, velocity and acceleration of the other two links can be plotted against input link position. These graphs are shown in figure 3.

International Journal of Science and Research (IJSR)
ISSN (Online): 2319-7064
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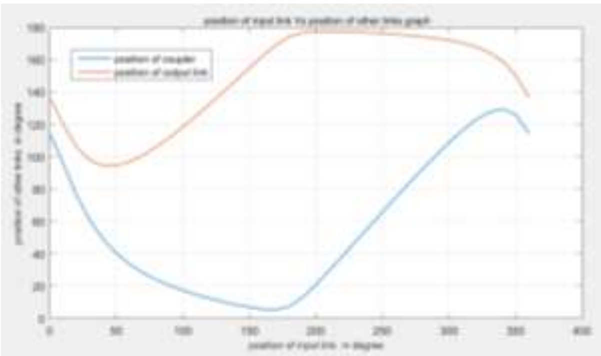
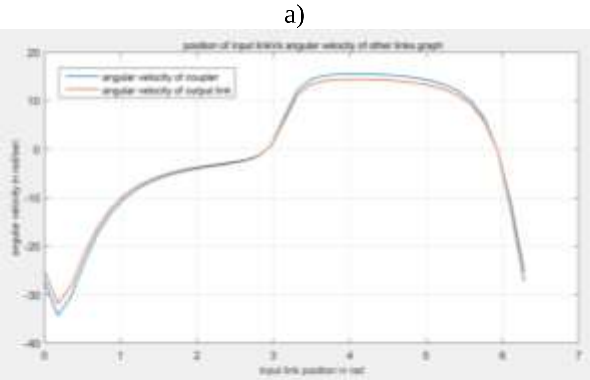
Table 2: Kinematic analysis result

θ2 in rad	θ3 in rad	θ4 in rad	ω3 in rad/sec	ω4 in rad/sec	α 3(1.0e+04 *) in rad/sec2	α 3(1.0e+03 *) in rad/sec2
0	1.9999	2.3876	-27.2639	-25.2759	0.6008	-0.612
0.1745	1.6744	2.0987	-34.3194	-31.817	0.4871	-0.4312
0.3491	1.3384	1.857	-30.3153	-28.1048	0.4724	-0.3796
0.5236	1.0654	1.7113	-23.1994	-21.5078	0.4578	-0.3569
0.6981	0.8604	1.6508	-17.2662	-16.0072	0.4249	-0.3286
0.8727	0.7069	1.651	-13.0514	-12.0997	0.3909	-0.3029
1.0472	0.5895	1.6925	-10.1453	-9.4056	0.3639	-0.2847
1.2217	0.4968	1.7621	-8.1217	-7.5295	0.3457	-0.2744
1.3963	0.4217	1.8511	-6.6814	-6.1942	0.336	-0.2716
1.5708	0.3591	1.954	-5.6306	-5.22	0.3346	-0.276
1.7453	0.3058	2.0668	-4.8441	-4.4909	0.342	-0.288
1.9199	0.2596	2.1868	-4.2383	-3.9292	0.3593	-0.3089
2.0944	0.2189	2.3121	-3.7529	-3.4793	0.3894	-0.3413
2.2689	0.1827	2.4411	-3.3368	-3.0935	0.4373	-0.39
2.4435	0.1507	2.5722	-2.9299	-2.7163	0.513	-0.4645
2.618	0.1232	2.7037	-2.4229	-2.2462	0.6365	-0.5833
2.7925	0.1025	2.8329	-1.5213	-1.4103	0.8483	-0.7845
2.9671	0.0967	2.9527	0.7684	0.7124	1.2041	-1.1199
3.1416	0.1306	3.0434	6.5082	6.0336	1.4788	-1.377
3.3161	0.2295	3.0855	12.2184	11.3275	1.1561	-1.0744
3.4907	0.3675	3.0979	14.4181	13.3668	0.7988	-0.7438
3.6652	0.5191	3.0996	15.164	14.0583	0.5923	-0.5549
3.8397	0.6755	3.097	15.4402	14.3144	0.4722	-0.4465
4.0143	0.8337	3.092	15.5265	14.3943	0.3979	-0.3807
4.1888	0.9922	3.0854	15.5102	14.3792	0.35	-0.3395
4.3633	1.1501	3.0773	15.4188	14.2945	0.3188	-0.3142
4.5379	1.3067	3.0677	15.255	14.1427	0.2996	-0.3003
4.7124	1.4612	3.0561	15.0056	13.9114	0.2897	-0.2956
4.8869	1.6127	3.0421	14.6406	13.573	0.2883	-0.2995
5.0615	1.7596	3.0248	14.1053	13.0767	0.2954	-0.3122
5.236	1.8997	3.0028	13.3008	12.331	0.3126	-0.3357
5.4105	2.0296	2.9737	12.0427	11.1646	0.3428	-0.3733
5.5851	2.1429	2.9333	9.9674	9.2407	0.3915	-0.4308
5.7596	2.2279	2.8738	6.3287	5.8673	0.4667	-0.5163
5.9341	2.2618	2.7804	-0.3685	-0.3416	0.5718	-0.6309
6.1087	2.2025	2.6267	-12.2098	-11.3195	0.6614	-0.7167
6.2832	1.9999	2.3876	-27.2639	-25.2759	0.6008	-0.612

Figure 3 below shows the graph of position of coupler and output link, angular velocity of coupler and output link and angular acceleration of coupler and output link with respect to input link position.



a)



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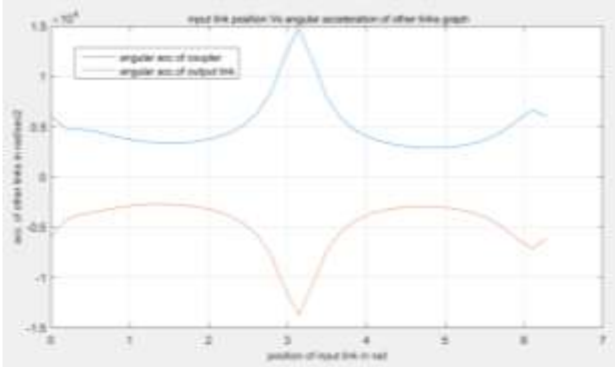

c)

Figure 3: Input link position with other links a) position graph b) angular velocity graph c) angular acceleration graph

4. Conclusion

In the first part of this paper the synthesis of four-bar linkage is shown. From each alternative deferent types of four-bar mechanism is shown. The three coupler position method is the best choice to design the four-bar mechanism. The analysis part shows the motion of the mechanism with respect to the input link position and angular velocity. The method allows to analysis the motion characteristics of any mechanism once the link lengths and the input link variables are known. The MATLAB programs written for the synthesis and analysis of four-bar linkage in this paper can be applied for any synthesis and analysis of four-bar linkage.

Author Profile

B. Diriba Hirpo is a honor of BSc degree in Industrial Engineering from Bahir Dar University of Engineering Faculty, Ethiopia and Post graduate student at Tianjin University of Technology and education, Tianjin, China.

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