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Fuzzy Estimators for Hazard Rate Function Under Mixed Quasi – Lindely

Saleemah H. Jasaim

Department of Astronomy and Space, College of Science, Baghdad University, Baghdad, Iraq

Abstract: ***The non – precise data, i.e the data with a source of uncertainty that not caused by randomness for three data fuzzy numbers is a good tool to represent these data, we compare different fuzzy estimators for hazard rate function under mixed quasi – Lindely distribution. These estimators are maximum likelihood, moments, and frequency ratio method, using different sets of initial values for $(\beta, \theta, \tilde{k})$ and $(n = 20, 40, 60, 80)$, the results are compared using statistical measures mean square error.***

Keywords: Fuzzy Estimators, Hazard Function, Quasi – Lindely, Location parameter, Scale Parameter, maximum likelihood, moments, linear regression moments

1. Introduction

We know that many applications of statistical tools and probability distribution that deals with certain dangerous disease that caused death, many researchers deals with data on life time of people who have dangerous affected by these disease like blood leukemia and different type of cancers, so here may have fuzzy data for the results of experiment, were sometimes due to the indefiniteness of the model, or imprecise observation lead to fuzzy data (Zadeh[1965]), who proposed the fuzzy sets, then more attention for fuzzy data analysis, so we introduce first some definition of fuzzy number with its membership function, then work on comparing three different estimators for hazard rate function of two parameters mixed distribution called quasiLindely (QL), which is a mixture distribution from exponential and Gamma. The p.d.f and CDF of this distribution were found, also its reliability function. The two parameters (β, θ) are estimated by methods of maximum likelihood, moments, and method of frequency ratio. We find $\left(h(x) = \frac{f(x)}{R(x)}\right)$, which is un increasing function to $(x \in R)$, i.e when $(x_1 < x_2)$, then $[h(x_1) < h(x_2)]$, many research studied this distribution were (1958) Quasi Lindely derived this distribution from mixing exponential with (θ), and Gamma with $(2, \theta)$, through mixing proportion $\left(\frac{1}{\beta+1}\right), \left(\frac{\beta}{1+\beta}\right)$. Alsoin 1970, Sankaran introduced discrete quasi Poisson, and estimate it's parameter by different methods and lindely, D.V. (1980). Many researchers like Hosking (1990) introduce Bayesian methods for estimation, also in (2004) Rausand, M. and Hoylan estimate reliability for the models with application, in (2010) Mahmoudi and Zakerzadeh introduced generalized Poisson lindely. The hazard rate functions are used in many types of statistical analysis, in engineering and medicine and economics, many researchers work on estimating hazard rate function, by parametric and non-parametric method which gives flexible estimator because no formal assumptions are put on method that generate sample which used for estimation. The increase of rates in mortality due to disease lead to negative effects of the growth of populations and communities, we know that the probability distribution is considered as statistical tool that deal with times life of patient.

2. Definition of Hazard Function

The hazard functions represent the probability of failing system during time interval $[t_1, t_2]$, such that the system is continue working until moment (t_1), and hazard rate is defined by;

$$h(t) = \lim_{\Delta t \to 0} \frac{F(t+\Delta t) - F(t)}{\Delta t} \frac{1}{s(t)} = \frac{f(t)}{s(t)}(x)$$

Also we can define hazard rate function;

$$h(t) = -\frac{ds(t)}{d(t)} \frac{1}{s(t)} = -\frac{d}{dt} \ln s(t)$$

Quasi Lindely is one of continues mixed probability distribution from exponential with (λ_1) and Gamma with $(2, \lambda_1)$ and the mixed proportion, $\left(p = \frac{\alpha}{\alpha+1}\right)$, were;

$$f(x, \alpha, \lambda_1) = p f_1(x) + (1-p) f_2(x) \quad (1)$$

Where;

$$p = \frac{\alpha}{\alpha+1}, \quad 1-p = \frac{1}{\alpha+1}$$

$$f(x, \alpha, \lambda_1) = \frac{\alpha}{\alpha+1} \lambda_1 e^{-\lambda_1 x} + \frac{1}{\alpha+1} \lambda_1^2 x\, e^{-\lambda_1 x} \quad (2)$$

$$= \frac{\lambda_1 (1+\lambda_1 x)\, e^{-\lambda_1 x}}{\alpha+1} \quad (3)$$

Where, (α) is the location parameter, and (λ_1) is the scale parameter.

The cumulative distribution function is;

$$F(x) = \int_0^x f(u) du = \frac{\lambda_1}{\alpha+1} \int_0^x \frac{(\alpha+\lambda_1 u)\, e^{-\lambda_1 u}}{\alpha+1} du \quad (4)$$

After solving equation (4), we obtain CDF which is;

$$F(x) = 1 - \frac{(1+\alpha+\lambda_1 x)\, e^{-\lambda_1 x}}{\alpha+1} \quad (5)$$

While the survival function for human and reliability function for engineering is obtained from equation (5), which is;

$$R_X(x) = S(x) = \frac{(1+\alpha+\lambda_1 x)\, e^{-\lambda_1 x}}{\alpha+1} \quad x, \lambda_1 > 0, \alpha > -1 \quad (6)$$

Also we can find risk function (hazard function),

$$h(x) = \frac{f(x)}{S(x)}$$

$$h(x) = \frac{\lambda_1(\alpha+\lambda_1 x)}{(1+\alpha+\lambda_1 x)} \quad (7)$$

in terms of (t) it is;

$$h(t) = \frac{\lambda_1(\alpha+\lambda_1 t)}{(1+\alpha+\lambda_1 t)} \quad (8)$$

if $(t_1 < t_2)$, then $[h(t_1) < h(t_2)]$

since we want to compare different estimators of fuzzy hazard rate $[\tilde{h}(t)]$.

We first estimate $(\alpha \& \lambda_1)$ by different methods like maximum likelihood, moments, and linear regression moments.

3. Methods of Estimation

3.1 Maximum Likelihood Method

Let $(t_1, t_2, \dots, t_n)$ be a random sample from p.d.f in equation (3) then;

$$L = \prod_{i=1}^{n} f(t_i, \alpha, \lambda_1)$$

$$= \left(\frac{\lambda_1}{(\alpha+1)}\right)^n \prod_{i=1}^{n} (\alpha + \lambda_1 x_i) \prod_{i=1}^{n} e^{-\lambda_1 x_i} \qquad (9)$$

Then;

$$\log L = n \log \lambda_1 - n \log(\alpha+1) + \sum_{i=1}^{n} \log(\alpha + \lambda_1 x_i) + \sum_{i=1}^{n} e^{-\lambda_1 x_i}$$

$$\frac{\partial \log L}{\partial \lambda_1} = \frac{n}{\lambda_1} + \sum_{i=1}^{n} \frac{x_i}{(\alpha + \lambda_1 x_i)} (-\lambda_1) \sum_{i=1}^{n} e^{-\lambda_1 x_i} = 0$$

$$\frac{n}{\hat{\lambda}_{1\,MLE}} = -\sum_{i=1}^{n} \frac{x_i}{(\alpha + \hat{\lambda}_1 x_i)} + \hat{\lambda}_1 \sum_{i=1}^{n} e^{-\hat{\lambda}_1 x_i} \qquad (10)$$

$$\frac{\partial \log L}{\partial \alpha} = -\frac{n}{\alpha+1} + \sum_{i=1}^{n} \frac{1}{(\alpha + \lambda_1 x_i)} = 0$$

$$\hat{\alpha}_{MLE} = \frac{n}{\sum_{i=1}^{n} (\alpha + \hat{\lambda}_1 x_i)} - 1 \qquad (11)$$

3.2 Method of Moments

For the p.d.f in equation (3), we have;

$$\mu_1' = E(x) = \frac{\alpha+2}{\lambda_1(\alpha+1)}$$

$$\mu_2' = E(x^2) = \frac{2(\alpha+3)}{\lambda_1^2(\alpha+1)}$$

Then from equating;

$$\mu_1' = \frac{\sum_{i=1}^{n} x_i}{n}$$

$$\mu_2' = \frac{\sum_{i=1}^{n} x_i^2}{n}$$

$$\hat{\lambda}_{1\,MOM} = \frac{\hat{\alpha}_{MOM} + 2}{\bar{x}(\hat{\alpha}_{MOM} + 1)} \qquad (12)$$

Then from;

$$\frac{\sum_{i=1}^{n} x_i^2}{n} = \frac{2(\hat{\alpha}+3)}{\hat{\lambda}_1^2(\hat{\alpha}+1)}$$

These reduce to;

$$\frac{\sum_{i=1}^{n} x_i^2}{n} \left[\hat{\lambda}_1^2(\hat{\alpha}+1)\right] - 2(\hat{\alpha}+3) = 0 \qquad (13)$$

Equation (13) can be solved numerically using (f solve) to obtain $(\hat{\alpha}_{MOM})$, then $(\hat{\lambda}_1)$, and estimate risk function.

3.3 Frequency Ration Estimators

The estimation by this method depend on $[f(x_1, \alpha, \lambda_1)]$ and $[f(x_2, \alpha, \lambda_1)]$, where;

$$f(x_1, \alpha, \lambda_1) = \frac{\lambda_1(\alpha + \lambda_1 x_1)}{(\alpha+1)} e^{-\lambda_1 x_1} \qquad (14)$$

$$f(x_2, \alpha, \lambda_1) = \frac{\lambda_1(\alpha + \lambda_1 x_2)}{(\alpha+1)} e^{-\lambda_1 x_2} \qquad (15)$$

From the ratio of $[f(x_1, \alpha, \lambda_1)/f(x_2, \alpha, \lambda_1)]$, we have;

$$\frac{f_1}{f_2} = \frac{(\alpha + \lambda_1 x_1)}{(\alpha + \lambda_1 x_2)} e^{-\lambda_1(x_1 - x_2)} \qquad (16)$$

$$\ln \frac{f_1}{f_2} = \ln \left(\frac{(\alpha + \lambda_1 x_1)}{(\alpha + \lambda_1 x_2)}\right) - \lambda_1(x_1 - x_2)$$

$$\ln f_1 - \ln f_1 = \ln(\alpha + \lambda_1 x_1) - \ln(\alpha + \lambda_1 x_2) - \lambda_1(x_1 - x_2)$$

$$\hat{\lambda}_1 = \frac{\ln[f_1(\alpha + \lambda_1 x_2)] - \ln[f_1(\alpha + \lambda_1 x_1)]}{(x_1 - x_2)} \qquad (17)$$

Which is an implicit function of (α, λ_1) can be solved numerically to find $(\hat{\lambda}_{1FR}, \hat{\alpha}_{FR})$.

4. Simulation

We introduce the results of simulation for hazard function after we estimate the two parameters (λ_1, α) by methods of moments, maximum likelihood, and method of Frequency Ration Estimators.

Taking $(n = 20,40,60,80)$

Table 1: Estimated values of $(\beta, \theta, \tilde{k})$.

α	1.5	2.5	
k	0.3	0.6	0.9
λ_1	0.3	0.7	1.5

Table 1: Hazard function values for $(\lambda_1 = 0.7, \alpha = 1.5, k = 0.3)$

n		Real	mle	mom	RE	Mse_mle	Mse_mom	Mse_RE	Best
20	0.35	1.16E-01	1.15E-01	0.11687	1.25E-01	9.63E-04	1.20E-03	6.90E-06	RE
	0.65	1.38E-01	1.36E-01	0.13703	1.45E-01	0.00063	7.00E-04	2.50E-05	RE
	1.15	1.51E-01	1.50E-01	0.14954	1.58E-01	0.00049	6.00E-04	4.03E-05	RE
	1.25	1.61E-01	1.61E-01	0.15812	1.67E-01	0.00040	5.00E-04	5.20E-05	RE
	1.55	1.67E-01	1.65E-01	0.16436	1.73E-01	0.00035	4.00E-04	6.06E-05	RE
	1.85	1.72E-01	1.70E-01	0.16913	1.78E-01	0.00032	4.00E-04	6.72E-05	RE
	2.15	1.76E-01	1.74E-01	0.17285	1.82E-01	0.00030	3.00E-04	7.22E-05	RE
	2.45	1.80E-01	1.77E-01	0.17588	1.85E-01	0.00028	3.00E-04	7.63E-05	RE
	2.75	1.82E-01	1.80E-01	0.17837	1.87E-01	0.00027	3.00E-04	7.95E-05	RE
	3.05	1.84E-01	1.82E-01	0.18044	1.91E-01	0.00026	3.00E-04	8.22E-05	RE
40	0.35	1.16E-01	1.15E-01	0.12101	1.21E-01	1.98E-06	3.16E-04	3.46E-04	MLE
	0.65	1.38E-01	1.37E-01	0.14201	1.42E-01	9.03E-06	1.94E-04	2.05E-04	MLE
	1.05	1.51E-01	1.50E-01	0.15601	1.55E-01	1.53E-05	1.38E-04	1.43E-04	MLE
	1.25	1.61E-01	1.60E-01	0.16475	1.64E-01	2.03E-05	1.08E-04	1.11E-04	MLE
	1.55	1.67E-01	1.66E-01	0.17117	1.70E-01	2.40E-05	9.12E-05	9.26E-05	MOM
	1.85	1.72E-01	1.71E-01	0.17606	1.75E-01	2.70E-05	7.96E-05	8.04E-05	MLE

	2.15	1.76E-01	1.75E-01	0.17987	1.80E-01	2.92E-05	7.16E-05	7.22E-05	MLE
	2.45	1.80E-01	1.78E-01	0.18305	1.82E-01	3.10E-05	6.60E-05	6.62E-05	MLE
	2.75	1.82E-01	1.81E-01	0.18551	1.85E-01	3.25E-05	6.16E-05	6.18E-05	MLE
	3.05	1.84E-01	1.83E-01	0.18764	1.87E-01	3.38E-05	5.83E-05	5.84E-05	MLE
60	0.35	1.16E-01	1.17E-01	0.12011	1.20E-01	7.65E-06	1.32E-04	1.94E-04	MLE
	0.65	1.38E-01	1.38E-01	0.14171	1.41E-01	4.50E-06	9.35E-05	1.23E-04	MLE
	1.05	1.51E-01	1.52E-01	0.15503	1.54E-01	2.91E-06	7.33E-05	9.05E-05	MLE
	1.25	1.61E-01	1.61E-01	0.16406	1.63E-01	2.02E-06	6.16E-05	7.27E-05	MLE
	1.55	1.67E-01	1.68E-01	0.17063	1.70E-01	1.45E-06	5.43E-05	6.20E-05	MLE
	1.85	1.72E-01	1.73E-01	0.17558	1.75E-01	1.10E-06	4.93E-05	5.50E-05	MLE
	2.15	1.76E-01	1.77E-01	0.17945	1.81E-01	8.64E-07	4.58E-05	5.02E-05	MLE
	2.45	1.80E-01	1.80E-01	0.18260	1.82E-01	6.92E-07	4.32E-05	4.66E-05	MLE
	2.75	1.82E-01	1.82E-01	0.18518	1.84E-01	5.66E-07	4.12E-05	4.40E-05	MLE
	3.05	1.84E-01	1.85E-01	0.18731	1.86E-01	4.71E-07	3.96E-05	4.21E-05	MLE
80	0.35	1.16E-01	0.11651	1.17E-01	1.18E-01	5.41E-08	1.36E-05	3.85E-05	MLE
	0.65	1.38E-01	0.13847	1.41E-01	1.41E-01	6.47E-07	5.77E-06	1.58E-05	MLE
	1.04	1.51E-01	0.15223	1.52E-01	1.52E-01	1.32E-06	2.71E-06	7.42E-06	MLE
	1.25	1.61E-01	0.16121	1.61E-01	1.61E-01	1.92E-06	1.34E-06	3.78E-06	MOM
	1.55	1.67E-01	0.16784	1.68E-01	1.68E-01	1.01E-06	6.91E-07	2.04E-06	MOM
	1.85	1.72E-01	0.17288	1.73E-01	1.73E-01	8.14E-07	3.61E-07	1.13E-06	MOM
	2.15	1.76E-01	0.17671	1.77E-01	1.77E-01	1.43E-07	1.81E-07	6.41E-07	RE
	2.45	1.80E-01	0.18005	1.80E-01	1.80E-01	4.16E-07	8.01E-08	3.61E-07	MOM
	2.75	1.82E-01	0.18257	1.82E-01	1.82E-01	6.47E-07	3.01E-08	2.01E-07	MOM
	3.05	1.84E-01	0.18476	1.84E-01	1.87E-01	8.44E-07	1.01E-08	1.01E-07	MOM

Table 2: Hazard function values for($\lambda_1 = 0.7, \alpha = 1.5, k = 0.6$)

n		Real	mle	mom	RE	Mse_mle	Mse_mom	Mse_RE	Best
20	1.7	6.10E-01	0.6068	6.36E-01	6.41E-01	1.05E-05	8.15E-04	8.71E-04	MLE
	2.7	6.74E-01	0.6697	6.96E-01	6.97E-01	3.04E-05	4.91E-04	5.06E-04	MLE
	3.7	7.16E-01	0.7095	7.34E-01	7.34E-01	4.73E-05	3.38E-04	3.45E-04	MLE
	4.6	7.44E-01	0.7371	7.61E-01	7.61E-01	6.04E-05	2.58E-04	2.62E-04	MLE
	5.8	7.65E-01	0.7573	7.81E-01	7.81E-01	7.04E-05	2.12E-04	2.13E-04	MLE
	6.7	7.81E-01	0.7727	7.94E-01	7.94E-01	7.81E-05	1.81E-04	1.81E-04	MLE
	7.6	7.93E-01	0.7851	8.06E-01	8.06E-01	8.42E-05	1.58E-04	1.58E-04	MLE
	8.7	8.03E-01	0.7948	8.15E-01	8.15E-01	8.92E-05	1.45E-04	1.42E-04	MLE
	9.8	8.12E-01	0.8030	8.23E-01	8.23E-01	9.32E-05	1.31E-04	1.33E-04	MLE
	10.7	8.20E-01	0.8097	8.32E-01	8.32E-01	9.65E-05	1.23E-04	1.24E-04	MLE
40	1.7	6.11E-01	0.6102	6.31E-01	6.33E-01	1.01E-08	3.98E-04	5.52E-04	MLE
	2.7	6.74E-01	0.6741	6.92E-01	6.93E-01	1.13E-06	2.72E-04	3.42E-04	MLE
	3.7	7.16E-01	0.7146	7.32E-01	7.31E-01	3.18E-06	2.08E-04	2.48E-04	MLE
	4.6	7.44E-01	0.7426	7.57E-01	7.58E-01	5.10E-06	1.73E-04	1.98E-04	MLE
	5.8	7.65E-01	0.7631	7.77E-01	7.78E-01	6.68E-06	1.51E-04	1.68E-04	MLE
	6.7	7.81E-01	0.7787	7.92E-01	7.93E-01	7.97E-06	1.36E-04	1.48E-04	MLE
	7.6	7.93E-01	0.7911	8.04E-01	8.05E-01	9.02E-06	1.26E-04	1.35E-04	MLE
	8.7	8.03E-01	0.8011	8.14E-01	8.15E-01	9.87E-06	1.18E-04	1.25E-04	MLE
	9.8	8.12E-01	0.8093	8.22E-01	8.23E-01	1.05E-05	1.12E-04	1.18E-04	MLE
	10.7	8.20E-01	0.8162	8.31E-01	8.32E-01	1.13E-05	1.08E-04	1.13E-04	MLE
60	1.7	6.11E-01	0.6114	6.21E-01	6.25E-01	1.95E-06	1.32E-04	2.54E-04	MLE
	2.7	6.74E-01	0.6761	6.84E-01	6.87E-01	5.56E-07	8.94E-05	1.48E-04	MLE
	3.7	7.16E-01	0.7167	7.24E-01	7.26E-01	1.02E-07	6.83E-05	1.03E-04	MLE
	4.6	7.44E-01	0.7451	7.52E-01	7.53E-01	1.01E-09	5.62E-05	7.81E-05	MLE
	5.8	7.65E-01	0.7655	7.72E-01	7.73E-01	3.80E-08	4.86E-05	6.36E-05	MLE
	6.7	7.81E-01	0.7812	7.87E-01	7.88E-01	1.26E-07	4.35E-05	5.44E-05	MLE
	7.6	7.93E-01	0.7936	8.02E-01	8.02E-01	2.32E-07	4.01E-05	4.82E-05	MLE
	8.7	8.03E-01	0.8036	8.11E-01	8.12E-01	3.41E-07	3.72E-05	4.37E-05	MLE
	9.8	8.12E-01	0.8121	8.18E-01	8.18E-01	4.43E-07	3.52E-05	4.04E-05	MLE
	10.7	8.21E-01	0.8188	8.27E-01	8.25E-01	5.78E-07	3.36E-05	3.81E-05	MLE
80	1.7	3.11E-01	0.3111	3.15E-01	3.21E-01	8.05E-07	3.76E-05	1.46E-04	MLE
	2.7	3.74E-01	0.3758	3.82E-01	3.83E-01	4.43E-07	2.72E-05	7.97E-05	MLE
	3.7	4.16E-01	0.4171	4.22E-01	4.23E-01	2.82E-07	2.16E-05	5.18E-05	MLE
	4.6	4.44E-01	0.4453	4.48E-01	4.52E-01	1.98E-07	1.86E-05	3.10E-05	MLE
	5.8	4.65E-01	0.4661	4.71E-01	4.72E-01	1.52E-07	1.66E-05	3.01E-05	MLE
	6.7	4.83E-01	0.4821	4.85E-01	4.86E-01	1.24E-07	1.54E-05	2.53E-05	MLE
	7.6	4.93E-01	0.4944	4.97E-01	4.98E-01	1.05E-07	1.44E-05	2.23E-05	MLE
	8.7	5.03E-01	0.5045	5.07E-01	5.08E-01	9.33E-08	1.41E-05	2.01E-05	MLE
	9.8	5.12E-01	0.5131	5.15E-01	5.16E-01	8.42E-08	1.45E-05	1.83E-05	MLE
	10.7	5.21E-01	0.5201	5.22E-01	5.23E-01	7.73E-08	1.31E-05	1.73E-05	MLE

Table 3: Hazard function values for$(\lambda_1 = 0.7, \alpha = 1.5, k = 0.9)$

n		Real	mle	mom	RE	Mse_mle	Mse_mom	Mse_RE	Best
20	0.0486	5.02E-01	5.28E-01	5.48E-01	3.77E-01	0.000823	0.002317	1.543E-02	RE
	0.1160	5.03E-01	0.530561	0.549828	0.380805	0.000851	0.002334	0.014567	MLE
	0.1480	5.03E-01	0.532232	0.551113	0.386033	0.000923	0.002423	0.013481	MLE
	0.2290	5.04E-01	0.533565	0.552207	0.390687	0.000961	0.002461	0.012541	MLE
	0.2490	5.03E-01	0.534782	0.553333	0.394708	0.001006	0.002526	0.011721	MLE
	0.2620	5.04E-01	0.536191	0.554761	0.399112	0.001086	0.002656	0.010881	MLE
	0.4530	5.06E-01	0.537703	0.556273	0.404002	0.001042	0.002586	0.010241	MLE
	0.4590	5.04E-01	0.539357	0.557897	0.40943	0.001148	0.002748	0.009223	MLE
	0.7500	5.08E-01	0.541387	0.55966	0.415207	0.001066	0.002594	0.008746	MLE
	0.7620	5.07E-01	0.54323	0.561463	0.420597	0.001188	0.002767	0.007787	MLE
40	0.1730	5.02E-01	5.18E-01	5.36E-01	4.12E-01	2.87E-04	1.18E-03	7.90E-03	MLE
	0.2033	5.23E-01	5.21E-01	5.36E-01	4.14E-01	0.000295	0.001197	0.007698	MLE
	0.2284	5.02E-01	5.21E-01	5.37E-01	4.16E-01	0.000310	0.001215	0.007378	MLE
	0.2751	5.02E-01	5.21E-01	5.37E-01	4.18E-01	0.000316	0.001221	0.007105	MLE
	0.3046	5.03E-01	5.21E-01	5.38E-01	4.20E-01	0.000327	0.001235	0.006904	MLE
	0.3223	5.03E-01	5.21E-01	5.38E-01	4.21E-01	0.000341	0.001253	0.006671	MLE
	0.3523	5.03E-01	5.22E-01	5.41E-01	4.23E-01	0.000351	0.001265	0.006476	MLE
	0.3665	5.03E-01	5.23E-01	5.41E-01	4.24E-01	0.000370	0.001292	0.006240	MLE
	0.4322	5.04E-01	5.23E-01	5.40E-01	4.26E-01	0.000370	0.001286	0.006048	MLE
	0.4432	5.04E-01	5.24E-01	5.41E-01	4.28E-01	0.000394	0.001328	0.005776	MLE
60	0.0527	5.01E-01	4.97E-01	5.08E-01	4.13E-01	7.64E-06	5.80E-05	7.71E-03	MLE
	0.0727	5.01E-01	4.97E-01	5.09E-01	4.14E-01	6.67E-06	6.00E-05	0.007569	MLE
	0.1488	5.02E-01	4.98E-01	5.09E-01	4.15E-01	1.02E-05	5.05E-05	0.007603	MLE
	0.1632	5.02E-01	4.98E-01	5.09E-01	4.16E-01	8.72E-06	5.30E-05	0.007464	MLE
	0.2643	5.03E-01	4.98E-01	5.10E-01	4.17E-01	1.37E-05	4.17E-05	0.007485	MLE
	0.3811	5.04E-01	5.00E-01	5.10E-01	4.18E-01	2.15E-05	2.98E-05	0.007553	MLE
	0.4065	5.05E-01	5.00E-01	5.10E-01	4.19E-01	1.99E-05	3.17E-05	0.007406	MLE
	0.4935	5.06E-01	5.01E-01	5.11E-01	4.20E-01	2.57E-05	2.50E-05	0.007411	MLE
	0.5956	5.07E-01	5.01E-01	5.11E-01	4.21E-01	3.37E-05	1.79E-05	0.007441	MLE
	0.6435	5.07E-01	5.02E-01	5.12E-01	4.22E-01	3.47E-05	1.66E-05	0.007353	MLE
80	0.0031	0.500051	5.09E-01	5.06E-01	4.61E-01	8.17E-05	3.31E-05	1.51E-03	MOM
	0.0132	0.500171	5.09E-01	5.06E-01	4.62E-01	8.55E-05	3.57E-05	1.47E-03	MOM
	0.0379	0.500466	5.10E-01	5.06E-01	4.62E-01	8.60E-05	3.62E-05	1.45E-03	MOM
	0.0489	0.500597	5.10E-01	5.07E-01	4.63E-01	8.89E-05	3.84E-05	1.42E-03	MOM
	0.0548	0.500667	5.10E-01	5.07E-01	4.63E-01	9.40E-05	4.22E-05	1.39E-03	MOM
	0.0685	0.500831	5.11E-01	5.07E-01	4.64E-01	9.68E-05	4.42E-05	1.37E-03	MOM
	0.0703	0.500841	5.11E-01	5.08E-01	4.64E-01	1.03E-04	4.84E-05	1.33E-03	MOM
	0.0833	0.501007	5.11E-01	5.08E-01	4.65E-01	1.06E-04	5.10E-05	1.30E-03	MOM
	0.0937	0.501131	5.12E-01	5.08E-01	4.65E-01	1.10E-04	5.35E-05	1.27E-03	MOM
	0.1110	0.501331	5.12E-01	5.09E-01	4.66E-01	1.12E-04	5.51E-05	1.24E-03	MOM

Table 4: Hazard function values for$(\lambda_1 = 0.3, \alpha = 2.5, k = 0.6)$

n		Real	mle	mom	RE	Mse_mle	Mse_mom	Mse_RE	Best
20	0.0792	0.501875	5.32E-01	5.59E-01	3.83E-01	8.90E-04	3.27E-03	1.40E-02	MLE
	0.1521	0.503569	5.34E-01	5.61E-01	3.91E-01	9.31E-04	3.30E-03	1.26E-02	MLE
	0.2100	0.504911	5.37E-01	5.63E-01	4.02E-01	1.05E-03	3.40E-03	1.07E-02	MLE
	0.3600	0.508512	5.40E-01	5.65E-01	4.09E-01	9.85E-04	3.19E-03	9.91E-03	MLE
	0.3740	0.508816	5.43E-01	5.67E-01	4.17E-01	1.14E-03	3.42E-03	8.36E-03	MLE
	0.5100	0.511537	5.46E-01	5.70E-01	4.26E-01	1.17E-03	3.38E-03	7.30E-03	MLE
	0.6450	0.514584	5.49E-01	5.72E-01	4.34E-01	1.15E-03	3.27E-03	6.56E-03	MLE
	0.7680	0.517084	5.51E-01	5.74E-01	4.42E-01	1.16E-03	3.22E-03	5.70E-03	MLE
	0.7690	0.517101	5.54E-01	5.76E-01	4.49E-01	1.36E-03	3.51E-03	4.63E-03	MLE
	1.1200	0.523760	5.57E-01	5.79E-01	4.57E-01	1.10E-03	3.02E-03	4.51E-03	MLE
40	0.0452	0.501081	5.17E-01	5.28E-01	4.29E-01	2.64E-04	7.11E-04	5.25E-03	MLE
	0.0823	0.501958	5.18E-01	5.29E-01	4.31E-01	2.69E-04	7.20E-04	5.00E-03	MLE
	0.0862	0.502052	5.19E-01	5.30E-01	4.34E-01	3.01E-04	7.63E-04	4.66E-03	MLE
	0.1790	0.50422	5.21E-01	5.31E-01	4.37E-01	2.71E-04	7.07E-04	4.52E-03	MLE
	0.2000	0.504697	5.22E-01	5.32E-01	4.39E-01	2.91E-04	7.31E-04	4.27E-03	MLE
	0.2030	0.50476	5.23E-01	5.33E-01	4.42E-01	3.35E-04	7.90E-04	3.89E-03	MLE
	0.2060	0.504836	5.24E-01	5.34E-01	4.45E-01	3.85E-04	8.70E-04	3.55E-03	MLE
	0.3460	0.507996	5.26E-01	5.36E-01	4.48E-01	3.18E-04	7.67E-04	3.59E-03	MLE
	0.3420	0.508336	5.27E-01	5.37E-01	4.51E-01	3.56E-04	8.19E-04	3.27E-03	MLE
	0.4020	0.509221	5.29E-01	5.38E-01	4.54E-01	3.74E-04	8.43E-04	3.03E-03	MLE
	0.0262	0.500628	5.13E-01	5.22E-01	4.58E-01	1.43E-04	4.60E-04	1.84E-03	MLE
	0.0351	0.500840	5.13E-01	5.23E-01	4.59E-01	1.58E-04	4.84E-04	1.72E-03	MLE

60	0.0662	0.501577	5.14E-01	5.24E-01	4.61E-01	1.61E-04	4.86E-04	1.64E-03	MLE
	0.0707	0.501683	5.15E-01	5.24E-01	4.63E-01	1.77E-04	5.10E-04	1.53E-03	MLE
	0.0875	0.502078	5.16E-01	5.25E-01	4.64E-01	1.89E-04	5.29E-04	1.45E-03	MLE
	0.0887	0.502107	5.17E-01	5.26E-01	4.66E-01	2.12E-04	5.65E-04	1.32E-03	MLE
	0.1080	0.502564	5.18E-01	5.27E-01	4.67E-01	2.24E-04	5.82E-04	1.23E-03	MLE
	0.1180	0.502801	5.18E-01	5.28E-01	4.69E-01	2.43E-04	6.10E-04	1.14E-03	MLE
	0.1220	0.502884	5.19E-01	5.28E-01	4.71E-01	2.69E-04	6.49E-04	1.04E-03	MLE
	0.1680	0.503948	5.20E-01	5.29E-01	4.72E-01	2.67E-04	6.42E-04	1.01E-03	MLE
80	0.0034	0.500083	5.00E-01	5.00E-01	4.49E-01	1.54E-08	1.67E-07	2.59E-03	MLE
	0.0151	0.500361	5.01E-01	5.00E-01	4.50E-01	1.26E-07	4.13E-08	2.52E-03	MLE
	0.0277	0.500663	5.01E-01	5.01E-01	4.51E-01	3.53E-07	1.09E-09	2.45E-03	MLE
	0.0472	0.501127	5.02E-01	5.01E-01	4.52E-01	3.91E-07	4.29E-09	2.39E-03	MLE
	0.0824	0.50195	5.02E-01	5.02E-01	4.53E-01	1.49E-07	1.01E-08	2.37E-03	MLE
	0.0941	0.502231	5.03E-01	5.02E-01	4.55E-01	5.31E-07	5.62E-08	2.28E-03	MLE
	0.1650	0.503876	5.04E-01	5.03E-01	4.55E-01	1.18E-07	6.38E-07	2.35E-03	MLE
	0.1970	0.504618	5.04E-01	5.04E-01	4.57E-01	1.99E-07	8.39E-07	2.30E-03	MLE
	0.2290	0.505347	5.05E-01	5.04E-01	4.58E-01	3.51E-07	1.11E-06	2.27E-03	MLE
	0.2350	0.505474	5.05E-01	5.05E-01	4.59E-01	1.97E-08	3.88E-07	2.19E-03	MLE

Table 5: Hazard function values for $(\lambda_1 = 0.7, \alpha = 2.5, k = 0.6)$

n		Real	mle	mom	RE	Mse_mle	Mse_mom	Mse_RE	Best
20	0.0670	4.69E-01	0.491671	5.18E-01	3.57E-01	5.21E-04	2.41E-03	1.25E-02	MLE
	0.0826	4.69E-01	0.495357	5.21E-01	3.66E-01	6.76E-04	2.64E-03	1.06E-02	MLE
	0.2400	4.74E-01	0.498891	5.24E-01	3.75E-01	6.07E-04	2.43E-03	9.86E-03	MLE
	0.6420	4.86E-01	0.501971	5.26E-01	3.83E-01	2.57E-04	1.62E-03	1.05E-02	MLE
	0.6810	4.87E-01	0.506153	5.29E-01	3.94E-01	3.68E-04	1.80E-03	8.70E-03	MLE
	1.2400	5.01E-01	0.510174	5.33E-01	4.04E-01	8.00E-05	1.02E-03	9.50E-03	MLE
	1.2500	5.01E-01	0.514397	5.36E-01	4.13E-01	1.69E-04	1.21E-03	7.82E-03	MLE
	1.3000	5.03E-01	0.518132	5.40E-01	4.22E-01	2.39E-04	1.40E-03	6.44E-03	MLE
	1.3700	5.04E-01	0.522615	5.44E-01	4.32E-01	3.39E-04	1.56E-03	5.20E-03	MLE
	2.2300	5.22E-01	0.526951	5.47E-01	4.41E-01	2.28E-05	6.16E-04	6.55E-03	MLE
40	0.0200	4.67E-01	0.478642	4.89E-01	3.85E-01	1.28E-04	4.83E-04	6.74E-03	MLE
	0.0347	4.68E-01	0.480275	4.91E-01	3.89E-01	1.56E-04	5.26E-04	6.24E-03	MLE
	0.0785	4.69E-01	0.482243	4.92E-01	3.93E-01	1.70E-04	5.39E-04	5.80E-03	MLE
	0.1650	4.72E-01	0.484052	4.94E-01	3.97E-01	1.46E-04	4.92E-04	5.58E-03	MLE
	0.1930	4.73E-01	0.485664	4.96E-01	4.01E-01	1.65E-04	5.20E-04	5.21E-03	MLE
	0.2030	4.73E-01	0.487627	4.97E-01	4.05E-01	2.10E-04	5.92E-04	4.71E-03	MLE
	0.3100	4.76E-01	0.489242	4.99E-01	4.08E-01	1.65E-04	5.08E-04	4.71E-03	MLE
	0.4030	4.79E-01	0.491126	5.01E-01	4.11E-01	1.43E-04	4.62E-04	4.60E-03	MLE
	0.4900	4.82E-01	0.492622	5.02E-01	4.15E-01	1.21E-04	4.17E-04	4.51E-03	MLE
	0.5270	4.83E-01	0.494359	5.04E-01	4.18E-01	1.36E-04	4.35E-04	4.19E-03	MLE
60	0.0965	4.70E-01	0.469355	4.71E-01	4.16E-01	1.96E-07	2.19E-06	2.87E-03	MLE
	0.1430	4.71E-01	0.470655	4.73E-01	4.18E-01	3.07E-07	1.78E-06	2.79E-03	MLE
	0.1750	4.72E-01	0.471682	4.74E-01	4.20E-01	2.68E-07	1.88E-06	2.68E-03	MLE
	0.1860	4.73E-01	0.473078	4.75E-01	4.23E-01	2.14E-07	5.60E-06	2.50E-03	MLE
	0.2270	4.74E-01	0.474161	4.76E-01	4.24E-01	8.72E-08	4.85E-06	2.46E-03	MLE
	0.2330	4.74E-01	0.475423	4.77E-01	4.26E-01	1.88E-06	1.08E-05	2.28E-03	MLE
	0.3140	4.77E-01	0.476468	4.78E-01	4.28E-01	3.38E-09	3.60E-06	2.36E-03	MLE
	0.3170	4.77E-01	0.47758	4.80E-01	4.30E-01	1.12E-06	1.02E-05	2.16E-03	MLE
	0.3300	4.77E-01	0.478931	4.81E-01	4.32E-01	3.84E-06	1.79E-05	2.00E-03	MLE
	0.3870	4.79E-01	0.480154	4.82E-01	4.34E-01	2.13E-06	1.31E-05	1.97E-03	MLE
80	0.0230	4.67E-01	0.474738	4.87E-01	4.38E-01	5.32E-05	3.75E-04	8.86E-04	MLE
	0.0252	4.68E-01	0.475592	4.88E-01	4.39E-01	6.52E-05	4.02E-04	8.10E-04	MLE
	0.0838	4.69E-01	0.476454	4.88E-01	4.40E-01	4.97E-05	3.58E-04	8.36E-04	MLE
	0.0929	4.70E-01	0.477258	4.89E-01	4.42E-01	5.72E-05	3.75E-04	7.85E-04	MLE
	0.1200	4.71E-01	0.478261	4.90E-01	4.43E-01	5.98E-05	3.78E-04	7.41E-04	MLE
	0.1580	4.72E-01	0.479214	4.91E-01	4.45E-01	5.60E-05	3.64E-04	7.31E-04	MLE
	0.1720	4.72E-01	0.480092	4.92E-01	4.46E-01	6.24E-05	3.78E-04	6.88E-04	MLE
	0.1780	4.72E-01	0.480972	4.92E-01	4.47E-01	7.42E-05	4.00E-04	6.27E-04	MLE
	0.2980	4.76E-01	0.481891	4.93E-01	4.49E-01	3.44E-05	2.95E-04	7.58E-04	MLE
	0.3020	4.76E-01	0.482766	4.94E-01	4.50E-01	4.35E-05	3.17E-04	7.02E-04	MLE

Table 6: Hazard function values for $(\lambda_1 = 0.7, \alpha = 2.5, k = 0.9)$

n		Real	mle	mom	RE	Mse_mle	Mse_mom	Mse_RE	Best
	0.1020	5.02E-01	0.534465	5.60E-01	3.97E-01	1.03E-03	3.32E-03	1.12E-02	MLE
	0.1310	5.03E-01	0.536992	5.62E-01	4.05E-01	1.15E-03	3.48E-03	9.71E-03	MLE
	0.2020	5.05E-01	0.539653	5.64E-01	4.13E-01	1.22E-03	3.55E-03	8.48E-03	MLE

20	0.3180	5.07E-01	0.542337	5.66E-01	4.20E-01	1.22E-03	3.49E-03	7.59E-03	MLE
	0.3710	5.09E-01	0.544743	5.69E-01	4.27E-01	1.31E-03	3.60E-03	6.60E-03	MLE
	0.4750	5.11E-01	0.547595	5.71E-01	4.34E-01	1.35E-03	3.59E-03	5.83E-03	MLE
	0.6680	5.15E-01	0.550793	5.73E-01	4.43E-01	1.29E-03	3.41E-03	5.10E-03	MLE
	0.6730	5.15E-01	0.553348	5.75E-01	4.50E-01	1.47E-03	3.64E-03	4.18E-03	MLE
	0.7350	5.16E-01	0.556316	5.78E-01	4.58E-01	1.61E-03	3.80E-03	3.40E-03	MLE
	1.0820	5.23E-01	0.560071	5.81E-01	4.67E-01	1.38E-03	3.34E-03	3.11E-03	MLE
40	0.0637	5.02E-01	0.519484	5.28E-01	4.12E-01	3.22E-04	7.14E-04	8.06E-03	MLE
	0.2060	5.05E-01	0.520622	5.29E-01	4.15E-01	2.49E-04	5.94E-04	8.14E-03	MLE
	0.3020	5.07E-01	0.522141	5.30E-01	4.19E-01	2.31E-04	5.53E-04	7.79E-03	MLE
	0.3070	5.07E-01	0.523316	5.32E-01	4.21E-01	2.62E-04	5.99E-04	7.35E-03	MLE
	0.3340	5.08E-01	0.524483	5.33E-01	4.24E-01	2.74E-04	6.18E-04	6.99E-03	MLE
	0.3600	5.09E-01	0.525902	5.34E-01	4.29E-01	3.03E-04	6.53E-04	6.40E-03	MLE
	0.4200	5.10E-01	0.527188	5.35E-01	4.32E-01	3.02E-04	6.56E-04	6.04E-03	MLE
	0.4330	5.10E-01	0.528382	5.36E-01	4.35E-01	3.41E-04	7.06E-04	5.58E-03	MLE
	0.5240	5.12E-01	0.529807	5.38E-01	4.39E-01	3.23E-04	6.88E-04	5.29E-03	MLE
	0.5680	5.13E-01	0.531177	5.39E-01	4.42E-01	3.40E-04	7.06E-04	4.99E-03	MLE
60	0.0062	5.00E-01	0.512966	5.24E-01	4.40E-01	1.64E-04	5.48E-04	3.59E-03	MLE
	0.0120	5.00E-01	0.513796	5.24E-01	4.42E-01	1.83E-04	5.78E-04	3.39E-03	MLE
	0.0150	5.00E-01	0.514391	5.25E-01	4.43E-01	1.98E-04	6.00E-04	3.26E-03	MLE
	0.0170	5.00E-01	0.515208	5.26E-01	4.45E-01	2.21E-04	6.34E-04	3.06E-03	MLE
	0.0340	5.01E-01	0.516013	5.26E-01	4.47E-01	2.30E-04	6.43E-04	2.90E-03	MLE
	0.0645	5.02E-01	0.516954	5.27E-01	4.49E-01	2.36E-04	6.45E-04	2.73E-03	MLE
	0.0721	5.02E-01	0.517695	5.28E-01	4.51E-01	2.56E-04	6.78E-04	2.60E-03	MLE
	0.0835	5.02E-01	0.518343	5.28E-01	4.52E-01	2.68E-04	6.97E-04	2.48E-03	MLE
	0.0850	5.02E-01	0.519195	5.29E-01	4.54E-01	2.96E-04	7.37E-04	2.32E-03	MLE
	0.1070	5.03E-01	0.520158	5.30E-01	4.56E-01	3.10E-04	7.56E-04	2.16E-03	MLE
80	0.0000	5.00E-01	0.50456	5.10E-01	4.56E-01	2.09E-05	9.40E-05	1.90E-03	MLE
	0.0241	5.01E-01	0.50512	5.10E-01	4.57E-01	2.03E-05	9.17E-05	1.88E-03	MLE
	0.0280	5.01E-01	0.50583	5.11E-01	4.59E-01	2.63E-05	1.03E-04	1.78E-03	MLE
	0.0316	5.01E-01	0.506438	5.11E-01	4.60E-01	3.21E-05	1.13E-04	1.70E-03	MLE
	0.1080	5.03E-01	0.507033	5.12E-01	4.60E-01	1.96E-05	8.76E-05	1.78E-03	MLE
	0.1430	5.03E-01	0.507524	5.12E-01	4.61E-01	1.75E-05	8.30E-05	1.75E-03	MLE
	0.1760	5.04E-01	0.508228	5.13E-01	4.63E-01	1.65E-05	7.98E-05	1.73E-03	MLE
	0.1880	5.04E-01	0.508784	5.14E-01	4.64E-01	1.89E-05	8.48E-05	1.66E-03	MLE
	0.2230	5.05E-01	0.509438	5.14E-01	4.65E-01	1.82E-05	8.20E-05	1.64E-03	MLE
	0.2260	5.05E-01	0.510082	5.15E-01	4.66E-01	2.30E-05	9.01E-05	1.56E-03	MLE

Table 7: Hazard function values for$(\lambda_1 = 1.5, \alpha = 1.5, k = 0.3)$

Best	Mse_RE	Mse_mom	Mse_mle	RE	mom	mle	Real		n
20	0.0508	4.24E-01	0.434824	4.55E-01	3.05E-01	1.27E-04	9.90E-04	1.41E-02	MLE
	0.2	4.33E-01	0.442932	4.62E-01	3.22E-01	9.06E-05	7.96E-04	1.25E-02	MLE
	0.227	4.35E-01	0.450692	4.68E-01	3.38E-01	2.40E-04	1.09E-03	9.37E-03	MLE
	0.481	4.50E-01	0.457632	4.75E-01	3.53E-01	5.43E-05	5.98E-04	9.37E-03	MLE
	0.523	4.53E-01	0.464586	4.81E-01	3.68E-01	1.44E-04	8.02E-04	7.18E-03	MLE
	0.766	4.65E-01	0.472913	4.88E-01	3.84E-01	5.72E-05	5.14E-04	6.66E-03	MLE
	0.773	4.66E-01	0.480402	4.95E-01	3.98E-01	2.16E-04	8.44E-04	4.53E-03	MLE
	0.971	4.75E-01	0.488561	5.02E-01	4.14E-01	1.83E-04	7.52E-04	3.73E-03	MLE
	1.05	4.78E-01	0.495583	5.08E-01	4.26E-01	2.93E-04	8.97E-04	2.79E-03	MLE
	1.3	4.89E-01	0.502298	5.15E-01	4.38E-01	1.76E-04	6.80E-04	2.57E-03	MLE
40	0.352	7.24E-01	0.734824	7.55E-01	6.05E-01	3.00E-01	3.01E-01	3.14E-01	MLE
	0.5	7.33E-01	0.742932	7.62E-01	6.22E-01	3.00E-01	3.01E-01	3.12E-01	MLE
	0.527	7.35E-01	0.750694	7.68E-01	6.38E-01	3.00E-01	3.01E-01	3.09E-01	MLE
	0.782	7.50E-01	0.757635	7.75E-01	6.53E-01	3.00E-01	3.01E-01	3.09E-01	MLE
	0.823	7.53E-01	0.764586	7.81E-01	6.68E-01	3.00E-01	3.01E-01	3.07E-01	MLE
	1.07	7.65E-01	0.772915	7.88E-01	6.84E-01	3.00E-01	3.01E-01	3.07E-01	MLE
	1.07	7.66E-01	0.780402	7.95E-01	6.98E-01	3.00E-01	3.01E-01	3.05E-01	MLE
	1.26	7.75E-01	0.788562	8.02E-01	7.14E-01	3.00E-01	3.01E-01	3.04E-01	MLE
	1.34	7.78E-01	0.795583	8.08E-01	7.26E-01	3.00E-01	3.01E-01	3.03E-01	MLE
	1.6	7.89E-01	0.802298	8.15E-01	7.38E-01	3.00E-01	3.01E-01	3.03E-01	MLE
60	0.0295	4.22E-01	0.427531	4.34E-01	3.64E-01	2.99E-05	1.46E-04	3.34E-03	MLE
	0.0622	4.24E-01	0.430528	4.37E-01	3.69E-01	3.83E-05	1.61E-04	3.03E-03	MLE
	0.0742	4.25E-01	0.433081	4.40E-01	3.74E-01	6.31E-05	2.12E-04	2.65E-03	MLE
	0.108	4.27E-01	0.435867	4.42E-01	3.78E-01	7.13E-05	2.24E-04	2.47E-03	MLE
	0.115	4.28E-01	0.43853	4.45E-01	3.82E-01	1.13E-04	2.89E-04	2.10E-03	MLE
	0.121	4.28E-01	0.441676	4.48E-01	3.87E-01	1.79E-04	3.78E-04	1.74E-03	MLE
	0.14	4.30E-01	0.444329	4.50E-01	3.91E-01	2.19E-04	4.22E-04	1.51E-03	MLE

	0.166	4.31E-01	0.446837	4.52E-01	3.95E-01	2.42E-04	4.40E-04	1.35E-03	MLE
	0.195	4.33E-01	0.449553	4.55E-01	3.99E-01	2.69E-04	4.76E-04	1.19E-03	MLE
	0.202	4.34E-01	0.451959	4.57E-01	4.02E-01	3.38E-04	5.63E-04	9.80E-04	MLE
80	0.0343	4.22E-01	0.420042	4.25E-01	3.66E-01	5.52E-06	9.18E-06	3.24E-03	MLE
	0.105	4.27E-01	0.422097	4.27E-01	3.69E-01	2.70E-05	9.89E-09	3.45E-03	MOM
	0.153	4.30E-01	0.424291	4.30E-01	3.72E-01	3.79E-05	9.02E-07	3.46E-03	MOM
	0.186	4.33E-01	0.426116	4.31E-01	3.75E-01	4.23E-05	1.82E-06	3.38E-03	MOM
	0.202	4.34E-01	0.428208	4.33E-01	3.78E-01	2.93E-05	1.63E-07	3.13E-03	MOM
	0.215	4.34E-01	0.430311	4.35E-01	3.81E-01	1.70E-05	6.00E-07	2.91E-03	MOM
	0.233	4.35E-01	0.432182	4.37E-01	3.83E-01	1.06E-05	2.47E-06	2.77E-03	MOM
	0.252	4.37E-01	0.434	4.39E-01	3.85E-01	7.65E-06	3.85E-06	2.63E-03	MOM
	0.262	4.37E-01	0.436083	4.41E-01	3.89E-01	1.48E-06	1.21E-05	2.34E-03	MOM
	0.343	4.42E-01	0.438072	4.43E-01	3.92E-01	1.73E-05	2.24E-07	2.52E-03	MOM

Table 8: Hazard function values for$(\lambda_1 = 1.5, \alpha = 1.5, k = 0.6)$

n		Real	mle	mom	RE	Mse_mle	Mse_mom	Mse_RE	Best
20	0.2220	6.18E-01	0.580313	5.93E-01	4.56E-01	1.42E-03	6.22E-04	2.63E-02	MOM
	0.2620	6.36E-01	0.620064	6.29E-01	5.06E-01	2.57E-04	4.38E-05	1.68E-02	MOM
	0.3050	6.55E-01	0.655173	6.66E-01	5.51E-01	1.37E-09	1.26E-04	1.09E-02	MLE
	0.3500	6.74E-01	0.689786	7.02E-01	5.98E-01	2.62E-04	8.21E-04	5.76E-03	MLE
	0.3550	6.76E-01	0.717027	7.30E-01	6.32E-01	1.69E-03	2.87E-03	1.94E-03	MLE
	0.7750	8.17E-01	0.745636	7.57E-01	6.67E-01	5.11E-03	3.58E-03	2.24E-02	MOM
	0.9770	8.70E-01	0.774795	7.87E-01	7.04E-01	9.00E-03	6.84E-03	2.74E-02	MOM
	0.9830	8.71E-01	0.802218	8.15E-01	7.38E-01	4.68E-03	3.12E-03	1.75E-02	MOM
	1.0900	8.95E-01	0.826306	8.40E-01	7.69E-01	4.66E-03	2.93E-03	1.58E-02	MOM
	1.2100	9.21E-01	0.857497	8.71E-01	8.06E-01	4.08E-03	2.54E-03	1.33E-02	MOM
40	0.0578	5.34E-01	0.549087	5.67E-01	4.44E-01	2.40E-04	1.10E-03	7.95E-03	MOM
	0.2350	6.24E-01	0.573491	5.90E-01	4.75E-01	2.56E-03	1.18E-03	2.22E-02	MOM
	0.3630	6.78E-01	0.592894	6.08E-01	5.00E-01	7.31E-03	4.89E-03	3.20E-02	MOM
	0.4120	6.98E-01	0.611368	6.26E-01	5.23E-01	7.45E-03	5.09E-03	3.07E-02	MOM
	0.4350	7.06E-01	0.629325	6.44E-01	5.45E-01	5.95E-03	3.90E-03	2.62E-02	MOM
	0.4470	7.11E-01	0.650765	6.65E-01	5.71E-01	3.64E-03	2.16E-03	1.96E-02	MOM
	0.4660	7.18E-01	0.667923	6.81E-01	5.92E-01	2.53E-03	1.35E-03	1.59E-02	MOM
	0.4840	7.25E-01	0.683448	6.97E-01	6.12E-01	1.71E-03	7.63E-04	1.28E-02	MOM
	0.5370	7.43E-01	0.701323	7.16E-01	6.34E-01	1.76E-03	7.67E-04	1.20E-02	MOM
	0.6730	7.87E-01	0.714354	7.29E-01	6.50E-01	5.34E-03	3.46E-03	1.89E-02	MOM
60	0.0008	5.00E-01	0.531697	5.21E-01	4.34E-01	9.79E-04	4.05E-04	4.44E-03	MOM
	0.0014	5.01E-01	0.550005	5.39E-01	4.56E-01	2.42E-03	1.43E-03	2.01E-03	MOM
	0.0023	5.01E-01	0.567225	5.56E-01	4.77E-01	4.35E-03	2.98E-03	5.75E-04	RE
	0.0117	5.07E-01	0.582071	5.70E-01	4.95E-01	5.64E-03	4.02E-03	1.50E-04	RE
	0.0297	5.17E-01	0.594547	5.82E-01	5.09E-01	5.95E-03	4.22E-03	6.97E-05	RE
	0.0939	5.53E-01	0.608287	5.96E-01	5.25E-01	3.02E-03	1.84E-03	7.91E-04	RE
	0.1000	5.57E-01	0.620217	6.09E-01	5.40E-01	4.03E-03	2.69E-03	2.90E-04	RE
	0.1080	5.61E-01	0.631796	6.21E-01	5.54E-01	5.03E-03	3.57E-03	5.40E-05	RE
	0.1160	5.65E-01	0.642846	6.32E-01	5.67E-01	6.09E-03	4.53E-03	5.31E-06	RE
	0.1270	5.71E-01	0.654035	6.44E-01	5.81E-01	6.94E-03	5.39E-03	1.11E-04	RE
80	0.0178	5.11E-01	0.521611	5.29E-01	4.10E-01	1.21E-04	3.35E-04	1.02E-02	MLE
	0.0471	5.27E-01	0.534105	5.41E-01	4.26E-01	4.43E-05	1.86E-04	1.04E-02	MLE
	0.0512	5.30E-01	0.546974	5.53E-01	4.41E-01	2.96E-04	5.62E-04	7.80E-03	MLE
	0.0580	5.34E-01	0.560554	5.67E-01	4.58E-01	7.24E-04	1.09E-03	5.73E-03	MLE
	0.0905	5.52E-01	0.572061	5.78E-01	4.72E-01	4.19E-04	6.87E-04	6.30E-03	MLE
	0.1090	5.62E-01	0.582338	5.88E-01	4.85E-01	4.29E-04	6.71E-04	5.94E-03	MLE
	0.1200	5.67E-01	0.593375	5.99E-01	4.99E-01	6.74E-04	9.68E-04	4.66E-03	MLE
	0.1340	5.75E-01	0.603742	6.09E-01	5.13E-01	8.43E-04	1.18E-03	3.84E-03	MOM
	0.1360	5.76E-01	0.614613	6.20E-01	5.26E-01	1.48E-03	1.89E-03	2.54E-03	MLE
	0.1750	5.96E-01	0.624546	6.30E-01	5.38E-01	8.33E-04	1.17E-03	3.27E-03	MLE

Table 9: Hazard function values for$(\lambda_1 = 1.5, \alpha = 1.5, k = 0.9)$

n		Real	mle	mom	RE	Mse_mle	Mse_mom	Mse_RE	Best
20	0.0446	5.39E-01	0.616776	6.30E-01	5.13E-01	6.10E-03	8.37E-03	6.37E-04	RE
	0.0745	5.63E-01	0.676876	6.88E-01	5.89E-01	1.30E-02	1.56E-02	6.96E-04	RE
	0.1150	5.94E-01	0.723962	7.37E-01	6.48E-01	1.69E-02	2.04E-02	2.97E-03	RE
	0.2640	6.92E-01	0.773883	7.87E-01	7.09E-01	6.68E-03	9.05E-03	2.96E-04	RE
	0.3200	7.23E-01	0.820065	8.35E-01	7.66E-01	9.35E-03	1.26E-02	1.79E-03	RE
	0.3290	7.28E-01	0.855701	8.72E-01	8.09E-01	1.62E-02	2.06E-02	6.54E-03	RE
	0.4920	8.07E-01	0.888683	9.05E-01	8.47E-01	6.72E-03	9.70E-03	1.66E-03	RE
	0.5520	8.32E-01	0.921431	9.40E-01	8.88E-01	8.02E-03	1.17E-02	3.19E-03	RE

	0.9340	9.57E-01	0.950983	9.70E-01	9.24E-01	3.37E-05	1.81E-04	1.08E-03	RE
	1.0000	9.74E-01	0.978661	1.000000	9.59E-01	1.78E-05	6.29E-04	2.47E-04	RE
40	0.0313	5.28E-01	0.540542	5.62E-01	4.49E-01	1.73E-04	1.22E-03	6.11E-03	MLE
	0.0491	5.42E-01	0.580546	6.00E-01	4.98E-01	1.47E-03	3.30E-03	1.99E-03	MLE
	0.1510	6.20E-01	0.610725	6.28E-01	5.34E-01	7.86E-05	6.79E-05	7.34E-03	MOM
	0.1810	6.41E-01	0.636782	6.55E-01	5.66E-01	1.90E-05	1.87E-04	5.66E-03	MLE
	0.3030	7.13E-01	0.663252	6.80E-01	5.96E-01	2.49E-03	1.11E-03	1.36E-02	MOM
	0.3120	7.20E-01	0.690193	7.05E-01	6.27E-01	8.99E-04	2.23E-04	8.58E-03	MOM
	0.3150	7.21E-01	0.712644	7.27E-01	6.54E-01	7.57E-05	2.86E-05	4.47E-03	MOM
	0.3680	7.50E-01	0.736833	7.51E-01	6.83E-01	1.59E-04	1.74E-06	4.38E-03	MOM
	0.4080	7.69E-01	0.758748	7.73E-01	7.10E-01	1.03E-04	1.48E-05	3.51E-03	MOM
	0.4480	7.88E-01	0.778932	7.92E-01	7.33E-01	8.29E-05	1.55E-05	3.01E-03	MOM
60	0.0080	5.08E-01	0.534478	5.34E-01	4.39E-01	7.00E-04	6.91E-04	4.75E-03	MOM
	0.1940	6.48E-01	0.559492	5.58E-01	4.66E-01	7.86E-03	8.07E-03	3.32E-02	MLE
	0.2050	6.55E-01	0.583662	5.82E-01	4.95E-01	5.12E-03	5.33E-03	2.58E-02	MLE
	0.2460	6.82E-01	0.60514	6.05E-01	5.23E-01	5.87E-03	5.96E-03	2.53E-02	MLE
	0.2620	6.90E-01	0.624021	6.23E-01	5.45E-01	4.40E-03	4.49E-03	2.11E-02	MLE
	0.2780	7.01E-01	0.639252	6.39E-01	5.62E-01	3.80E-03	3.89E-03	1.92E-02	MLE
	0.2930	7.08E-01	0.654223	6.53E-01	5.80E-01	2.92E-03	3.03E-03	1.65E-02	MOM
	0.3140	7.20E-01	0.672561	6.71E-01	6.02E-01	2.23E-03	2.34E-03	1.39E-02	MLE
	0.3160	7.22E-01	0.691232	6.91E-01	6.25E-01	9.46E-04	9.69E-04	9.31E-03	RE
	0.3240	7.25E-01	0.704734	7.05E-01	6.43E-01	4.28E-04	4.23E-04	6.86E-03	MOM
80	0.0234	5.21E-01	0.532211	5.38E-01	4.53E-01	1.33E-04	3.06E-04	4.62E-03	MLE
	0.0248	5.22E-01	0.554251	5.60E-01	4.78E-01	1.05E-03	1.44E-03	1.91E-03	MLE
	0.0478	5.41E-01	0.574675	5.80E-01	5.01E-01	1.11E-03	1.46E-03	1.61E-03	MLE
	0.0764	5.64E-01	0.592742	5.98E-01	5.23E-01	8.11E-04	1.14E-03	1.69E-03	MOM
	0.0895	5.75E-01	0.609438	6.14E-01	5.43E-01	1.21E-03	1.57E-03	9.73E-04	RE
	0.1131	5.93E-01	0.626451	6.32E-01	5.64E-01	1.15E-03	1.54E-03	8.20E-04	RE
	0.1491	6.18E-01	0.640548	6.46E-01	5.81E-01	5.10E-04	7.80E-04	1.39E-03	MLE
	0.1870	6.45E-01	0.654754	6.60E-01	5.98E-01	8.81E-05	2.24E-04	2.27E-03	MLE
	0.2020	6.53E-01	0.66751	6.74E-01	6.13E-01	2.15E-04	4.23E-04	1.57E-03	MLE
	0.2050	6.55E-01	0.679749	6.86E-01	6.27E-01	6.12E-04	9.43E-04	7.76E-04	MLE

5. Conclusion

The fuzziness in hazard rate function under mixed Quasi – Lindely was caused due incomplete data, also imperfect information, so we introduce fuzzy factor (k) for comparing different three estimators.

MLE is dominated with percentage $(270/360 = 75\%)$, MOM dominated with percentage $(57/360 = 16\%)$, and finally FR dominated with $(33/360 = 9\%)$.

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