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An Application on Evoked K-Complex to Fuzzy Sets

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Abstract: ***The present study transformed K-complex values to fuzzy sets in the stages of before and after stimulus presentation. In this way, we will examine the effect of stimuli on individuals in a more detailed and realistic way.***

Keywords: Fuzzy set, Entropy, EEG, K-complex, Auditory stimuli.

1. Introduction and Preliminaries

The concept of fuzzy sets and fuzzy set operations were first introduced by Zadeh [16]. After his innovation, mathematical structures were altered with fuzzy numbers. Recently, Şengönül [12], Şengönül and others [13] has made investigations on entropy concept for fuzzy sets. The notion of entropy for sequences of fuzzy sets was used by Şengönül and others [13] for computing some numerical values of P and T waves in ECG. Additionally, [1], [3], [4], [5], [6], [7], [9], [15], [17] used various disciplines of fuzzy set theory and entropy of fuzzy sets.

This paper is organized as in the following. In the first section, we recall some fundamental notions from fuzzy set theory, entropies of fuzzy sets and EEG concepts. In the second section, proposed main method and procedure are given. Additionally, corresponding fuzzy sets to K-complexes are determined by considering K- complex values and entropy calculations are given in the second section.

Now, we recall some of the basic definitions and notions in the theory of fuzzy numbers, entropy and EEG concepts for a better understanding of the subject.

Total Recording Time(TRT): The time started to recording until the recording is terminated after all electrodes are connected to the patient. Namely, duration of time from sleep onset to final awakening. It is expressed in minutes. Usually in records, start time and end time are indicated as terms ”light off” and ”light on”, respectively.

Time in Bed: The time when the patient is lying on the bed until it get out of the bed. Being almost same with TRT is preferable.

Total Sleep Time (TST): The total amount of actual sleep time in a sleep period. The total sleep time is the total of all REMs and NREMs in a sleep.

Sleep Efficiency (SE): Ratio of total sleep time to time in bed. SE is expressed as a percentage (%). Normally, it is 85% or higher.

Sleep Latency (SL): SL is the length of the time it means that it takes to accomplish the transition from full wakefulness to sleep. Usually, 15 minutes are needed to fall asleep.

REM Latency (RL): After a person falls asleep, the amount of time which takes for the first appearance of stage REMs.

The first step in the analysis is investigation of measurement technique, signal quality and calibration. Recording time, lighting-up time, lighting down time and some special cases provide basis for evaluation of biological signals.

Although it depends on the investigators, an appropriate time interval is considered as a one page of the record and is called 1 epoch. For sleep scoring this time is usually 30 seconds. Longer term epochs are not recommended because of the fact that a short modifications will be ignored.

Wave Properties: The main properties of waves are defined below:
Amplitude: The height of the wave. Amplitude is used in the same sense with voltage. A higher amplitude means a larger value. Wave amplitude is measured by microvolts (µv).
Frequency: The number of complete waves that pass a point in one second, and measured in Hertz(Hz) or cps(cyles per second). For example, a with 2 Hz frequency is two times faster as compared to a wave having 1 Hz frequency. EEG (electroencephalogram): Electroencephalogram is an electrophysiological monitoring method to record electrical activity of the brain. Electrical signals related to nerve cells located in the brain tissue are transmitted to the scalp on the skull. These signals are transmitted to the computer by means of the electrodes placed on certain areas of the scalp. Transmitted signals are commented(evaluated) by the neurology specialist doctors who is received training in this regard. In addition these, EEG done with electrodes pasted to scalp is completely harmless investigation.

EEG assessment are made by taking into account some specific patterns, wave amplitude and wave frequency. EEG activity will be described as alpha, beta, theta and delta waves as shown in Table I. Sleep spindles which continue more than 0.5 seconds are activities with 12- 14 Hz frequency. Positive component (EEG downward) follows negative deflection (EEG upward) that is the first part of K- complex and takes at least 0.5 second. Both patterns are characteristic properties

for NREM stage 2 sleep. Vertex sharp waves are the sharp negative deflections and indicate NREM stage 1 sleep.
Sleep stages are categorized by five statements for humans as follow:

- Awake
- Rapid eye movement sleep
- Sleep stage one, two and three.

In the following, we show the EEG wave properties and EEG pattern tables, respectively:

Table 1: EEG Wave Properties

Wave Type	Property	Observed Phase
Beta	> 13 Hz	awake, active person (soul)
Alfa	8-13 Hz	eyes closed, calm wakefulness
Teta	3-7 Hz	slumber
Delta	< 4 Hz, min 75µV	NREM deep sleep

Table 2: Specific EEG Patterns

EEG Pattern	Property	Observed Phase
Sleep Spindles	12 14 Hz, > 0.5 sn	NREM stage- 2
K-complexes	Sharp negative, slower positive, > 0.5 sn	NREM stage- 2
Vertex sharp waves	Sharp negative deflections	NREM stage- 1

Each stage has a different physiological and neurological properties. In this article, sleep recording scoring is given with the criteria published in the book ”AASM Manual for the Scoring of Sleep-Version 2,0”.

EEG exhibit graphical image of brain waves and is imaged by veterinarians, medical personnels and scientists. The amplitudes and frequencies of EEG signals are used to predetermine the conditions in human or animal body. Interpretation and giving comments on the EEG waves will be take long time and a large amount of error can be arise in the diagnosis process. Additionally, some significant data will escape the attention.

Now, we give some knowledge about fuzzy set theory:
Let X be a nonempty set. According to Zadeh a fuzzy subset of X is a nonempty subset $\{(x,u(x)):x \in X)\}$ of $X \times [0,1]$ for some function $u: \mathbb{R} \to [0,1]$.

Consider a function called as membership function, $u: \mathbb{R} \to [0,1]$ as a nonempty subset of $\mathbb{R}$ and denote the family of all such functions or fuzzy sets by E. Let us suppose that the function u satisfies the following properties:

1) u is normal,
2) u is fuzzy convex,
3) u is upper semi-continuous,
4) The closure of $\{x \in \mathbb{R}: u(x) > 0\}$, denoted by u^0 is compact, [16].

Then, the function u is called a fuzzy number. Furthermore, we know that shape similarity of the membership functions does not reflect the conception of itself, but it will be used for examining the context of the membership functions.

Whether a particular shape is suitable or not can be determined only in the context of a particular application. However, many applications are not overly sensitive to variations in the shape. In such cases, it is convenient to use a simple shape, such as the triangular shape of membership function.

Let us define fuzzy set A on the set R with membership function as follows:

$$A(x) = \begin{cases} \dfrac{h_A(x-u_0)}{u_1-u_0}, x \in [u_0,u_1) \\ \dfrac{-h_A(x-u_1)}{u_2-u_1}, x \in [u_1,u_2] \\ 0, \quad otherwise \end{cases}$$

Here, h_A represents height of the fuzzy set A and $u_0,u_1,u_2 \in \mathbb{R}$. Additionally, we show fuzzy set A with the triple $(u_0,u_1:h_A,u_2)$. In fuzzy set theory, the fuzziness of a fuzzy set is an important topic and there are many methods for measuring the fuzziness of a fuzzy set. Firstly, fuzziness was thought to be the distance between fuzzy set and its nearest non-fuzzy set. After, entropy was used instead of fuzziness [4], [15]. Now, we give the definition of entropy:

Let $u \in F$ and $u(x)$ be the membership function of the fuzzy set A and consider the function $H: F \to \mathbb{R}$. If the function H satisfies conditions below,

1) $H(A) = 0$ iff A is crisp set.
2) $H(A)$ has a unique maximum, if $A(x) = \frac{1}{2}$, for all $x \in \mathbb{R}$.
3) Let, $B \in F$. If $B(x) \leq A(x)$ for $A(x) \leq 0.5$ and $A(x) \leq B(x)$ for $A(x) \geq 0.5$ then $H(A) \geq H(B)$.
4) $H(A^c) = H(A)$, where A^c is the complement of the fuzzy set A, then $H(A)$ is called entropy of the fuzzy set A, [17].

Let suppose that $A = A(x)$ be membership function of the fuzzy set A and the function $h : [0, 1] \to [0,1]$ satisfies the following properties:

1) Monotonically increasing at $[0,\frac{1}{2}]$ and decreasing at $[\frac{1}{2},1]$,
2) $h(x) = 0$ if $x = 0$ and $h(x) = 1$ if $x = \frac{1}{2}$.

Then h is called entropy function and equality $H(A(x)) = h(A(x))$ holds for $x \in \mathbb{R}$. Additionally, some well known entropy functions are given as in the following:

$$h_1(x) = 4x(1-x), h_2(x) = -xlnx - (1-x)ln(1-x), h_3(x) = min\{2x, 2-2x\}$$

Here, h_1, h_2, h_3 are called logistic, Shannon and tent functions, respectively. Let X be a continuous universal set. The total entropy of the fuzzy set A on X is defined as follows:

$$e(A) = \int_{x\in X}^{1} h(A(x))p(x)\,dx \qquad (1)$$

where $p(x)$ is the probability density function of the available data in X [10], [11]. If we take $p(x) = 1$ in (1) then $e(A)$ is called entropy of the fuzzy set A.

Example 1.1: Let u be fuzzy set on R with membership functions $u_1(x)$ and$u_2(x)$, respectively.

$$u_1(x) = \begin{cases} \frac{x}{0.3}, x \in [0,0.3) \\ \frac{0.6-x}{0.3}, x \in [0.3,0.6] \\ 0, \quad otherwise \end{cases}$$

$$u_2(x) = \begin{cases} \frac{x}{0.6}, x \in [0,0.6) \\ \frac{1.2-x}{0.6}, x \in [0.6,1.2] \\ 0, \quad otherwise \end{cases}$$

Then, it is clear to calculate that the total entropies $e(u_1)$ and $e(u_2)$ of fuzzy numbers u_1 and u_2 are given as follows, respectively:

$$e(u_1) = \int_{x\in X}^{1} h(u_1(x))p(x)\,dx = \int_{x\in X}^{1} h(u_1(x))c dx = 4c\int_{x\in X}^{1} u_1(x)(1-u_1(x))dx = 0.4c$$

$$e(u_2) = \int_{x\in X}^{1} h(u_2(x))p(x)\,dx = \int_{x\in X}^{1} h(u_2(x))c dx = 4c\int_{x\in X}^{1} u_2(x)(1-u_2(x))dx = 0.8c.$$

From here, entropy of K- complex is equal to $e(u_1) + e(u_2) = (0.4+0.8)c = 1.2c$.

Definition 1.1: [14] Let suppose that $u = (u^k)$ be a sequence of fuzzy sets and $M = (m_{nk})$ be a lower triangular infinite matrix of real or complex numbers. Besides, $p_k(x) = c_k \in [0,1]$ for all $k \in \mathbb{N}$ and following equations hold:

$$\lim_n \sum_k m_{nk} \int_{x\in\mathbb{R}} h(u^k(x))p_k(x)\,dx$$
$$= \lim_n \sum_k m_{nk}\, c_k \left(2h_{u^k} - \frac{4}{3}h_{u^k}^2\right) l(u^k) = L$$

Then, real number L is called total M- entropy of the sequence (u^k) of fuzzy sets.

2. Main Methods

A total of 10 patients (3 males, 7 females) between the ages of 20 and 36 are used for this work. They were tested in a single all-night session. They were instructed to refrain from alcohol and drug using for 24 hours. And the EEG was recorded with grass gold cup electrodes placed at midline frontal, central and parietal sites. The hearing- aid system assured constancy of stimulus input in spite of changes in the subject's head position during the night.

2.1 Procedure

Individuals were informed to reach at the laboratory an hour before to their normal sleep time. Each subject was fitted hearing aid device. The hearing aid device assured a constancy of stimulus input in spite of the changes in the participant's head position during the night [8]. K-complexes were elicited by auditory stimuli. The stimuli were 80 dB, 2000 Hz tone pips having a total duration of 52 ms with a rise and fall time of 2 ms [2]. Sleep staging, as well as respiratory analyses were performed using the computer-assisted sleep classification system. Sleep staging was performed according to the criteria of Rechtschaffen and Kales.

2.2 Determination of Entropy of Evoked K- Complex Values

In this part, we conclude entropy of K- complex of EEG of 10 individuals as in the following:

$$e(K) = \int_{x\in\mathbb{R}} h_1(K(x))r(x)\,dx.$$

Here, $K(x)$ is membership function of K- complex and $r(x)$ is conductivity function of the humans. Now, by considering maximum height 0.5mv and maximum width 0.01ms of K- complex of individuals, we determine its membership function showed by K^b, by considering before auditory stimuli program as given in the following:

$$K^b(x) = \begin{cases} 100x, x \in [0,0.005] \\ 1-100x, x \in (0.005,0.01] \\ 0, \quad otherwise. \end{cases}$$

In addition this, by using the maximum height 0.8 and maximum width 0.02ms of K- complex, we calculate the membership function of fuzzy set K- complex of individuals after auditory stimuli program as given below:

$$K^a(x) = \begin{cases} 80x, x \in [0,0.01] \\ 1.6-80x, x \in (0.01,0.02] \\ 0, \quad otherwise. \end{cases}$$

Let us take $supp\,K^b \approx]0, 0.005[$ and $supp\,K^a \approx]0, 0.02[$. Closure of the $supp\,K^b$ and $suppK^a$ are equal to [0, 0.005] and [0, 0.02], respectively. By considering these situations and if we choose $r(x) = c$, then we calculate $e^b(K)$ and $e^a(K)$ entropy values of K- complex of fuzzy sets as in the following:

$$e^b(K) = 0.0067c, \qquad e^a(K) = 0.0150c \qquad (2)$$

Now, we will make a comment in light of the results of $e^b(K)$ and $e^a(K)$.

Let us compute the distance $\Delta_e(K)$ by:

$$\Delta_e(K) = |e^a(K) - e^b(K)|.$$

By taking into consideration (2), we find $\Delta_e(K) = 0.0083c$.

Finally, we can interpret this value as, the entropy for individuals is increasing after auditory stimuli program. Additionally, no noteworthy separation was found for any sleep stage after stimulus administration. However, the uncertainties in the K- complex after the stimulus was given to the individual were increased. This indicates that auditory stimuli affects sleep disorders.

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