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Mathematical Model of Stability Analysis of a Disturbs System (2)

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Abstract: ***A model for thermal convection loop is considered, and the basic governing differential equations are introduced with heat transfer by conduction, convection, and radiation. Study state solutions are obtained. The stability of this model has been investigation. The differential equations governing such a stability problem are introduced, and then have been solved for fixed amplitude. It has been shown that smaller value for viscosity coefficient and thermal conductivity coefficient and larger values for thermal expansion coefficient can reduce system stability, heat transfer by radiation increase effect of thermal diffusivity coefficient to reduce system stability.***

Keywords: convection, conduction, radiation, differential equations, viscosity, stability, thermal diffusivity

1. Introduction

An introduction to fluid flow was presented, then the flow in a tube at the level of the polar coordinates will be illustrated with an illustrative diagram and the mathematical equations governing the fluid flow and an explanation of the physical concepts. Also the presentation of the basic equations of the thermal convection loop. Then the methodology of the research can be summaries such that After the generation of the governing partial differential equations and the relative boundary conditions of this model, we can separate the governing equations to two parts. The first is the steady state problem and its differential equations are solved by the linear shooting method with some manipulations and the second part is concerned with the stability problem which will treated by some theorem of algebra.

Also it is a useful action scientifically to mention some resent work such that, diffusion approximation in multidimensional radiative transfer problems [2] and the role of heat transfer in sunlight to fuel conversion using high temperature solat thermochmical reactors [4]. The vapor flow effect on falling film evaporation of R134 outside horizontal tube bundle, [3], also the effect of heat and mass transfer and Electrochemistry, on Performance in Solid Oxide Full Cell Stacks [1].

As mentioned before the transfer of heat energy in fluids is particularly important in studies of fluid dynamics, and has gained the attention of many researchers because of the applications of the broad process.

In the field of applied mathematics, the study of the behavior of fluid in its gyration and heat transfer in it based on the transformation of the physical model into a mathematical model is represented by equations representing the behavior of fluid, where all flow situations are subject to certain relationships and marginal conditions.

Natural has attracted a lot of interest from researchers because of its extensive practical applications

2. Methodology

The Model and Governing Equations

Equation of State:
It is well known that there is a functional relationship between the pressure P, density ρ and the temperature T^* of a gas. Then the state equation is given by:

$$P=\rho T^* R, \quad \text{where R is the gas constant.} \quad (1)$$

The Equation of Continuity
The conservation of the mass is given by:

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho \vec{v}) \quad (2)$$

Equations of Motion
The conservation of momentum gives the equation of motion in the vector form as:

$$\rho \frac{D\, V^{\rightarrow}}{Dt} = -P\nabla p^{\rightarrow} + B^{\rightarrow} + \mu \nabla^2 V^{\rightarrow} \quad (3)$$

Energy Equation
The conservation of energy gives the equation:

$$\rho\, c_p \frac{DT}{DT} = -P\nabla.V^{\rightarrow} + K\nabla^2 T - \nabla.q^{\rightarrow} \quad (4)$$

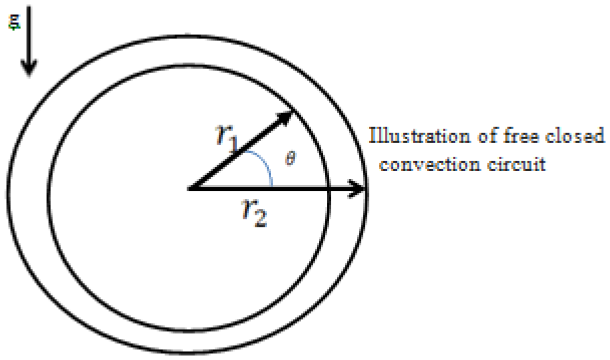
The radiative transfer equation in terms of the radiative flux $\underline{q}$ and radiative density Σ are

$$\alpha C\Sigma + div\underline{q} = 4\tilde{\sigma}\alpha T^4, \quad (5)$$

$$\nabla C\Sigma + 3\alpha \underline{q} = 0. \quad (6)$$

Where c is the speed of light and $\tilde{\sigma}$ is Stefan's constant.

After clarifying the basic equations we choose the thermal convection loop for study and analysis. The figure below is clarified of this circuit.



The thermal convection loop which under study, is a closed free thermal convection loop in which the fluid is revolving continuously around the tubes and this cannot be happen in the free open thermal convection loops. The fluid in the free open thermal convection loops changed through external continuer.

The fluid used is a viscous fluid and the tube is a tube located in the vertical direction? The forces that influence this fluid Including forces related to land acceleration, gravitational acceleration and the buoyancy force while the movement of fluid resulting from the presence of a decline in temperature because of the presence of cold fluid on the top and the last warm down. The latter is warm at the bottom and thus the buoyancy force leads to the generation of fluid movement, viscosity, thermal diffusion and they hinder movement.

After this presentation of the model we have to write more simplification of the differential equations and the related boundary conditions, which leads to the non-dimensional forms with the boundary conditions and the definition of the parameters.

The governing partial differential equations will be divided into two part, one is the steady state and the other of the disturb part. In the end of result we are writing the conclusion and recommend for further research.

Limit conditions which achieve the basic equations of model as following:

$$T = T_2 \ and \ V_r = V_g = 0 \ when \ r = r_2, 0 \leq g \leq 2\pi$$

$$T = T_1 \ and \ V_r = V_g = 0 \ when \ r = r_1, 0 \leq g \leq 2\pi$$

Non-linear equations and factors:
To find the non-linear equations we will define the following non-linear values:

$$\Sigma = \frac{C\Sigma_1}{\sigma T_1^4}, \ \hat{q} = \frac{q}{\sigma T_1^4}, \ \hat{r} = \frac{r}{r_1}, \ \hat{p} = \frac{\bar{p}}{p_0 v_0^2}, \ \hat{T} = \frac{T}{T_1},$$

$$\hat{t} = \frac{\mu}{p_0 r_1^2}, \quad \hat{v} = \frac{r_1 p_0}{\mu}\bar{v}$$

Where the Σ_1 is radiation density, σ is Stefan constant , C is the speed of light, also non-linear numbers: Grash of number $G_r = \frac{g p_0^2 r_1^3 \beta T_1}{\mu^2}$, Prandtl number $p_r = \frac{\mu c_p}{k}$

The continuation equation becomes the movement and energy on sequence:

$$\hat{\nabla}.\hat{V} = 0$$

$$\frac{\partial \hat{V}}{\partial \hat{t}} + (\hat{V}.\hat{\nabla})\hat{V} = -\hat{\nabla}\hat{P} + \hat{\nabla}^2\hat{V} + \frac{G_r}{\beta T_1}(\hat{V} - 1)G_r$$

$$\left[\frac{\partial \hat{V}}{\partial \hat{t}} + \hat{V}.\hat{\nabla}\hat{T}\right] = \frac{1}{p_r}\nabla^2\hat{T} - \alpha\hat{\nabla}\hat{q}$$

Where $\alpha = \frac{\sigma T_1^3 r_1}{\mu c_p}$ and according to the equations of radiative transfer Which can be roughly written in terms of radiation flux $\hat{q}$ and radiation density Σ_1 , this be done through differential approximation:

$$\varepsilon C\Sigma_1 + \nabla\bar{q} = 4\sigma\varepsilon T^4$$

$$\nabla C\Sigma_1 + \nabla\bar{q} = 0$$

Where ε *absorption coefficient (11).*

Can write previous equations in the non-linear form (19):

$$W\Sigma + \nabla\bar{q} = 4W\hat{T}^4$$

$$\nabla\Sigma + 3W\bar{q} = 0$$

Where the W is Bouguer number.

In the state, when the temperature difference between the fluid and the wall is small, the quantity of T^4 that appeared in the previous equations can be expressed as follows:

$$T^4 \approx T_1^4 + 4(T - T_1)T_1^3$$

Then the radiative equation becomes:

$W\Sigma + \nabla\bar{q} = 16W\widehat{T - 12}W$ and $\nabla\Sigma + 3W\bar{q} = 0$

Where Bouguer number W is very small compare to one. This property is known Optically thin limit, then the energy become:

$$\left[\frac{\partial \hat{T}}{\partial \hat{t}} + \hat{V}.\hat{\nabla}\hat{T}\right] = \frac{1}{p_r}\nabla^2\hat{T} - \alpha 16\hat{T}W + \alpha 12W$$

For purpose of study the model through non-linear equation, we need the following limit conditions:

$$\hat{T} = \frac{T_2}{T_1} \ and \ \hat{V}_r = \hat{V}_g = 0 \ when \ \hat{r} = \frac{r_2}{r_1}, 0 \leq g \leq 2\pi$$

$$\hat{T} = 1 \ and \ \hat{V}_r = \hat{V}_g = 0 \ when \ \hat{r} = 1, 0 \leq g \leq 2\pi$$

Stable state solutions

For the purpose of conducting a preliminary study to analyze stability, it should be noted that the basic variables that represent the model are$\hat{T}, \hat{p}, \widehat{v}_g, \hat{v}_r$ has been harassed and how much is known to be written as follows:

$$\hat{v}_g = v_1(\hat{r}) + v_2(\hat{r}, g, \hat{t}) \ ,$$

$$\hat{v}_r = u_1(\hat{r}) + u_2(\hat{r}, g, \hat{t})$$

$$\hat{T} = \hat{T}_1(\hat{r}, g) + \hat{T}_2(\hat{r}, g, \hat{t}),$$

$$\hat{p} = \hat{p}_1(\hat{r}, g) + \hat{p}_2(\hat{r}, g, \hat{t})$$

We compensate for these quantities in the equations that control flow, and quantitiesv_1, u_1, p_1, $\hat{T}_1$ representing the steady state quantities. After compensating these quantities with basic flow equations and separating the boundary that includes steady state quantities, we get steady state equations.

3. Steady State Solution

From continuity equation produce $u_1 = 0$, and by deriving complex of kinetic equation for every $\hat{r}$ and g on sequence then solute the produced equations graphically, we note that T_1can be write as follow:

$$\hat{T}_1 = F(\hat{r}) + gG(\hat{r}) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.1)$$

Then we get on:

$$\frac{d^3 v_1}{d\hat{r}^3} + \frac{2d^2 v_1}{\hat{r} d\hat{r}^2} - \frac{1}{\hat{r}^2}\frac{dv_1}{d\hat{r}} + \frac{G_r}{\hat{r}} G(\hat{r}) + \frac{v_1}{\hat{r}^3} = 0 \quad (3.2)$$

Where the found of G function from kinetic equation where use integration by sequences (15) by imposing the approximate solution of the equation$\breve{G}$ by follow form (20): $\breve{G} = \sum_{n=0}^{\infty} A_n \hat{r}^{m+n}$, and the solution be:

$$G{=}A\bar{G} \text{ whereas } : \bar{G} = I_0(t) = \sum_{m=0}^{\infty} \frac{(t/2)^{2m}}{m!\Gamma(m+1)}$$

It is a function of Bessel modified (Modified Bessel) [10] Of type 1 of class 0 of variable t Which is known as: $t = \sqrt{p_r \alpha 16 w \bar{r}}$ And A constant , And to find A we observe limit conditions in the paragraph (2.1) where we get [19]:

$$\bar{r} = \frac{r_2}{r_1} \quad \text{when } \hat{T}_1 = 1 + 9\,\frac{T_2}{T_1} \ldots\ldots\ldots (3.3)$$

Where it results from a comparison (3.3) with (3.1) that $G = \frac{T_2}{T_1}$ when $\bar{r} = \frac{r_2}{r_1}$. Thus we get the A=3.02776593496074 when values $\sigma = 5.67e-8$,$\mu = 0.406, r_1 = 7,\ T_2 = 20$,$k = 3.028$ K=3.028e-6,$r_2 = 8$ the solution becomes as follows G=3.02776593496074$\breve{G}$.

And to resolve the final value issue (Boundary value problem) Of the equation (3.2) The linear correction method was used (linear shooting method) [3], Where they were converted for two of the initial value (Initial value problem) each one is solved in a Rang - Kuta manner

Rk-45[14] after rank reduction, clarify the function draw v1 in figure (2.1):

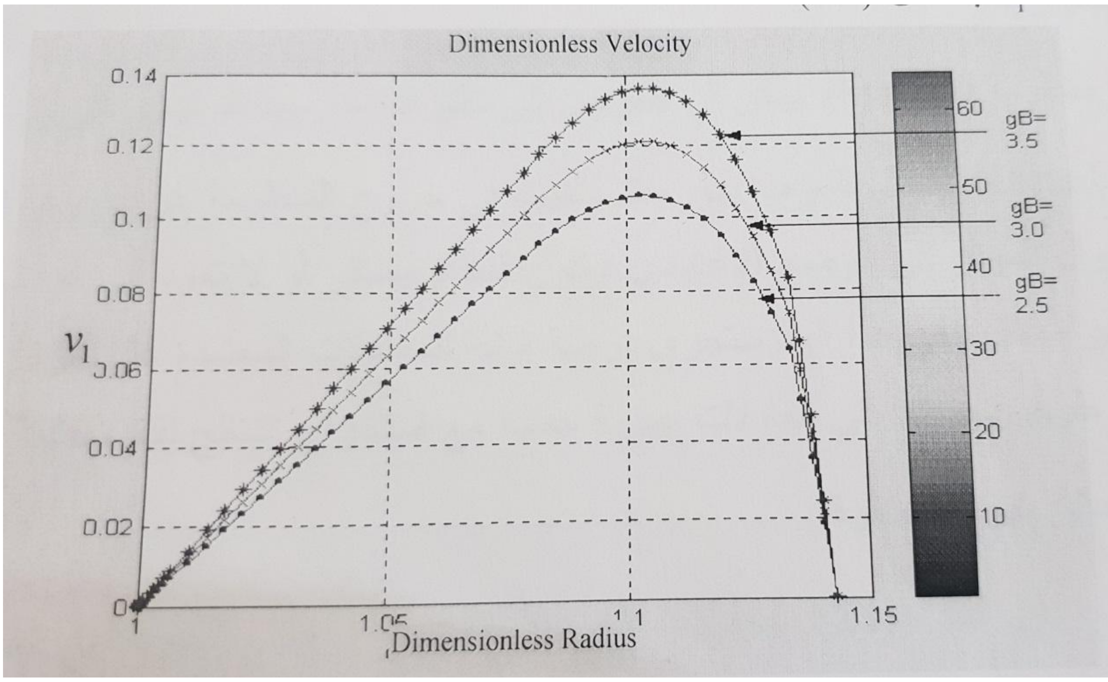


Figure 3.1

Figure shows curve function at values $R_2 = 8\ ,R_1 = 7, T_1 = 20, \mu = 3$ and for different values of gβ.

To find a heat function T Of the equation (3.1) to find the function F, which gives the energy equation, such that

$$\hat{r}^2 F''(\hat{r}) + \hat{r} F'(\hat{r}) - P_r\, \alpha 16 W \hat{r}^2 F(\hat{r}) = P_r\, v_1 \hat{r} G(\hat{r}) - P_r \hat{r}^2\ \alpha\ 12\, W \ldots.. (3.4)$$

The complementary function from the left side of the equation (3.4) was found to be equal to zero. To find the integration of the equation (3.4), each boundary of the right side of the equation is processed on its own; the heat function diagram$\hat{T}_1 = F + 9G$ is shown below

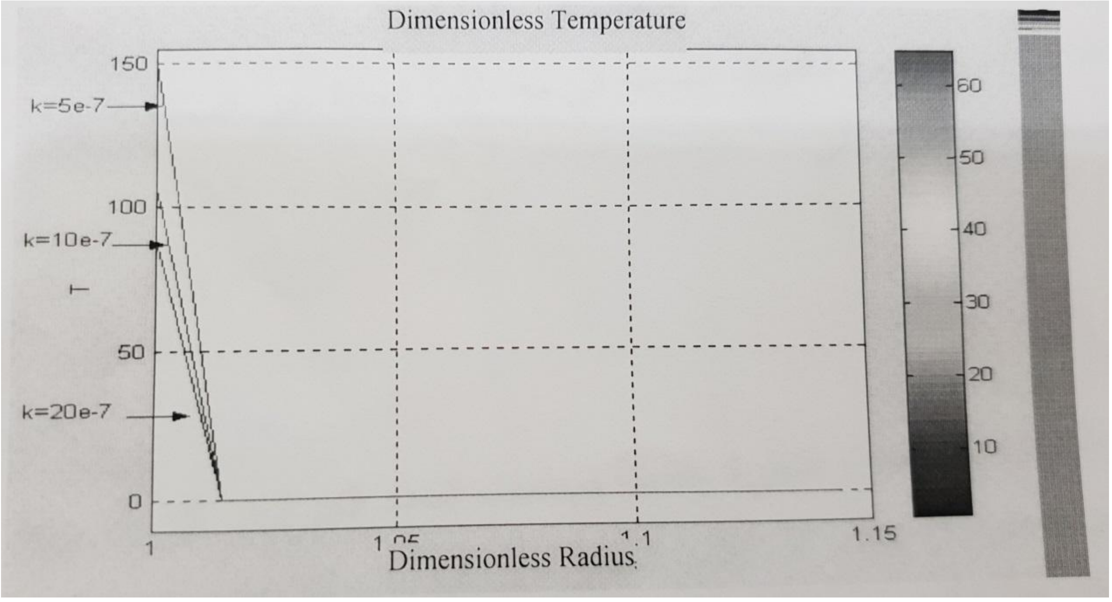


Figure 3.2

Figure shows curve heat function at values $\mu = 0.406$, $\sigma = 5.67e - 8$, $T_1 = 60$, $T_2 = 20$, $R_1 = 7$, $R_2 = 8$ for different values of K.

4. Stability Analysis

Any system can be subject to external influences that may cause system disruption, and this in turn may cause the exit from the development to another stage and may return after the demise of the impact to the normal situation, the system is stable or not return to the normal situation, the system is unstable. In this section, a preliminary study of time-dependent equations will be conducted in terms of stability, by converting them into linear algebraic equations with extract results for different values of parameters.

4.1 Unstable state equations

After compensating the quantities in the paragraph (3.1)with basic flow equations and the separation of the boundaries that include the amounts of the unstable situation with neglect of the nonlinear boundary we get the equations of continuity, motion and energy that represent the unstable state.

4.2 Stability Analysis

To study the stability of model disturbance by studying the unstable state equations, we imagine the bidirectional disorder r, ϑ, the basic variables of unstable state equations are as follows

$$u_2 = A_1 e^{i(\hat{r}k_1 + \vartheta k_2) + c\hat{t}},$$
$$v_2 = A_2 e^{i(\hat{r}k_1 + \vartheta k_2) + c\hat{t}},$$
$$p_2 = A_3 e^{i(\hat{r}k_1 + \vartheta k_2) + c\hat{t}},$$
$$T_2 = A_4 e^{i(\hat{r}k_1 + \vartheta k_2) + c\hat{t}}$$

Whereas k_1, k_2 real values are not linear to wavelength in both directions, c the speed of the wave is a complex value where $c_1, c_2 \in \Re c = c_1 + ic_2$

And that the positive or negative for c_1 value is the result of the growth of disturbance (disturbance) or disintegration of the sequence, when $c_1 < 0$ the system is stable and when $c_1 > 0$ the system is unstable as the A_1, A_2, A_3, A_4 constants represent the amplitude of the router and the compensation of the previous quantities in the unstable state equations can be written the unstable state coefficient in the following form: $\begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix} = 0,$

Whereas $a_i, i = 1,2, \ldots .9$ variable quantities for thermal expansion parameters β are viscosity μ and thermal diffusion k. It is known that the previous equation system has a non-trivial solution if the matrix of parameters is zero on this basis it produces:

$$a_1 + a_5 + a_9 - a_9 a_4 a_2 - a_1\, a_5 a_3 = 0 \quad (4.1)$$

To find out the effect of the change in the values of the parameters on the stability of the system, the roots of the equation (4.1) were found for different parameters of thermal expansion β, viscosity μ and thermal diffraction k.

4.3 Effect of thermal expansion coefficient

Since g it is constant ground acceleration in quantity g β,And by observing the roots of the equation(4.1) and for different values of g β, the observed that the increase g β, is more negative the real part of the roots ,and this means the departure of roots from the zero (the point of transition to non-stability), which increases the stability of the system by increasing the value g β and as in the following table:

Root	g β e-19	k_2	k_1
-9.437+6.4 i	0.01	3	2
1e8-0.978+5.23 i	7	3	2
1e3-23.971+0.5 i	1.0 e-8	3	3
1e7-0.38+0.8 i	1	3	3

A table showing the polynomial roots (4.1) of different values of thermal expansion coefficient

4.4 Effect of viscosity factor

The coefficient of viscosity μ affects the movement of fluid, which affects the stability of the system. And by observing the polynomial roots (4. 1) and for values different fromμ, and it was observed that by increasing the values ofμ, the negative of the true part of the roots decrease, as shown in the table:

Roots	μ	k_2	k_1
1e5 – 8.61 + 3.1i	4.39 e-4	3	2
1e2 – 24.06 + 9.0i	439	3	2
- 4.998 + 9.41i	0.439	2	3
- 2.39 – 8.6i	4790	2	3

4.5 Effect of Thermal Diffusion Coefficient

The coefficient of thermal diffusion k to have an effect on the flow property is similar to the effect of the viscosity coefficient of μ by observing the polynomial roots (4.1) for different values of K. It was observed that increasing the values to reduce true part negative of the roots, thus reducing the stability of the system, As shown in the following table:

Roots	K	k_2	k_1
-5.32 + 0.9i	7	3	2
-1.8+ 8.2i	13	3	2
-9.67 + 5.1i	1	2	3
-0.97 – 0.04i	6.5	2	3

5. Conclusions and Recommendations

5.1 Conclusions

The effect of the change in parameter values affects the properties of the fluid used thus affecting flow behavior and heat transfer, which gives different results with respect to the stability of the system. And from (1 .1) it can be observed that for a known change in temperature, increase the value of the coefficient of thermal expansion $\beta$and lead to decreasing density, and can be expressed mathematically as:

$$p = p_0(1 - \beta \uparrow (T - T_1)) \downarrow \Longrightarrow p \downarrow$$

It is this change in density that causes buoyancy that causes fluid movement and thus affects the speed and stability of the system. By observing the figure (3.1) it can be concluded that increasing the value of the β thermal expansion coefficient increases the velocity of the fluid and thus predicts the increased velocity of the fluid to large values of the β thermal expansion coefficient.

The effect of thermal diffusion coefficient k is the opposite of the effect of β thermal expansion coefficient on heat transfer. This was observed from Figure (3.1). It was observed that the increase in the value of the thermal diffusion coefficient, as a little, causes the heat to decrease significantly. Results obtained from the addition of radiation heat transfer with previous results, the effect of thermal diffusion coefficient k on the heat function curve was observed.

The stability of the system can be expected to increase the small values of theμ coefficient, because the effect of μ viscosity is the opposite effect of the β thermal expansion factor, since increasing the μ viscosity value reduces fluid velocity because fluid viscosity is a fluid-resistant fluid.

The results obtained in this study are as follows:

1) The solution was found for the system of remote equations: Navier - Stocks equations (motion equation), the equation of continuity, and the equation of energy convection method, in addition to the transmission method of radiation.
2) The stability of the system of unstable equations was analyzed, and the effect of the β thermal expansion parameters, the μ viscosity coefficient, and the coefficient of thermal diffusion on the stability of the unstable state equations system were observed. Generally, the small values μ and the thermal diffusion coefficient K and the large values of the β- Leading to the stability of the system and by comparing the results obtained and previous studies were limited to the transmission of heat in conduction and convection, it is concluded that the addition of the method of transmission of radiation increases the impact of the coefficient of thermal diffusion K, which led to the possibility of obtain a more stable system with a small reduction in the value of the thermal diffusion coefficient k.

5.2 Recommendations

For the purpose of modernization and development, it is possible to study the following:
First: Analysis of the stability of a variable wave capacity.
Second: Study of thermal convection of two concentric cylinders with stability analysis.

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