

An Improved Ant Colony System Algorithm for Solving Shortest Path Network Problems

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Abstract: Shortest Path Problems (SPP) are concerned with finding a path with minimum distance from one or more sources to one or more destinations through a network. With the increasing application of shortest path algorithms to network problems in real life, researchers and practitioners have begun to look outside the traditional algorithms, such as label setting and label correcting, which have some deficits compared with an algorithm such as Ant Colony. In this paper, an improved ant colony system meta-heuristic algorithm for solving SSP has been presented with modifications made in the following areas: the introduction of dynamic programming into the heuristic information, and the application of a ratio approach to the local pheromone update process. A hypothetical network problem of ten nodes with twenty edges was used as a test case. The results show that the improved ant colony algorithm outperforms the existing one in terms of the number of iterations required to converge to optimality.

Keywords: Shortest path, Network, Label-setting, Label-correcting, Ant colony system, Algorithm, Dynamic programming.

1. Introduction

A network is a collection of entities connected by some relationship, which can be represented as a graph. Shortest Path Problems (SPP) are concerned with finding a path with minimum distance from one or more sources to one or more destinations through a network (Yongtaek et al, 2010; Paraveen et al, 2013).

Classical examples of SPPs are the Travelling Salesman Problem (TSP), the Knapsack Problem (KP) and the Capacitated Vehicle Routing Problem (CVRP). There is also an increasing application of SPP in real life. Researchers in algorithms for solving SPPs have begun to look outside the traditional algorithms such as label setting and label correcting, which have some deficits in handling large scale and complicated network problems, to novel meta-heuristic algorithms. To address the stated shortfalls of the exact algorithms, the ant colony algorithm was introduced by Dorigo (1992). It has been the focal point for researchers with the hope of finding a better algorithm for solving large scale and complicated networks problems (Gulben and Orhan, 2015). Some current contributors to the literature of ant colony algorithms are Mikita et al (2014) who introduced the Elitist strategy and punishment strategy pheromone update where extra pheromone is added to the edges that belong to the best paths whilst pheromone are subtracted from edges belonging to the worst paths per iteration, Anuj et al (2012) incorporated a genetic algorithm to the ant colony algorithm, Gulben and Orhan (2015) developed the ant colony system algorithm with parametric analysis and Xiaomeng (2016) proposed the pheromone update increment as the inverse of the pheromone of the path constructed by the ant.

The current research develops an improved ant colony system algorithm for solving SPPs. The proposed algorithm has been applied to a network of ten (10) nodes with 20 edges. Ant colony algorithm is used to solve the problem because it was primarily designed to handle network problems and has demonstrated its ability in obtaining the best solution for large network problems (Gulben and

Orhan, 2015). The ant Colony algorithm was inspired by the socialist lifestyle exhibited by ants in search of food and other provisions (Dhiman et al., 2014). They determine the shortest path from their source of food to nest through a chemical called pheromone. The ants drop this pheromone along their paths as they carry the food to the nest. The path that accumulates more of the pheromone gives the shortest path. For example in Figure 1, the first ant walks randomly until it discovers the food source (F); it then returns to the nest (N), depositing a quantity of pheromone along the path. Other ants follow one of the paths (as in 2) at random, also depositing a quantity of pheromone in their trail. As the ants on the shortest path lay pheromone faster, this path will contain more pheromone than the other paths, thus making it more desirable (as in 3) to subsequent ants. Therefore, the shortest path is likely to be followed by many ants as it is regularly reinforced with a greater quantity of pheromone. The pheromone of the longer paths evaporate with time, therefore making them less attractive to other ants (Syed and Sindhanaiselvin, 2013).

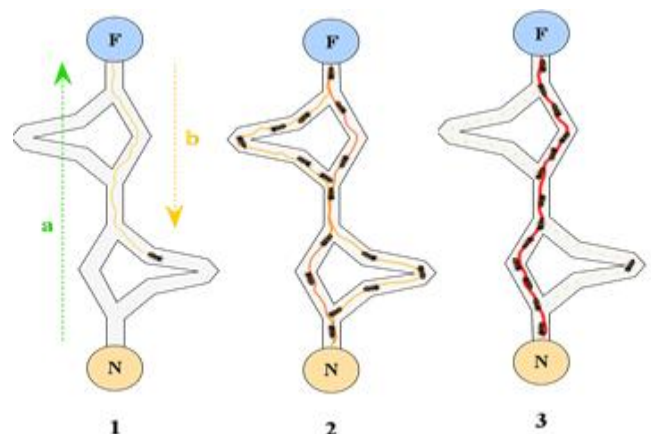


Figure 1: The behavior of real ants on a path with time (Syed and Sindhanaiselvin, 2013)

The ant colony algorithm is one in which a number of artificial ants are considered at each iteration. Iteration refers to the total number of steps taken by all the ants to accomplish a feasible solution (Manuel, 2004). Each ant

builds a solution by moving from one node to another on the graph network with the constraint of not coming back to the same node which it had been to before in its movement (Gianni, 2004). At each step of the construction of the solution, the decision of an artificial ant moving from node i to node j is based on a stochastic process which is influenced by the pheromone trail, provided node j has not been visited before by the ant (Dorigo et al, 2006).

Like real ant colonies, artificial ant algorithms are made-up of entities cooperating to find a solution to the problem under consideration. Although the complex nature of each artificial ant makes it capable of finding a feasible solution, high quality solutions are the result of cooperation. Ants cooperate by means of the information they simultaneously read on the problem states. Stigmergy is a mechanism of communication by modifying the environment. Both real and artificial ants modify some aspects of their environment. As real ants deposit pheromone on the path they visit, artificial ants change some numeric information of the problem states (Dorigo et al, 1999). This information takes into account the ant's current performance and can be obtained by any ant accessing the state. In ant algorithms, the only means of communication among the ants is by the pheromone. They affect the way the problem environment is perceived by the ants as a function of the past history. Also an evaporation mechanism, similar to real pheromone evaporation, modifies the pheromone (Blum, 2005). The objective of both real and artificial ants is to find a shortest path joining an origin (nest) to destination (food) site. They both move step-by-step through adjacent states of the problem. Artificial ants, as real ones, construct solutions by applying a probabilistic decision to move through adjacent states (Dhiman et al, 2014).

The three main types of meta- heuristics ACO algorithms are: (i) Ant System (AS) by Dorigo (1992) (ii) Max-Min Ant System (MMAS) by Hoos and Stutzle (1996) (iii) Ant Colony System (ACS) by Dorigo and Gambardella (1997). The three meta-heuristics ACO algorithms differ by the way they update the pheromone trail values. The current work is situated within ACS. In the next section, the characteristics of the ACS and the modifications for improvement made to it by the authors is presented; this is followed by a discussion of a test case. The final section concludes the paper.

2. The ACS and Improvements

2.1 The Ant Colony System Algorithm

The seven main steps in ACS are: (i) setting parameters (ii) initializing pheromone trails. (iii) calculating the heuristic Information. (iv) building of the ant solution by using the stochastic state transition rule. (v) carrying out the local pheromone update. (vi) applying local search to improve solution constructed by an ant. (vii) Updating the global pheromone information.

Initialization of pheromone: This is the initial pheromone $\tau_{ij}(0)$ that is deposited along the route (i, j) at the beginning of the search, which is usually a small positive constant

value. The purpose of the initial pheromone is to make sure that every edge has at least a little probability of being selected (Gianni, 2004). The initial pheromone is defined (see Oto et al, 2014) as:

$$\tau_{ij}(0) = \frac{1}{n}$$

where n is the number of number nodes.

Heuristic information phase: Heuristic information, known as visibility measure is an additional information available to the ant algorithm that helps to streamline the search procedure: it is usually referred to as a local information. Heuristic information tells how attractive a route is; which assists the ants in the construction of shortest path (Mavrovouniotis PhD thesis, 2013). It is defined as a reciprocal of the distance between two nodes as:

$$\eta_{ij} = \frac{1}{d_{ij}}$$

Pheromone update rule: The updating of pheromone along a trail is carried out after all the ants have constructed the solutions. However, different researchers have come up with other updating schemes. The three aspects under pheromone update are: the amount of pheromone to be laid, the amount be evaporated, and the ants that are permitted to lay their pheromone (Nada, 2009).The two pheromone updates are the local pheromone update and global pheromone update.

Local Update: The local updating rule is applied on the edges whilst constructing the solutions, in order to reduce the quantity of pheromone on them (Nada, 2009). Since its introduction by Dorigo and Gambardella (1997), a number of people have come up with different approaches for calculating it.

Chia-Ho et al (2009) proposed a formula for calculating the local pheromone update as:

$$\tau_{ij}(t+1) = (1-\rho)\tau_{ij}(t) + \rho\tau_{ij}(0) \text{ if } \{edge(i,j) \in R_k\}$$

Where ρ is the rate of evaporation of the pheromone along (i, j) trail.

Global Update: Global updating is performed by all ants that have completed their schedule. It is to increase the content of the pheromone on the path to facilitate the convergence of the algorithm. Yu et al (2009) proposed a formula for calculating the pheromone trail increment as:

$$\Delta\tau_{ij}^k = \begin{cases} \frac{Q}{K \times L} \times \frac{D^k - d_{ij}}{m^k \times D^k} & \text{if link } (i, j) \text{ is on the } k\text{th route} \\ 0 & \text{otherwise} \end{cases}$$

where Q is a constant value; D^k is length of the k^{th} path (solution); L is the sum of the length of all the paths generated i.e. $\sum_k D^k$; k is the name of the path; m^k is the number of customers in the k^{th} path; $m^k > 0$; K is the number of paths generated and $K > 0$.

Probabilistic Decision Rule phase: In constructing the feasible solutions, the decision by the ants to move from one node to the next is based on a stochastic probability rule.

The probabilistic decision rule by Majid et al. (2011) is given as:

$$P_{ij}^k(t) = \begin{cases} 1 & \text{if } \{q \leq q_0 \text{ and } j = j^*\} \\ 0 & \text{if } \{q \leq q_0 \text{ and } j \neq j^*\} \\ \frac{(\tau_{ij}(t))^\alpha (\eta_{ij}(t))^\beta}{\sum_{r \in N_i^k} (\tau_{ir}(t))^\alpha (\eta_{ir}(t))^\beta} & \text{otherwise} \end{cases} \quad j^*: \text{ yields } \arg \max_{r \in N_i^k} \left((\tau_{ij}(t))^\alpha (\eta_{ij}(t))^\beta \right)$$

and it is used to identify the unvisited node in N_i that

$$\eta_{ij} = \frac{1}{(d_{0i} + d_{ij})}$$

maximizes $P_{ij}^k(t)$ (Majid et al., 2011).

$N_i^k \in N_i$ is the set of nodes which are the neighbors of node i and are not yet visited by ant k (nodes in N_i^k are obtained from those in N_i by the use of ant k 's private memory H_i^k (which stores nodes already visited by the ant)).

N_i is the set of nodes which are directly linked to node i by an edge (i.e. the neighbours of node i).

$\tau_{ij}(t)$ is the quantity of pheromone trail laid along the edge linking node i and node j .

$\eta_{ij}(t)$ is the heuristic information for the ant visibility measure.

α is the parameter to control the influence of τ_{ij}

β is the parameter to control the influence of η_{ij}

q_0 is the pre-defined parameter ($0 \leq q_0 \leq 1$).

q is the uniformly distributed random number to determine the relative importance of exploitation versus exploration, $q \in [0, 1]$.

2.2 Proposed Improvement of the Ant Colony System Algorithm

The key contributions made to this algorithm falls under initial pheromone trail, heuristic information, local pheromone update and global pheromone update. The initial pheromone trail plays a vital role in the beginning stage of the construction of solutions by ants. However, the uniform values used as initial pheromone trails by researchers (e.g. Blum, 2005) do not fairly discriminate among the routes for the ants to make an informed choice, resulting in many ants generating solutions that are far from the best solution. Therefore, the new initial pheromone trail has been proposed to address this challenge. They are as follow:

The proposed initial pheromone trail is given as:

$$\tau_{ij}(0) = \frac{1}{(n-1)\bar{L}_{ij}}$$

n is the number of nodes (i.e. the size of the problem)

\bar{L}_{ij} is the ratio of the distance linking node i to node

$$j, \bar{L}_{ij} = \frac{d_{ij}}{\sum d_{ij}}$$

Heuristic information tells how attractive a route is, which assists ants in the construction of shortest path. The heuristic information in the literature is always calculated based on the current node, i where the ant is and the next node j . However, the current approach does not provide the ant with information on the nature of the path from source to the current node i . It is for this reason that, the new heuristic information has been proposed to solve the problem. Here the heuristic information is not dependent only on the usual distance from i to j but also on the distance from source to i .

The proposed heuristic information is therefore given as:

where:

d_{0i} is the minimum distance from the source node to the present node i .

d_{ij} is the distance from node i to node j .

The aim of the local pheromone update is to reduce the content of the pheromone along the routes to encourage other ants to generate new paths. However, care has to be taken so that paths which are far from the optimal are not created. Therefore, in this new approach, the reduction in the pheromone level during the construction of paths is such that smaller quantity is taken from the shortest routes compared to that taken on longer routes. This departs from the one in the literature (Chia-Ho et al, 2009) where a constant value is deducted along all the routes.

The proposed local pheromone trail update is therefore given as:

$$\tau_{ij}(t+1) = (1-\rho)\frac{\tau_{ij}(t)}{L_{ij}} + \rho\tau_{ij}(0)$$

Where

$$L_{ij} = \sum \tau_{ij}(t)$$

The proposed increment pheromone trail update is given as:

$$\tau_{ij}(t+1) = (1-\rho)\tau_{ij}(t) + \frac{1}{\rho} \sum_{k=1}^m \Delta\tau_{ij}^k$$

In the model for increment in the global update by Yu et al (2009), K is the number of paths and L the sum of the length of all paths generated and used for updating all the paths no matter the quality of it. This does not result in quick convergence to optimality. In the proposed model, the update is carried out based on the quality of the path generated. Hence it enhances the speed of the search for optimal solution.

The proposed increment is therefore given as:

$$\Delta\tau_{ij}^k = \begin{cases} \frac{1}{\bar{L}} \cdot \frac{L^k - d_{ij}}{L^k}, & \text{if } (i,j) \in U_i \text{ or } I_i \\ 0, & \text{otherwise} \end{cases}$$

d_{ij} is the distance of the edge (i, j)

L^k is the total distance of the k^{th} solution generated by ant k

\bar{L} is the ratio of the distance of the feasible solutions generated.

The probability decision rule is the same as the already stated one proposed by Majid et al (2011) with the proposed initial pheromone trail, heuristic information, local pheromone update trail and global pheromone trail update embedded in it.

3. A Test Problem

Let $G(V, E)$ be undirected graph consisting of an indexed set of nodes V with $n = |V|$ and a spanning set of edges (arcs) E with $m = |E|$, where n and m are the numbers of nodes and edges (arcs), respectively. Each arc is represented as a pair of nodes, thus from node i to node j , and denoted by (i, j) . Each arc is associated with distance.

The optimization model with distance as objective is given as:

$$\min \sum_{(i,j) \in E} d_{ij} x_{ij}$$

subject to: $x_{ij} \in \{0, 1\}, \forall (i, j)$
 $i = 1, 2, \dots, n - 1; j = 1, 2, \dots, n$

where:

d_{ij} is the distance from node i to node j ;

x_{ij} is a binary decision variable.

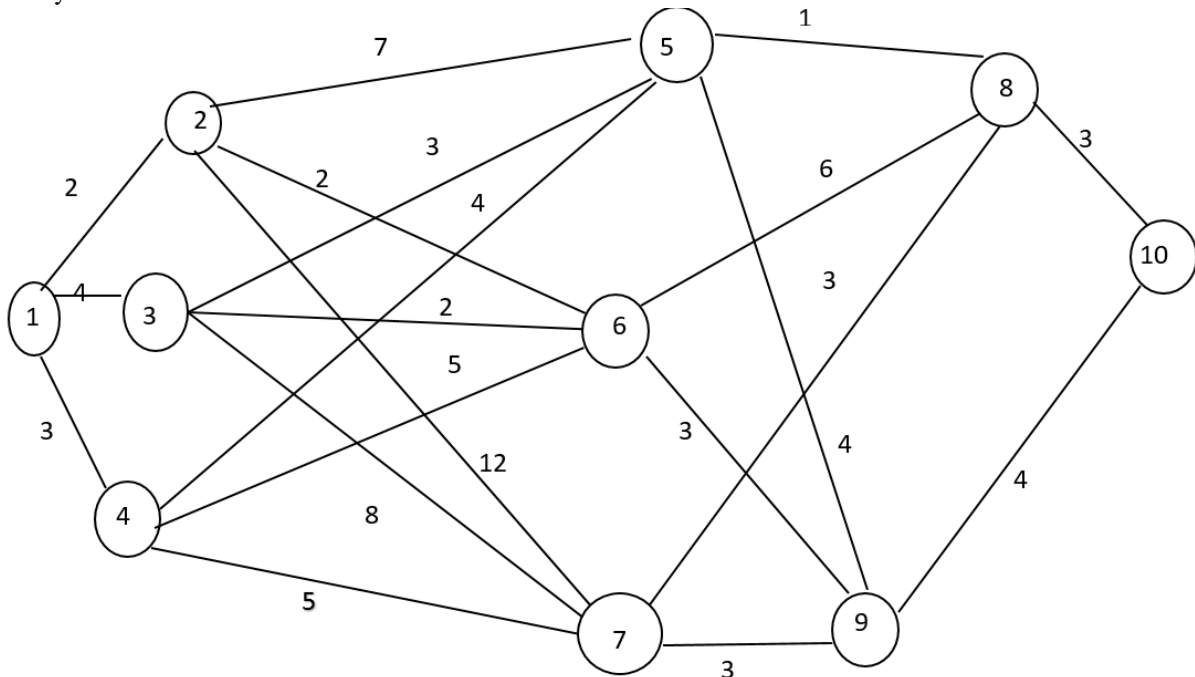


Figure 2: A network of 10 nodes showing the distances between them

3.1 Results and Discussions

Table 1 presents the results of the network problem considered. In the table, the first and the second columns are the names of the paths and their composites (path sets) whereas the third column gives the corresponding distances. Also, the last row is the number of iterations required by the existing Ant Colony System Algorithm (E) and the Improved Ant Colony System Algorithm (I) respectively to converge to the optimal solution.

Table 1: Results of the network problem considered

Paths Name	Path set	Distance (km)
P_1	1 - 3 - 7 - 8 - 10	18
P_2	1 - 2 - 5 - 8 - 10	13
P_3	1 - 2 - 5 - 9 - 10	17
P_4	1 - 3 - 6 - 8 - 10	15
P_5	1 - 3 - 6 - 9 - 10	13
P_6	1 - 3 - 5 - 8 - 10	11
P_7	1 - 3 - 5 - 9 - 10	15
P_8	1 - 2 - 6 - 8 - 10	13

P_9	1 - 3 - 7 - 9 - 10	19
P_{10}	1 - 4 - 6 - 8 - 10	17
P_{11}	1 - 4 - 7 - 8 - 10	14
P_{12}	1 - 4 - 5 - 9 - 10	15
P_{13}	1 - 4 - 5 - 8 - 10	11
P_{14}	1 - 2 - 6 - 9 - 10	11
P_{15}	1 - 4 - 6 - 9 - 10	15
P_{16}	1 - 2 - 7 - 9 - 10	21
P_{17}	1 - 2 - 7 - 8 - 10	18
The existing ACS takes 140 iterations to converge to optimal whilst the improved ACS takes 129		

From Table 1, the paths 1 - 3 - 7 - 8 - 10 (P_1) and 1 - 2 - 7 - 8 - 10 (P_{17}) each cover a total distance of 18km. Also, 1 - 2 - 5 - 8 - 10 (P_2), 1 - 3 - 6 - 9 - 10 (P_5) and 1 - 2 - 6 - 8 - 10 (P_8) each have a total distance of 13km. Besides, 1 - 2 - 5 - 9 - 10 (P_3) and 1 - 4 - 6 - 8 - 10 (P_{10}) a total distance of 17km. The same interpretation could be given to the remaining paths (table). The total minimum distance to be covered is 11km, associated with the paths 1 - 3 - 5 - 8 - 10 (P_6), 1 - 4 - 5 - 8 - 10 (P_{13}) and 1 - 2 - 6 - 9 - 10 (P_{14}).

10(P_{14}). Thus, the problem has multi-optimal paths. The existing ACS algorithm converges to optimality after 140 iterations whilst the improved ACS algorithm takes 129 iterations. Therefore, the improved Ant Colony System algorithm outperformed the existing one in terms of the number of iterations required to converge to optimality.

4. Conclusion

Improvements have been made to the existing Ant Colony System optimization algorithm and the algorithm applied successfully to a shortest path derived from a hypothetical network problem. The improved ant colony system algorithm outperformed the existing ant colony system algorithm in terms of the number of iterations required to converge to optimality. Future work would consider a real network under possibly multi-objective optimization framework.

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