

# Evaluation of the Student's Performance Using Fuzzy System

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**Abstract:** Examination is one of the common assessment methods to assess the level of knowledge of students. Assessment methods probably have a greater influence on how and what students learn than any other factor. Assessment is used to discriminate not only between different students but also between different levels of thinking. Due to the increasing trends in class sizes and limited resources for teaching, the need arises for exploring other assessment methods. Here, we propose an evaluation method considering the importance, difficulty, and complexity of questions.

**Keywords:** Fuzzy Evaluation System, Fuzzy Membership Function

## 1. Introduction

Evaluations of the student's performance have a greatest influence on how and what students learn than any other factors. Students are usually preoccupied with what constitutes the assessment in their chosen field and therefore assessment usually drives student learning. Evaluation of the students determines student approaches to learning (Boud, 1988). Evaluation process sends messages to students to define and priorities what is important to learn and ultimately how they spend their time leaning it. Assessment can be used to, as far as possible, create positive incentives for teachers to teach well, and for students to study well (Wiliam, 2011). However, despite its importance, 'assessment remains the aspect of the curriculum teaching and learning practices that is least amenable to change' (Scarino, 2013). Despite the challenges of making changes to assessment, there has been a need for 'change' due to the increasing trends in class sizes and limited resources for teaching (Donnelly, 2014).

Evaluation of students' learning achievement is the process of determining the performance levels of individual's students in relationship to education objectives. A high quality evaluation system certifies, supports, and improves individual achievement and ensures that all students receive fair treatment in order not to limit student's present and future opportunities. Thus, the system should regularly be reviewed and improved to ensure that it is proper, fair and beneficial to all students. It is also desirable that the system has transparency and automation in the evaluation.

Since its introducing in 1965 by Lotfi Zadeh (1965) the fuzzy set theory has been widely used in solving problems in various fields, and recently in education evaluation. Biswas (1995) presented two methods for student's answer scripts evaluation using fuzzy sets and a matching function: a fuzzy evaluation method. Chen and Lee (1999) present two methods for applying fuzzy sets to overcome the problem of rewarding two different fuzzy marks the same total score which could arise from Biswas' method. Echauz and Vashtsevanos (1995) proposed a fuzzy structure model for education garding system with its algorithm to aggregate different test score in order to produce a single score for

individual student. He also proposed a method to build the membership functions (MFs) of several linguistic values with different weights. Wilson. Karr and Freeman (1998) presented an automatic grading system based on fuzzy rules and genetic algorithms. Ma and Zhou (2000) proposed a fuzzy set approach of their to assess the outcome of student-centered learning using the evaluation student's answer scripts using fuzzy numbers associated with degree of confidence of the evaluator. From the previous studies, it can be found that fuzzy numbers, fuzzy sets, fuzzy rules and fuzzy systems are used for various educational grading systems.

Weon and Kim (2001) presented an evaluation strategy based on fuzzy MFs. They pointed out that system for student's achievement evaluation should consider the three important factors of the questions which the students answer: the difficulty. The importance and the complexity, Weon and Kim used singleton function to describe the factors of each question reflecting the effect of the three factors individually, but not collectively. Bai and Chen (2008b) pointed out that difficulty factors is a very subjective parameter and may cause an argument about fairness in evaluation.

Bai and Chen (2008a) proposed a method to automatically construct the grade MFs of fuzzy rules for evaluation student's learning achievement. Bai and Chen (2008b) proposed a method for applying fuzzy MFs and fuzzy rules for the same purpose. To solve the subjectivity of the difficulty factor of Weon and Kim's method (2001), they obtained the difficulty as a function of accuracy of the student's answer scripts and time consumed to answer. However, their method still has the subjectivity problem, since the results in scores and ranks are heavily depend on the values of several weight which are determined by the subjective knowledge of domain experts.

Here, we propose an evaluation method considering the importance, difficulty, and complexity of questions based on Mamdani's fuzzy inference (Mamdani, 1974) and center of gravity defuzzification which is an alternative to Bai and Chen's method (2008b). The transparency and objective nature of the fuzzy system makes it easy to understand and

explain the result of evaluation and thus to persuade the students.

## 2. A Review of Evaluation Methods using Membership Function and Fuzzy Rules

In this paper, we consider the same situation as in Bai and Chen's (2008b). Assume that there are  $n$  students to answer  $m$  questions. Accuracy rates of student's answer scripts (student's score in each question divided by the maximum score assigned to this question) are the basis for evaluation. We get an accuracy rate matrix of dimension  $m \times n$ ,  $A=[a_{ij}]$ ,  $m \times n$ ,

Where  $a_{ij} \in [0,1]$  denotes the accuracy rate of student  $j$  on question  $i$ . Time rates of student (the time consumed by a student to solve a question divided by the maximum time allowed to solve this equation) is another basis to be considered in evaluation. We get a time rate of matrix of dimension  $m \times n$ ,

$$T=[t_{ij}], m \times n$$

Where  $t_{ij} \in [0,1]$  denotes the time rate of student  $j$  on question  $i$ . We are given a grade vector.

$$G=[g_i], m \times 1,$$

Where  $g_i \in [1, 100]$  denotes the assigned maximum score of question  $i$  satisfying

$$\sum_{i=1}^m g_i = 100$$

Based on the accuracy rate matrix  $A$  and the grade vector  $G$ , we obtain the total score vector of dimension  $n \times 1$ ,

$$S=A^T G=[s_j], n \times 1,$$

Where  $s_j \in [0, 100]$  is the total score of student  $j$  which is obtained by

$$s_j = \sum_{i=1}^m a_{ij} \cdot g_i$$

The classical rank of students is then obtained by sorting values of  $S$  in a descending order.

**Example 1:** Assume that 10 students laid to an exam of 5 questions and the accuracy rate matrix, the time rate matrix and grade vector are given as follows:

$$A = \begin{bmatrix} 0.59 & 0.35 & 1 & 0.66 & 0.11 & 0.08 & 0.84 & 0.23 & 0.04 & 0.24 \\ 0.01 & 0.27 & 0.14 & 0.04 & 0.88 & 0.16 & 0.04 & 0.22 & 0.81 & 0.53 \\ 0.77 & 0.69 & 0.97 & 0.71 & 0.17 & 0.86 & 0.87 & 0.42 & 0.91 & 0.74 \\ 0.73 & 0.72 & 0.18 & 0.16 & 0.5 & 0.02 & 0.32 & 0.92 & 0.9 & 0.25 \\ 0.93 & 0.49 & 0.08 & 0.81 & 0.65 & 0.93 & 0.39 & 0.51 & 0.97 & 0.61 \end{bmatrix}$$

$$A = \begin{bmatrix} 0.7 & 0.4 & 0.1 & 1 & 0.7 & 0.2 & 0.7 & 0.6 & 0.4 & 0.9 \\ 1 & 0 & 0.9 & 0.3 & 1 & 0.3 & 0.2 & 0.8 & 0 & 0.3 \\ 0 & 0.1 & 0 & 0.1 & 0.9 & 1 & 0.2 & 0.3 & 0.1 & 0.4 \\ 0.2 & 0.1 & 0 & 1 & 1 & 0.3 & 0.4 & 0.8 & 0.7 & 0.5 \\ 0 & 0.1 & 1 & 1 & 0.6 & 1 & 0.8 & 0.2 & 0.8 & 0.2 \end{bmatrix}$$

$G^T = [10 \ 15 \ 20 \ 25 \ 30]$ , we denote transpose of  $V$  as  $V^T$ .

Important of the questions is an important factor to be considered. We have  $l$  levels of importance to describe the degree of importance of each question in the fuzzy domain. The domain expert determine the importance matrix of dimension  $m \times l$

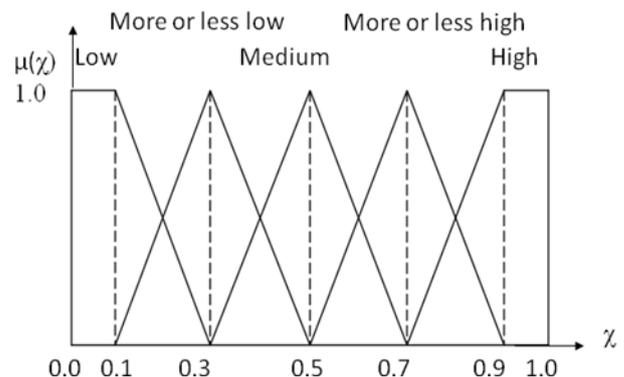
$$P = [P_{ik}], m \times l,$$

Where  $P_{ik} \in [0, 1]$  denotes the degree of membership of question  $i$  belonging to the importance level  $k$ . In this paper, five levels (fuzzy sets) of importance ( $l = 5$ ) are used;  $k = 1$  for linguistic term "low",  $k = 2$  for "more or less",  $k = 3$  "medium",  $k = 4$  "more or less high", and  $k = 5$  for "high". Their MFs are shown in fig. 2.1. we note that the same five fuzzy sets are applied to the accuracy, the time rate, the difficulty, the complexity and the adjustment of questions. The values of  $P_{ik}$ 's are obtained by the fuzzification once crisp values are given for the importance of questions by domain expert.

Complexity of the question which indicates the ability of students to give correct answers is also an important factor to be considered. The domain expert determine the fuzzy complexity matrix of dimension  $m \times l$ ,

$$C = [C_{ik}], m \times l,$$

Where  $c_{ik} \in [0,1]$  denotes the degree of membership of question  $i$  belonging to the complexity level  $k$ .



**Figure 1:** Membership functions of the fuzzy sets "low", "more or less low", "medium", "more or less high" and "high".

### Example 2

For the above example we get the following by domain expert:

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0.33 & 0.67 & 0 & 0 \\ 0 & 0 & 0 & 0.15 & 0.85 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0.07 & 0.93 & 0 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 0.85 & 0.15 & 0 & 0 \\ 0 & 0 & 0.33 & 0.67 & 0 \\ 0 & 0 & 0 & 0.69 & 0.31 \\ 0.56 & 0.44 & 0 & 0 & 0 \\ 0 & 0 & 0.7 & 0.3 & 0 \end{bmatrix}$$

Total score is then obtained as

$$S^T = [67.60 \ 54.05 \ 38.40 \ 49.70 \ 49.70 \ 48.80 \ 46.10 \ 52.30 \ 85.95 \ 49.70],$$

And thus the classical rank of students is then becomes:

$$S_9 > S_1 > S_2 > S_8 > S_4 = S_{10} > S_6 > S_7 > S_3.$$

Bai and Chen's method (2008b) uses 3 steps to evaluate student's answer script. In the first step, using the accuracy rate vector of dimension  $m \times 1$ ,

$$\bar{A} = [a_i], m \times 1,$$

Where  $a_i$  denotes the average accuracy rate of question  $i$  which is obtained by

$$a_i = \sum_{j=1}^n a_{ij} / n, \dots\dots\dots(1)$$

And the average time rate vector of the same dimension,

$$\bar{T} = [t_i], m \times 1,$$

Where  $t_i$  denotes the average time rate of question  $i$  which is obtained by

$$t_i = \sum_{j=1}^n t_{ij} / n,$$

We obtained the fuzzy accuracy rate matrix of dimension  $m \times l$

$$FA = [fa_{ik}], m \times l,$$

Where  $fa_{ik} \in [0, 1]$  denotes the membership value of the average accuracy rate of question  $i$  belonging to level  $k$ , and the fuzzy time rate matrix of dimension  $m \times l$ ,

$$FT = [ft_{ik}], m \times l,$$

Where  $ft_{ik} \in [0, 1]$  denotes the membership value of the average time rate of question  $i$  belonging to level  $k$ , respectively

**Example 3**

In the above example, we get

$$\bar{A}^T = [0.45 \ 0.31 \ 0.711 \ 0.47 \ 0.637],$$

$$\bar{T}^T = [0.057 \ 0.48 \ 0.31 \ 0.50 \ 0.57].$$

Based on the fuzzy MFs in fig. 1, we obtain the fuzzy accuracy rate matrix and the fuzzy time rate matrix:

$$FA = \begin{bmatrix} 0 & 0.25 & 0.75 & 0 & 0 \\ 0 & 0.95 & 0.05 & 0 & 0 \\ 0 & 0 & 0 & 0.945 & 0.055 \\ 0 & 0.15 & 0.85 & 0 & 0 \\ 0 & 0 & 0.315 & 0.685 & 0 \end{bmatrix},$$

$$FT = \begin{bmatrix} 0 & 0 & 0.65 & 0.35 & 0 \\ 0 & 0.1 & 0.9 & 0 & 0 \\ 0 & 0.95 & 0.05 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0.65 & 0.35 & 0 \end{bmatrix}.$$

In the second step, based on the fuzzy accuracy rate matrix,  $FA$ , fuzzy time rate matrix,  $FT$ , and the fuzzy rules,  $\mathfrak{R}_D$ , given in the form of IF-THEN rules, we obtain the fuzzy difficulty matrix of dimension  $m \times l$ ,

$$D = [d_{ik}], m \times l,$$

Where  $d_{ik} \in [0, 1]$  denotes the membership of difficulty of question  $i$  belonging to level  $k$ . When the level of accuracy,  $l_A$ , and the level of time,  $l_T$ , are given, the level of difficulty,  $l_D$ , is determined by the relation given by fuzzy rules,

$$l_D = \mathfrak{R}_D(l_A, l_T).$$

Denoting the weight of the accuracy rate and time rate, which are determined by domain expert, by  $w_A$  and  $w_T$  ( $w_A + w_T = 1$ ), respectively. The value of  $d_{ik}$  is obtained by

$$d_{ik} = \max_{\{\mathfrak{R}_D(l_A, l_T)\}} \{w_A \cdot fa_{i,l_A} + w_T \cdot ft_{i,l_T}\}.$$

Next, based on the fuzzy difficulty matrix,  $D$ , fuzzy complexity matrix,  $C$ , their weights,  $w_D$  and  $w_C$  ( $w_D + w_C = 1$ ), respectively, and the fuzzy rules,  $\mathfrak{R}_{AC}$ , we obtain the cost matrix of dimension  $m \times l$ , in the same manner

$$AC = [ac_{ik}], m \times l,$$

Where  $ac_{ik} \in [0, 1]$  denotes the degree of membership of the cost of question  $i$  belonging to level  $k$ , which is a measure of cost for students to answer question  $i$ .

Based on the fuzzy cost matrix,  $AC$ , fuzzy importance matrix,  $P$ , their weights,  $w_{AC}$  and  $w_P$  ( $w_{AC} + w_P = 1$ ), respectively, and the fuzzy rules,  $\mathfrak{R}_W$ , we obtain the adjustment matrix of dimension  $m \times l$

$$W = [w_{ik}], m \times l,$$

Where  $w_{ik} \in [0, 1]$  denotes the degree of membership of adjustment required by question  $i$  belonging to level  $k$ .

Then we use the following formula to obtain the adjustment vector,

$$\bar{W} = [w_i], m \times 1,$$

Where  $w_i \in [0, 1]$  denotes the final adjustment value required by question  $i$  obtained by

$$w_{i_1} = \frac{0.1 \times w_{i_1} + 0.3 \times w_{i_2} + 0.5 \times w_{i_3} + 0.7 \times w_{i_4} + 0.9 \times w_{i_5}}{0.1 + 0.3 + 0.5 + 0.7 + 0.9}, \dots\dots\dots(2)$$

Where 0.1, 0.3, 0.5, 0.7 and 0.9 are the centres of the fuzzy MFs shown in Fig. 2.1.

**Example 4**

Assume that we are given the rule base for  $\mathfrak{R}_D$  in Table 2.1 (a),  $\mathfrak{R}_{AC}$  and  $\mathfrak{R}_W$  in table 2.1 (b), respectively. The difficulty level of 1 ( $l_D = 1$ ) for question 1, for example, is obtained from  $\mathfrak{R}_D(4, 1)$ ,  $\mathfrak{R}_D(5, 1)$  and  $\mathfrak{R}_D(5, 2)$ . By setting  $w_A = 0.6$  and  $w_T = 0.4$ :  
 $d_{11} = \max [(0.6 \cdot fa_{14} + 0.4 \cdot ft_{11}), (0.6 \cdot fa_{15} + 0.4 \cdot ft_{11}), (0.6 \cdot fa_{15} + 0.4 \cdot ft_{12})]$   
 $= \max [(0.6 \cdot 0 + 0.4 \cdot 0), (0.6 \cdot 0 + 0.4 \cdot 0), (0.6 \cdot 0 + 0.4 \cdot 0)]$   
 $= \max [0, 0, 0] = 0.$

**Table 1:** A fuzzy rules base to infer the difficulty, cost and adjustment

(a) Difficulty						(b) Cost							
		Difficulty							Cost				
Accuracy	Time Rate	1	2	3	4	5	Difficulty	Complexity	1	2	3	4	5
1		3	4	4	5	5	1		1	1	2	2	3
2		2	3	4	4	5	2		1	2	2	3	4
3		2	2	3	4	4	3		2	2	3	4	4
4		1	2	2	3	4	4		2	3	4	4	5
5		1	1	2	2	3	5		3	4	4	5	5

(Where 1 is “low”, 2 is “more or less low”, 3 is “medium”, 4 is “more or less high”, and 5 is “high”)

In the same manner, we obtain  $d_{12} = 0.45$ ,  $d_{13} = 0.71$ ,  $d_{14} = 0.59$  and  $d_{15} = 0.15$ . thus, we obtain

$$D = \begin{bmatrix} 0 & 0.45 & 0.71 & 0.59 & 0.15 \\ 0.04 & 0.57 & 0.61 & 0.93 & 0.57 \\ 0.567 & 0.947 & 0.567 & 0.567 & 0 \\ 0 & 0.51 & 0.91 & 0.51 & 0.09 \\ 0.411 & 0.671 & 0.551 & 0.411 & 0.14 \end{bmatrix}$$

By setting  $w_D = 0.7$  and  $w_C = 0.3$ , we obtain the fuzzy cost matrix.

$$AC = \begin{bmatrix} 0.315 & 0.752 & 0.668 & 0.497 & 0.413 \\ 0.399 & 0.651 & 0.651 & 0.852 & 0.651 \\ 0.663 & 0.663 & 0.87 & 0.756 & 0.49 \\ 0.525 & 0.805 & 0.637 & 0.637 & 0.357 \\ 0.47 & 0.68 & 0.596 & 0.498 & 0.288 \end{bmatrix}$$

By setting  $w_{AC} = 0.5$  and  $w_P = 0.5$ , we obtain the fuzzy adjustment

$$T = \begin{bmatrix} 0.376 & 0.376 & 0.658 & 0.876 & 0.750 \\ 0.365 & 0.661 & 0.661 & 0.761 & 0.426 \\ 0.331 & 0.435 & 0.756 & 0.860 & 0.803 \\ 0.903 & 0.819 & 0.679 & 0.403 & 0.319 \\ 0.340 & 0.805 & 0.763 & 0.714 & 0.249 \end{bmatrix}$$

For the example, the adjustment vector  $\bar{W}$  is obtained as:

$$\bar{W}^T = [0.706 \quad 0.592 \quad 0.747 \quad 0.497 \quad 0.552]$$

We note that Bai and Chen (2008b) applied their method only to the students with the same total score. Now, we apply the Bai and Chen’s method (2008) to the students with the same total score. Assume that there are  $q$  students with an equal total score. Rearranging these students from student 1 to student  $q$  and the corresponding original  $a_{ij}$ , we obtain

$$SOD = \begin{bmatrix} S_1 & S_2 & S_3 & S_4 & S_5 & S_6 & S_7 & S_8 & S_9 & S_{10} \\ 139.761 & -7.920 & -145.065 & -41.844 & -70.916 & -51.783 & -72.921 & -41.723 & 337.845 & -45.434 \end{bmatrix}$$

and a new rank is then obtained as:

$$S_9 > S_1 > S_2 > S_8 > S_4 > S_{10} > S_6 > S_5 > S_7 > S_3.$$

### 3. Three Node Fuzzy Evaluation System

Bai and Chen’s method (2008b) has seven weights which are determined subjectively by domain expert. Quite different ranks can be obtained depending on its values. By using their method, the examiners could not verify easily

$$EA = [a_{ij}], m \times q.$$

In the third step, the sum of differences (SOD) for students with the same total score is computed using the formula:

$$SOD_j = \sum_{k=1}^q \sum_{i=1}^m (a_{ij} - a_{ip}) \cdot \tilde{G}.$$

Where  $\tilde{G} = [\hat{g}_i]$ ,  $\hat{g}_i = 0.5 + w_i$ . A new rank is then obtained by sorting the values of  $SOD_j$  in a descending order.

#### Example.5

For this example, students  $S_4$ ,  $S_5$  and  $S_{10}$  have obtained the same total score (i.e.,  $q = 3$ ), therefore,

$$EA = \begin{bmatrix} S_4 & S_5 & S_{10} \\ 0.66 & 0.11 & 0.24 \\ 0.04 & 0.88 & 0.53 \\ 0.71 & 0.17 & 0.74 \\ 0.61 & 0.5 & 0.25 \\ 0.81 & 0.65 & 0.61 \end{bmatrix}$$

For this example, the SOD vector is obtained as:

$$SOD = \begin{bmatrix} S_4 & S_5 & S_{10} \\ 3.266 & -5.455 & 2.189 \end{bmatrix}$$

The new rank of students with the same total score is then becomes:

$$S_4 > S_{10} > S_5,$$

and thus the new ranking order of all students becomes:

$$S_9 > S_1 > S_2 > S_8 > S_4 > S_{10} > S_5 > S_6 > S_7 > S_3.$$

how new ranks are obtained and to persuade skeptic students. Naturally, there is no determine the optimum values of weights. To reduce the degree of subjectivity in this method, we propose a method applying the most commonly used Mamdani’s fuzzy inference mechanism (Mamdani, 1974) and center of gravity defuzzification technique. The same model of Bai and Chen’s method (2008b) will be used and for the sake of comparison the same numerical example will be used.

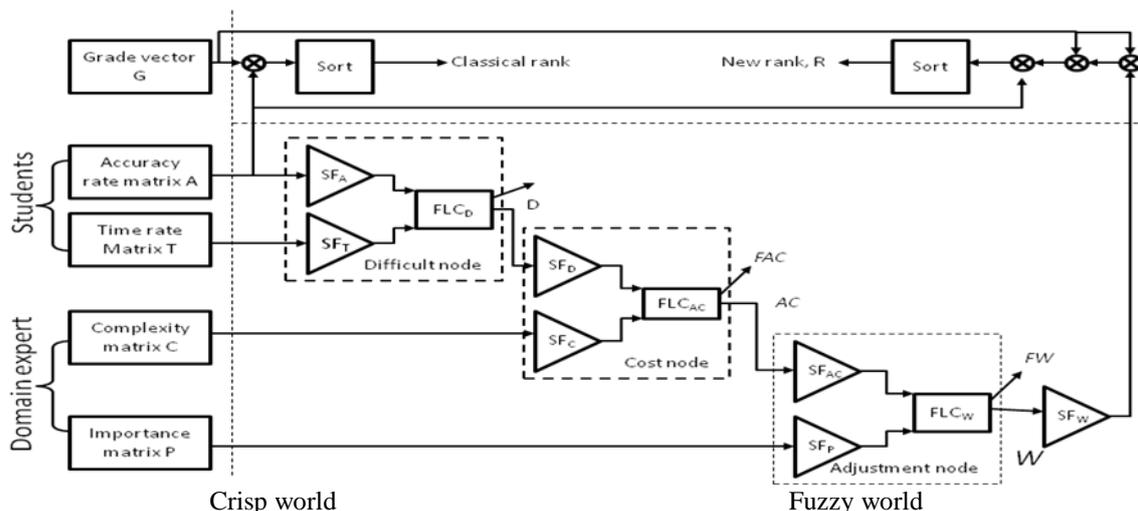
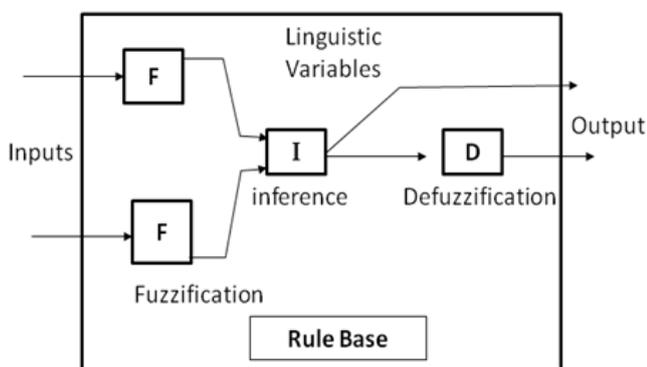


Figure 2: Block diagram of the three nodes fuzzy evaluation system

The block diagram of proposed system is shown in fig. 2. Bai and Chen's model (2008b) can be considered as a simple specific case of the block diagram. The system consists of three nodes, the difficulty node, the cost node, and the adjustment node. Input to the system, in the left part of figure, is given either by exam result or domain expert. Each node of the system behaves like a fuzzy logic controller (FLC) with two scable inputs and one output, as in Fig. 3. It maps a two-to-one relation by inference through a given rule base. The inputs are fuzzified based on the defined levels (Fuzzy sets) in Fig. 1. In the first node, both inputs are given by exam result, where in the later nodes, one input is the output of its previous node and the other is given by domain expert. The output of each node can be in the form of a crisp value (defuzzified) or in the form of linguistic variable (MFs). Each node has two scale factors, we can adjust the effects of inputs by varying the values of scale factors. In this paper, we let the SFs have the same value of unity to reflect the equal influence of each input on the output.



**Figure 3:** Node representations

In the proposed method, each fuzzy node proceeds in the steps:

**Step 1:** The first step is the fuzzification, where inputs are converted into membership degrees in the fuzzy sets in Fig. 1. Triangular MF is the commonly used due to its simplicity and easy computation.

**Step 2:** The second step is the application of the rule base, usually in the form of IF-THEN rules, through inference. We use Mamdani's max-min inference mechanism which is the most commonly used inference mechanism to produce fuzzy sets for defuzzification (1974). In Mamdani's max-min mechanism, implication is modelled by means of minimum operator and the resulting output MFs are combined using maximum operator. The inference mechanism can be written into

$$\alpha_{ik} = \max \{ \min (f_{i,l_1}, f_{i,l_2}) \} \quad (3)$$

Where  $\alpha_{ik}$  is the output of inference of question  $i$  in fuzzy set  $k$ . We have obtained a matrix of dimension  $m \times l$ ,  $\alpha = [\alpha_{ik}], m \times l$ .

**Step 3:** The third step is the defuzzification which is the process for converting fuzzy output values into a single crisp value or final decision. In this paper, the COG method is applied. The crisp value of question  $i$  is obtained by

$$y_i = \int_x x \cdot \mu(x) dx / \int_x \mu(x) dx, \quad \dots(4)$$

Where integrals are taken over the entire range of the output and  $\mu(x)$ . By taking the center of gravity, conflicting rules essentially are cancelled and a fair weighting is obtained.

Each of the three nodes follows the above scheme. The difficulty node has two inputs, the accuracy rate and the time rate and one output of the difficulty. The cost node has two inputs, the difficulty and complexity and one output of the cost. The adjustment node has two inputs, the cost and the importance and one output of the adjustment.

The adjustment vector,  $W$ , is then used to obtain the adjusted grade vector of dimension  $m \times l$ ,

$$\tilde{G} = [\tilde{g}_i], m \times l,$$

Where  $\tilde{g}_i$  is the adjusted grade of question  $i$ ,

$$\tilde{g}_i = g_i \cdot (1+w_i), \quad \dots(5)$$

and we obtain to its total grade by using the formula:

$$\tilde{g}_i = \tilde{g}_i \cdot \sum_j g_j / \sum_j \tilde{g}_j, \quad \dots(6)$$

Then we obtain the adjusted total scores of students by,

$$\tilde{S} = A^T \tilde{G}, \quad \dots(7)$$

The new rank of students is then obtained by sorting values of  $\tilde{S}$  in a descending order.

#### 4. Conclusion

Here, a fuzzy logic evaluation system for student's learning achievement is proposed. The proposed method considers the importance, complexity and difficulty of the question for student's answer script as a factors of evaluation. The system has been represented as a block diagram of three fuzzy logic controllers. Each logic controller generates an output from two inputs using Mamdani's max-min inference mechanism and the centre of gravity defuzzification.

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