Mathematical Analysis of Steady State Lateral Load Transfer on a Vehicle

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Abstract: ***This paper analyses lateral acceleration and steady state lateral load transfer on a vehicle during cornering. The three components of load transfer (un-sprung, kinematics and elastic) have been analyzed individually, Calculations and graphs/plots are made with PTC MathCad Prime 4.0. The paper covers effect of each components as a function of some other variables like lateral acceleration, roll center heights and varying roll stiffness. Un-sprung component provides the least opportunity to control the lateral load transfer while elastic component provides the maximum room for enhancing performance of the vehicle. Kinematic component can be altered through changes in the suspension geometry andweight distribution.***

Keywords: roll center, roll angle, lateral loading, roll moment, stiffness

1. Introduction

In the modern age of comfort and speed; suspension plays a significant role in the dynamics of a vehicle. Suspension is an arrangement of wheels, axle, springs, dampers struts, shock absorbers and linkages that connect the vehicle to its wheels and allows the relative motion between the two. They allow comfort for the passengers by nullifying the bumps on the uneven roads, increase the control that is increase in driver interaction and vehicle response, and they adhere contact of the wheels with the roads all the time. A well-designed suspension system guarantees minimum wear on the tires and other parts of the suspension system. The primary functions of any suspension are to absorb road impacts and smoothen the ride, to minimize pitch and roll i.e. to maintain an even keel for the body, and reduce impact stress on various components of the vehicle.

1.1 Vehicle dynamics and suspension requirements

The dynamic forces acting on the vehicle-drive torque, steering force, braking torque- depend on tire to road friction. If the dynamic load exceeds the friction between the tire and road then the control is lost because of slip or skidding of the wheel.Ideally the tire should always be in contact with the road but is very rarely practically possible as the factors such as aerodynamic design, road conditions and the environmental factors play a major role. Small deflections are absorbed by the tires where as for the larger bumps proper suspension is to be designed.

The vehicle dynamics plays an important role in the development of the vehicle industry. It is a science related to analytical and experimental study of responses of vehicle under motion. A vehicle in motion is under complex loading and forces such as centrifugal force, inertia, friction, traction and kinetic energy. The study includes most of the primary components of the vehicle such as tires, suspension, braking, vehicle aerodynamics, and steering and traction control.

1.2 What is vehicle body roll?

Vehicle suspension allows vertical relative motion between wheels and the body. This in turn during cornering creates a centrifugal force on vehicle CG while computing a lateral motion. This result in tilting of its body towards the direction of centrifugal force. This tilting of vehicle body is known as roll.

1.3 Roll center and roll axis

The roll centers are defined by the suspension geometry, there are lot many types of suspension and the front and rear roll centers of a vehicle depend on suspension configuration. The line that connects front and rear roll centers is called as roll axis. Roll center is the point at which the roll of the body occurs-sprung mass along with suspension rolls about in a plane which is perpendicular to the longitudinal direction of the vehicle which contains left and right wheels. The significance of roll center is seen when it does not coincide with CG of the vehicle, for such a scenario a moment arm is created. While computing a corner the size of this moment arm, stiffness of spring and anti-roll bars plays a significant role in determining the roll of the vehicle.[2]

1.4 Roll stiffness

While cornering the vehicle has a constant lateral acceleration and a centrifugal force acts on the CG of the vehicle. In most of the cases there is a difference in position of CG and roll axis of the vehicle which results in cause of roll moment about the roll axis resulting in constant roll angle. When the vehicle body rolls one of the spring of the suspension undergoes compression and other elongates which produces an equilibrium moment resisting the rolling moment produced by centrifugal force. The magnitude of the moment caused due to spring action (compression and elongation) per unit roll angle is called roll stiffness (which can also include tire stiffness in conjunction).

2. Lateral Load Transfer

Lateral load transfer also known as lateral weight transfer is the magnitude change of the vertical loads on the tires that occurs because of the lateral acceleration imposed on the center of mass of a vehicle. In simple terms, the magnitude of weight (ΔW) increase on outer tires and decrease on inner tires during cornering defines lateral load transfer.

The total lateral load transfer acting on a vehicle can be calculated from its FBD (free body diagram)

Refer fig. Refer fig.1 (rear view of the car) in a right-handed turn let A_y is the lateral acceleration, W is the weight of the car h is the height of CG, d is the width of the track W_L and W_R be the vertical loads on left and right tires respectively

Figure 1: FBD of rear view of car

[2]The total lateral load transfer can be given by the formula

$$\Delta W = \frac{W A_y h}{t} \quad (1)$$

The total lateral load W can be varied by changing the above parameters. One may lower his CG by reducing the height h or by changing the track width t. But there are limitations in doing so as it is difficult to redesign the car. Hence an alternative way is to redistribute the lateral load acting on the vehicle.

Vehicle variable values.
Total Mass – 1200 kg (sprung 1200kg + 180 un-sprung)
CG – 0.3m. Roll center variation – 0.125mm- 0.3 m
Roll stiffness – 1,50,000 Nm/rad to 2,50,000 Nm/rad
Lateral acceleration – 1 – 1.3g

2.1 Lateral Load Transfer from Unsprung Mass

Unsprung mass is the mass of all the components that are not supported by the suspension. For instance, the mass of suspension, tires wheel hub assembly etc. contribute to be the unsprung mass. One of the primary components of load transfer is the one caused by the unsprung mass. A lateral acceleration is caused at the CG of the vehicle at the time of corner which generates a centrifugal force. This force creates a moment which would be the result of it times the unsprung CG height. When this moment is divided by the axle track width we get the load transfer component [2]

$$\Delta W_{RC} = \frac{W_{sa} A_y Z_{RC}}{t} \quad (2)$$

$$W_{ua} := 1765.8$$

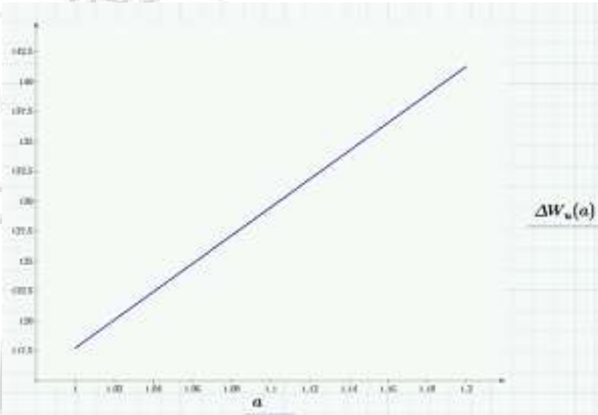
$$a := 1, 1.05 .. 1.2$$

$$Z_{ua} := 0.1$$

$$t := 1.5$$

$$\Delta W_u(a) := \frac{W_{ua} \cdot a \cdot Z_{ua}}{t}$$

$$\Delta W_u(a) = \begin{bmatrix} 117.72 \\ 123.606 \\ 129.492 \\ 135.378 \\ 141.264 \end{bmatrix}$$



Plot 1: Un-sprung mass- load transfer vs. lateral acceleration

Lateral load transfer due to un-sprung mass is difficult to control. We need to alter the mass/CG, which involves adding, removing or relocating mass in context of the un-sprung mass.

Changing the mass would directly affect the tire contact patch, weight distribution dynamics and overall vehicle dynamics.

2.2 Load Transfer -Kinematic Load Transfer Component

Load transfer from direct force arises from coupling effect that the roll centers have, ratio of forces on sprung mass to un-sprung mass. This is known as kinematic load transfer. The lateral force acting on the sprung mass which generates a moment on the tires.
W_{sa} = Sprung weight distribution to the axle
Z_{rc}=Roll center height from the track
Sprung weight distribution:

$$\frac{\textit{Distance from sprung weight CG to the opposite axel}}{\textit{wheel base of vehicle}}$$

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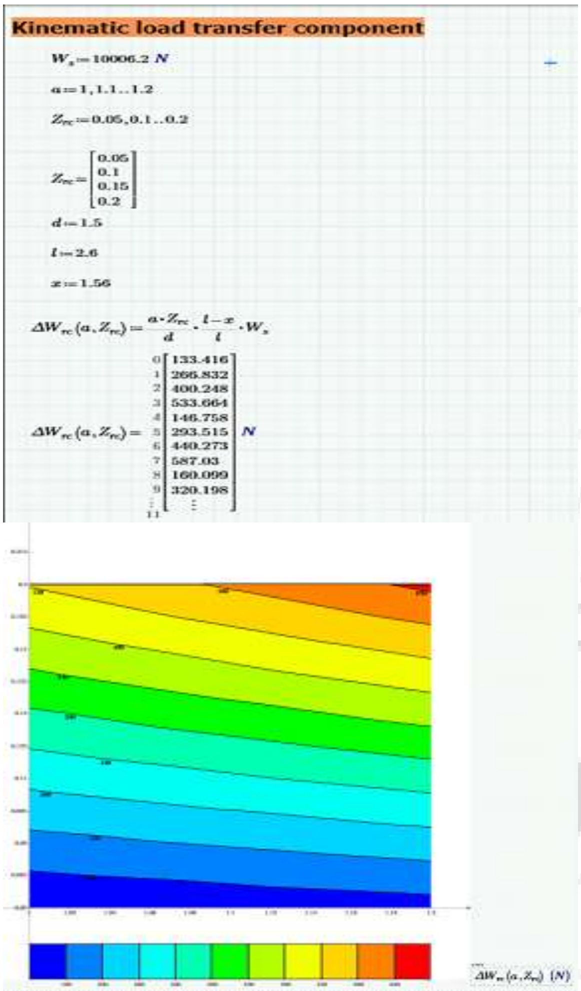
$$W_{sa} = \frac{l-x}{l} W_s \tag{3}$$

x=distance from the CG to the axle being analyzed
l= wheel base
[4]The expanded equation:

$$\Delta W_{RC} = \frac{A_y Z_{RC}}{t} \cdot \frac{l-x}{l} W_s \tag{4}$$



Plot 2: Load transfer as a function of acceleration and roll center height.

We cannot directly conclude load transfer dynamics through kinematic component. When front roll centre height increases, the lateral weight transfer goes down on the rear axle, going up on the front. On the other hand, increase in rear roll centre height, lateral load transfer shoots up on the rear axle and goes to low on the front axle.

Whenever we increase roll centre height in one axle, we always increase the overall lateral load transfer on that axle and decreasing it on the opposite axle. hence the total gain in roll centre height is higher than the decrease in the roll moment arm.The change of roll moment arm with roll centre heights is strongly dependant on the wheelbase and weight distribution between the front and rear axles.

We should analyse the elastic component subsequently.

2.3 Load Transfer Due to Roll Angle (or Elastic Load Transfer Component

The centrifugal forces induced from the inertia generate a moment hence making the sprung mass to roll towards the outer side while negotiating a corner. This results in the compression of the outside spring and elongation of the inside spring of the suspension

Springs are devices that take forces when they are displaced; the force that arises on each of the spring generates a moment which eventually resists the rotation of the body. The tires react to the generated forces in the springs which contribute to the lateral load transfer which is also known as elastic weight transfer component. The roll of the chassis shifts the CG of the sprung mass sideways which arises another moment that add to the lateral load transfer.

While negotiating a corner the sprung mass of the vehicle rolls by Ø (roll angle), roll rate (roll stiffness) $K_Ø$of the vehicle reacts to the roll angle change. On independent suspension roll stiffness varies with width of track and vertical stiffness of suspension. The total roll stiffness is the sum of roll stiffness of front and rear axles.

Assuming chassis to be a rigid body the roll angle will be same for both rear and front suspensions. The roll resistant moment is given by[4]

$$M_\phi = K_\phi \phi = (K_{\phi F} + K_{\phi R})\phi \tag{5}$$

Assuming
- No roll is produced on application of lateral force on roll axis.
- CG and roll centers lie on the central line of the vehicle.
- Roll rates (front and rear) are measured separately.

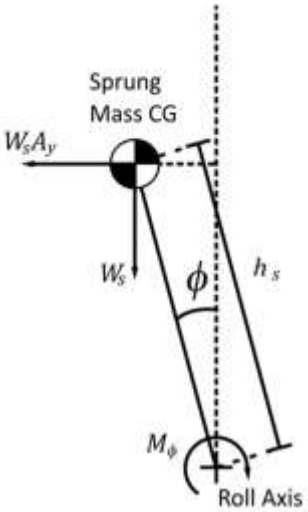


Figure 2: Roll load geometry

Considering the moment equilibrium about the roll axis O

$$M\phi - W_s h_s sin\phi - W_s A_y h_s cos\phi = 0 \quad (6)$$

As Ø is very small we get

$$M\phi = W_s h_s \phi + W_s A_y h_s \quad (7)$$

$M\phi$ is the roll resistance moment.

$W_s h_s sin\phi$ is roll moment by gravity due to shift of CG.

Solving for Ø and dividing by ay we get roll stiffness

$$\frac{\phi}{A_y} = \frac{W_s h_s}{K_{\phi F} + K_{\phi R} - W_s h_s} \quad (8)$$

Now $K_{\O sa}$ gives roll resistance moment and sprung CG side shift for a single axle

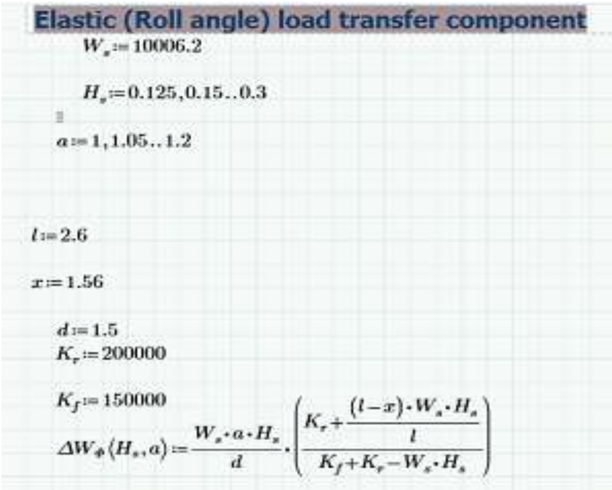
$$M_{\phi sa} = \frac{K_{\phi a} W_s h_s A_y}{K_{\phi F} + K_{\phi R} - W_s h_s} \rightarrow \begin{cases} K_{\phi a} = K_{\phi F}, \text{ front axle} \\ K_{\phi a} = K_{\phi R}, \text{ rear axle} \end{cases} \quad (9)$$

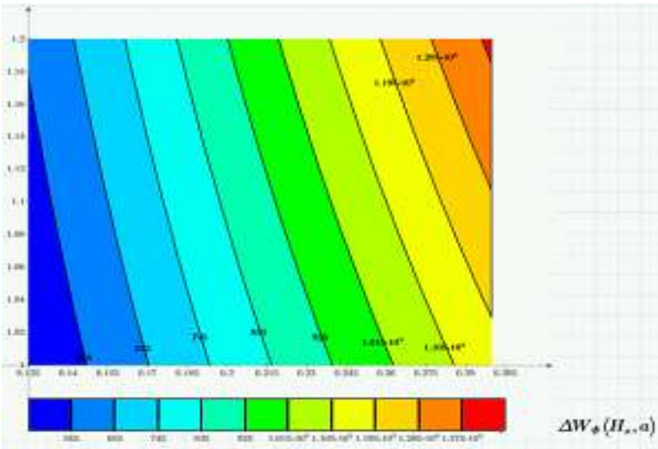
This component only resists roll angle and in the same way the weight shift component will be

$$M_{CG} = W_{sa} h_s \phi = \frac{l - x}{l} \cdot W_s h_s \phi \quad (10)$$

The total moment from roll angle on single axle [4]

$$\Delta W_\phi = \frac{W_s A_y h_s}{t} \cdot \left(\frac{K_{\phi a} + (l - x) W_s h_s / l}{K_{\phi F} + K_{\phi R} - W_s h_s} \right) \quad (11)$$





Plot 3: Lateral load transfer as a function of roll center height and lateral acceleration

The total lateral load transfer on an axle is obtained by adding the three components of lateral load transfer.[4]

$$\Delta W_a = \frac{W_s A_y}{t} \left[h_s \left(\frac{K_{\phi a} + W_s h_s (l - x)/l}{K_{\phi F} + K_{\phi R} - W_s h_s} \right) + \frac{l - x}{l} \cdot Z_{RC} \right] + \frac{W_{ua} A_y}{t} Z_{ua} \quad (12)$$

$$\Delta W(a,h_s,Z_{rc}) := \frac{W_s \cdot a \cdot h_s}{t} \cdot \left(\frac{K_r + \frac{(l-x) \cdot W_s \cdot h_s}{l}}{K_f + K_r - W_s \cdot h_s} \right) + \left(\frac{a \cdot Z_{rc}}{t} \cdot \frac{l-x}{l} \cdot W_s \right) + \left(\frac{W_{ua} \cdot a \cdot Z_{ua}}{t} \right)$$

$$\Delta W(a,h_s,Z_{rc}) = \begin{bmatrix} 730.527 \\ 863.943 \\ 997.359 \\ 1.131 \cdot 10^3 \\ 827.105 \\ 960.521 \\ 1.094 \cdot 10^3 \\ 1.227 \cdot 10^3 \\ 923.918 \\ 1.057 \cdot 10^3 \\ 1.191 \cdot 10^3 \\ 1.324 \cdot 10^3 \\ \vdots \end{bmatrix}$$

Figure 3: Total lateral load transfer matrix

3. Analysis of various lateral force components

3.1Analysis of direct lateral force component:

This component can be changed by varying weight distribution of the vehicle or load centre height. Weight distribution can be altered by varying the position of CG in the longitudinal direction, but this in turn may have some undesirable changes on the overall design of the vehicle.

Change load transfer from direct lateral force component:
This can be done by altering the roll center height. This is a complex process and requires redesign of suspension, changing the various parameters such as camber, caster toe angles etc. it can be altered by changing the suspension pickups so that the arms will be at different position and orientation.
It is not correct to conclude that increase in load centre height will increase the lateral load transfer. Increasing the load centre height on one axle, we increase lateral load transfer from the direct lateral force component and decrease lateral load transfer from the roll angle component

Let h_s be the perpendicular distance between CG and the sprung mass of the axis. The total resultant will depend on roll centre height and roll stiffness and will require a deeper analysis.

Figure 3: Geometry of roll moment

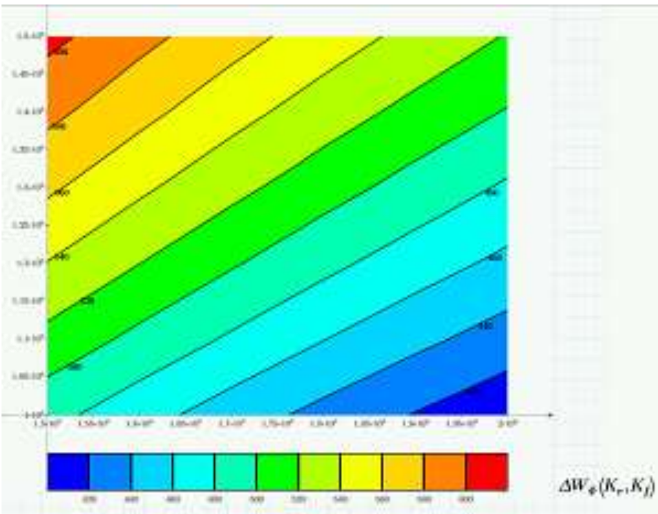
Increasing the load centre height in one axle decreases lateral weight transfer on opposite axle. This is caused because rising the roll centre in any axle will near about the roll axis to sprung weight i.e.CG. This in turn decreases the roll angle component but the roll centre height of opposite axle will not be raised direct lateral force component will not increase. This results in reduction of weight transfer on that axle.[1]

3.2 Elastic load transfer component:

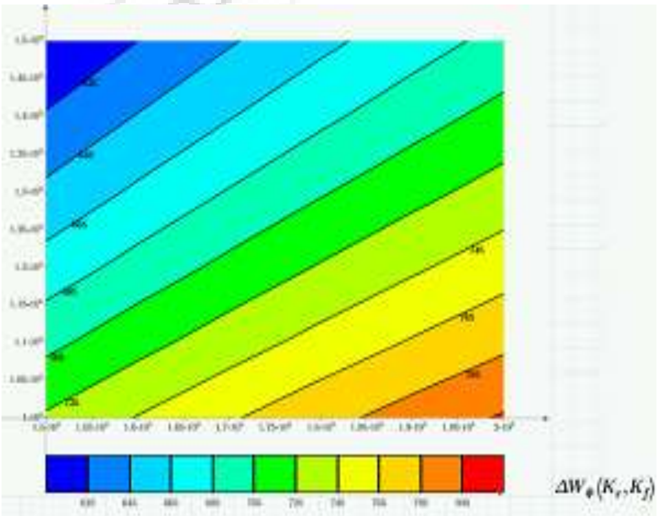
Ø has negligible value, the gravity component $W_s h_s \to 0$
Remaining roll angle component will be[4]

$$h_s \left(\frac{K_{\phi a}}{K_{\phi F} + K_{\phi R}} \right) \qquad (13)$$

The roll angle lateral load transfer component in one track will be a function of the ratio of roll stiffness of the track and the total roll stiffness vehicle when the roll moment of the arm is kept constant.The formula above gives the elastic lateral load transfer component.



Plot 4: Front Lateral load transfer sensitivity as a function of Front and rear roll stiffness



Plot 5: Rear Lateral load transfer sensitivity as a function of Front and rear roll stiffness

Analysing the plots above, if we keep the rear roll stiffness constant and keep on increasing the front roll stiffness, the front lateral weight transfer sensitivity (FLWTS) keeps on increasing, which means the lateral weight transfer per unit acceleration increases.

If we keep the front roll stiffness constant and keep on increasing the rear roll stiffness, the rear lateral weight transfer sensitivity (RLWTS) keeps on increasing, which means the lateral weight transfer per unit acceleration increases.[4]

4. Conclusion

We studied steady state lateral load transfer equations and plots using PTC MathCad Prime 4.0. information can be summarized as below:

Un-sprung weight component – not beneficial as a modification tool because of the after effect that it has on comfort, weight distribution, steering effects.

Direct force component or kinematic component –contributes in handling when the roll axis is close to the sprung CG, eventually the influence of roll component is lessened. Increasing the load centre height in one axle decreases lateral weight transfer on opposite axle.

Roll angle component or elastic component – utmost useful component, since it is the easiest to change and enhance vehicle handling, cornering stability etc. when antiroll devices are present.

Load transfer sensitivity (load/unit acceleration) is a function of front and rear roll stiffness.

5. Future work scope for paper

Calculations performed in this paper are done for steady-state lateral load transfer. In future, dampers along with springs are to be analysed as transient lateral load transfer is crucial in vehicle dynamics.
Vehicle and suspension simulation software could be used for real time dynamic analysis.

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