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Odds Exponential Log-Logistic Distribution - An Economic Reliability Test Plan

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Abstract: ***The odds exponential log logistic distribution introduced by Rosaiah et al. (2017) is considered as a probability model for the life time of the product. Sampling plans in which items that are put to test, to collect the life of the items in order to decide upon accepting or rejecting a submitted lot, are called reliability test plans. In this paper, we determine the minimum life test termination time, for various specified acceptance number, sample size and producer's risk. The preferability of the present reliability test plan over similar plans exists in the literature is established with respect to termination time of the experiment.***

Keywords: Odds exponential log logistic distribution, Economic reliability test plan, Experimental time

1. Introduction

Reliability study plays a vital role in the quality control analysis. On the basis of this study, an experimenter can save his time and cost to reach result which is to accept the submitted lot or to reject it. If the genuine products are rejected on the basis of sample information, this error is called type-I error (producer's risk α). On the other hand, if the genuine products are not accepted, this error is known as type-II error (consumer's risk β). If a decision to accept or reject the lot are subjected to the risks associated with the two types of errors, this procedure is termed as 'reliability test plan' or 'acceptance sampling based on life test'; for more details, see Duncan [6] and Stephens [26].

Skewed probability distributions are the basis for reliability test plans. These distributions are used to find the reliability sampling plans which will be more economical for the experimenter. Epstein and Sobel [7] was the first who considered truncated life tests in the exponential distribution. Gupta [8] suggested that for a skewed distribution, the median represents a better quality parameter than the mean. On the other hand, for a symmetric distribution, mean is preferable to use as a quality parameter. Balakrishnan *et al.* [4] proposed the acceptance sampling plans could be used for the quantiles and derived the formulae. Aslam and Shahbaz [3] considered generalized exponential distribution to explain economic reliability test plan. Aslam [1] also presented economic reliability test plan for a generalized Rayleigh Distribution. Lio *et al.* [12, 13] considered acceptance sampling plans based on the truncated life tests to Birnbaum-Saunders distribution and Burr type XII for percentiles and they proposed that the acceptance sampling plans based on mean may not satisfy the requirement of engineering on the specific percentile of strength or breaking stress. Mugahal *et al.* [14] introduced economic reliability group acceptance sampling plans for lifetimes of Marshall-Olkin extended distribution. Aslam *et al.* [2] discussed the economic reliability group acceptance sampling plan for Pareto second kind distribution. Rao *et al.* [19] explained the economic reliability test plan on the basis of Marshall-Olkin extended exponential distribution. Rao *et al.* [20] discussed the reliability test plans for type-II exponentiated log-logistic distribution. Rao *et al.* [21] presented economic reliability test plan for a generalized log-logistic distribution. Balamurali *et al.* [5] developed acceptance sampling plans based on median life for Fréchet distribution. Ramkumar and Sajana [17] considered economic reliability test plan for four parametric Burr distribution. Rosaiah *et al.* [23, 24] have considered economic reliability test plan for Type-I and Type-II generalized half logistic distribution. Rao [18] considered the economic reliability test plan based on truncated life tests for Marshall-Olkin extended Weibull Distribution.

The probability distributions/models have been in use to develop the reliability test plans. Under these plans we can find the termination time of experiment. These distributions can be used to find the best reliability sampling plans which are more economical for the experimenter. Kantam *et al*. [9] used the log-logistic distribution to develop the acceptance sampling plans. Kantam *et al.* [10] also studied the reliability test plans using the log- logistic distribution. In reliability test plan, we terminate the experiment if the termination time t ends or the r^{th} failure occurs if we put the nr sample units on test, whichever occurs first.

These are the reasons motivate to develop economic reliability test plans based on odds exponential log logistic distribution (OELLD). The rest of the paper is organized as follows. In Section 2, we describe concisely the OELLD. The design of proposed economic reliability test plans is discussed in Section 3. In Section 4, we present the description of the proposed plan and obtain the necessary results and an example with real data set is also given as an illustration. Finally, conclusions are made in Section 5.

2. The Odds Exponential Log Logistic Distribution

In this section, assuming that the life time of a product follows odds exponential log logistic distribution (OELLD).

The OELLD was introduced and studied quite extensively by Rosaiah *et al*. [25]. The probability density function (pdf), cumulative distribution function (cdf) and hazared function (hf) of OELLD are respectively given as follows:

$$f(t;\sigma,\lambda,\theta)=\frac{\theta}{\lambda\sigma}(t/\sigma)^{\theta-1}e^{-\frac{1}{\lambda}(t/\sigma)^{\theta}},\quad t>0;\ \sigma,\lambda,\theta>0 \qquad (1)$$

$$F(t;\sigma,\lambda,\theta)=1-e^{-\frac{1}{\lambda}(t/\sigma)^{\theta}},\quad t>0;\ \sigma,\lambda,\theta>0 \qquad (2)$$

$$h(t;\sigma,\lambda,\theta)=\frac{\theta}{\lambda\sigma^{\theta}}t^{\theta-1} \qquad (3)$$

where σ, λ are the scale parameters and θ is the shape parameter.

3. An Economic Reliability Test Plan for OELLD

Rao *et al*. [22] obtained the minimum sample sizes required to make a decision about the lot, given the waiting time in terms of σ_0 (i.e., t/σ_0) and acceptance number *c*, some risk probability, say *α,* based on median/percentile for OELLD and also determined the minimum sample size necessary to ensure a percentile life when the life test is terminated at a pre-assigned time t_q^0 and when the observed number of failures does not exceed a given acceptance number *c*. The decision procedure is to accept a lot only if the specified percentile of lifetime can be established with a pre-assigned high probability *α,* which provides protection to the consumer. For a ready reference brief contents of Rao *et al*. [22] are given in the following lines. For a specified σ_0 of σ, to determine the smallest positive integer n for given values of α (0 < α < 1), t/σ_0 and c such that

$$\sum_{i=0}^{c}\binom{n}{i}p^i(1-p)^{n-i}\ge 1-\alpha \qquad (4)$$

holds, where $p=F(t;\sigma,\lambda,\theta)$ given by (2) which indicates the failure probability before time *t*, and depends on the ratio t/σ_0. It is sufficient to specify this ratio for designing the experiment.

The minimum values of n satisfying the inequality (4) are obtained by Rao *et al.* [22] for $\lambda=2$ and θ=2; $\alpha=0.25, 0.10, 0.05, 0.01$ and $t/\sigma_0=0.8, 0.9, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5$ which are reproduced in Table-1 for ready reference.

In the present investigation, inequality (4) can be considered in a different way. Let us fix *n* and let *r* be a natural number less than *n*, so that as soon as the r^{th} (*r=c+1*) failure is observed, the process is stopped and the lot is rejected. Given $\sigma>\sigma_0$, the probability of such a rejection should be as small as possible. That is

$$\sum_{i=r}^{n}\binom{n}{i}p^i(1-p)^{n-i}\le 1-\alpha \qquad (5)$$

Taking *n* as a multiple of *r* say *kr* (*k* =1, 2, ...), inequality (5) can be regarded as an inequality in a single unknown in terms of t/σ_0 with known θ and λ. With the choice of *r, k, α*, inequality (5) can be solved for the earliest p say p_0 from which the value of t/σ_0 can be obtained by inverting the $F(t;\sigma,\lambda,\theta)$ given by (2). The specified population average in terms of σ_0 can be used here to get the value of *t* called the termination time. These are presented in Table 2 for various values of *n, r*=1(1)10, $\lambda=2$ and θ=2 at $\alpha=0.25, 0.10, 0.05, 0.01$, as similarly Table 3 represent various values of *n, r*=1(1)10 and for the estimated values of λ and θ at $\alpha=0.25, 0.10, 0.05, 0.01$. The comparison between the present sampling plan with that of Rao *et al*. [22] is presented in Table 6 and Table 7 for $\alpha=0.05$ and 0.01.

The operating characteristic function of sampling plan (*n, c,* t/σ_0) gives the probability of accepting the lot as

$$L(p)=\sum_{i=0}^{c}\binom{n}{i}p^i(1-p)^{n-i} \qquad (6)$$

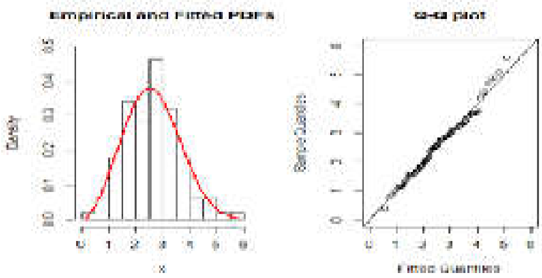
where $p=F(t;\sigma,\lambda,\theta)$. It is considered as a function of σ, the lot quality parameter. It can be seen that the operating characteristic is an increasing function of σ, for given *α*, t/σ_0. Values of the operating characteristic as a function of σ/σ_0 for a few sampling plans are given in Table 4 and 5.

4. Real Data example

In this section, we present the application of the proposed OELLD distribution considered by Lemonte [11] for a real data set to illustrate its potentiality. The following real data set corresponds to an uncensored data set from Nichols and Padgett [15] on breaking stress of carbon fibres (in Gba):
0.39,0.81,0.85,0.98,1.08,1.12,1.17,1.18,1.22,1.25,1.36,1.41, 1.47,1.57,1.57,1.59,1.59,1.61,1.61,1.69,1.69,1.71,1.73,1.80, 1.84,1.84,1.87,1.89,1.92,2.00,2.03,2.03,2.05,2.12,2.17,2.17, 2.17,2.35,2.38,2.41,2.43,2.48,2.48,2.50,2.53,2.55,2.55,2.56, 2.59,2.67,2.73,2.74,2.76,2.77,2.79,2.81,2.81,2.82,2.83,2.85, 2.87,2.88,2.93,2.95,2.96,2.97,2.97,3.09,3.11,3.11,3.15,3.15, 3.19,3.19,3.22,3.22,3.27,3.28,3.31,3.31,3.33,3.39,3.39,3.51, 3.56,3.60,3.65,3.68,3.68,3.68,3.70,3.75,4.20,4.38,4.42,4.70, 4.90,4.91,5.08,5.56.

The goodness of fit for our model by plotting the superimposed for the data shows that the OELLD is a good fit and also goodness of fit is emphasized with Q-Q plot, displayed in the following diagrams. The maximum likelihood estimates of the two-parameters of OELLD for the breaking stress of carbon fibres are $\hat{\lambda}=20.4032$ and $\hat{\theta}=2.7932$ and using Kolmogorov-Smirnov test, we found that the maximum distance between the data and the fitted of the OELLD is 0.0604 with p-value is 0.8582.

Estimated density and Q-Q plot for OELLD.



5. Comparative Study and Conclusions

In order to compare the proposed economic reliability test plan with that of Rao *et al*. [22] the entries common for both approaches are presented for $\lambda = 2$ and $\theta = 2$; $\alpha = 0.05,\ 0.01$ in Table 6 and 7. The entries given in the first row are belongs to the proposed test plan and those given in the parenthesis are obtained by Rao *et al*. [22]. All the entries are presented in Table 6 and 7 shows that for given values of n, $r(r = c+1)$, the values of t/σ_0 is uniformly smaller for the proposed economic reliability test plans as compare to the existing sampling plan of Rao *et al*. [22]. Hence the resulting sampling plan can be used to reduce the experimental time.

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Table 1: Minimum sample size necessary to assert the average life to exceed specified average life σ_0 with probability $1-\alpha$ and the corresponding acceptance number c, using binomial probability for $\lambda = 2$ and θ=2 in OELLD [Rao et al. (2016)]

	t/σ_0							
C	0.8	0.9	1	1.5	2.0	2.5	3	3.5
	α=0.25							
0	4	3	2	1	1	1	1	1
1	7	6	5	3	2	2	2	2
2	10	9	7	4	3	3	3	3
3	14	11	10	6	4	4	4	4
4	17	14	12	7	6	5	5	5
5	20	16	14	8	7	6	6	6
6	23	19	16	10	8	7	7	7
7	26	22	18	11	9	8	8	8
8	29	24	21	12	10	9	9	9
9	32	27	23	14	11	10	10	10
10	35	29	25	15	12	11	11	11
	α=0.10							
0	6	5	4	2	1	1	1	1
1	10	8	7	4	3	2	2	2
2	14	11	9	5	4	3	3	3
3	17	14	12	7	5	4	4	4
4	21	17	14	8	6	5	5	5
5	24	20	17	9	7	6	6	6
6	27	23	19	11	8	7	7	7
7	31	25	21	12	10	9	8	8
8	34	28	24	14	11	10	9	9
9	37	31	26	15	12	11	10	10
10	41	33	28	16	13	12	11	11
	α=0.05							
0	7	6	5	2	2	1	1	1
1	12	9	8	4	3	2	2	2

2	16	13	11	6	4	3	3	3
3	20	16	13	7	5	5	4	4
4	23	19	16	9	6	6	5	5
5	27	22	18	10	8	7	6	6
6	30	25	21	12	9	8	7	7
7	34	28	23	13	10	9	8	8
8	37	31	26	15	11	10	9	9
9	41	33	28	16	12	11	10	10
10	44	36	30	17	13	12	11	11
	α=0.01							
0	11	9	7	3	2	2	1	1
1	16	13	11	5	4	3	2	2
2	21	17	14	7	5	4	3	3
3	25	20	17	9	6	5	4	4
4	29	23	19	10	7	6	5	5
5	33	27	22	12	9	7	7	6
6	37	30	25	13	10	8	8	7
7	40	33	27	15	11	9	9	8
8	44	36	30	16	12	10	10	9
9	48	39	33	18	13	11	11	10
10	51	42	35	19	14	13	12	11

Table 2: Life test termination time in units of scale parameter t/σ_0 in OELLD for $\lambda=2$ and $\theta=2$

r	*n*=2*r*	3*r*	4*r*	5*r*	6*r*	7*r*	8*r*	9*r*	10*r*
α=0.25									
1	0.5364	0.4381	0.3793	0.3395	0.3100	0.2868	0.2685	0.2530	0.2401
2	0.7464	0.5929	0.5072	0.4505	0.4092	0.3776	0.3524	0.3316	0.3143
3	0.8395	0.6598	0.5616	0.4975	0.4513	0.4160	0.388	0.3649	0.3454
4	0.8935	0.6980	0.5927	0.5243	0.4753	0.4378	0.4081	0.3838	0.3631
5	0.9294	0.7234	0.6134	0.5422	0.4911	0.4522	0.4215	0.3962	0.3750
6	0.9553	0.7416	0.6283	0.5549	0.5025	0.4627	0.4310	0.4052	0.3832
7	0.9750	0.7554	0.6394	0.5645	0.5112	0.4706	0.4383	0.4118	0.3899
8	0.9905	0.7664	0.6484	0.5724	0.5181	0.4769	0.4441	0.4173	0.3949
9	1.0032	0.7753	0.6556	0.5785	0.5236	0.4818	0.4488	0.4218	0.3990
10	1.0139	0.7828	0.6617	0.5838	0.5284	0.4863	0.4527	0.4254	0.4025
α=0.10									
1	0.3249	0.2654	0.2295	0.2055	0.1879	0.1739	0.1624	0.1534	0.1453
2	0.5547	0.4408	0.3773	0.3351	0.3045	0.2810	0.2622	0.2468	0.2340
3	0.6699	0.5267	0.4485	0.3973	0.3605	0.3323	0.3100	0.2915	0.2761
4	0.7403	0.5787	0.4918	0.4351	0.3943	0.3631	0.3386	0.3183	0.3014
5	0.7889	0.6144	0.5212	0.4607	0.4173	0.3844	0.3581	0.3367	0.3186
6	0.8246	0.6406	0.5429	0.4795	0.4343	0.3998	0.3724	0.3503	0.3313
7	0.8524	0.6609	0.5597	0.4943	0.4476	0.4118	0.3838	0.3608	0.3414
8	0.8747	0.6774	0.5732	0.5061	0.4581	0.4215	0.3927	0.3690	0.3491
9	0.8931	0.6909	0.5842	0.5157	0.4667	0.4295	0.4000	0.3759	0.3557
10	0.9086	0.7022	0.5937	0.5238	0.4741	0.4363	0.4062	0.3818	0.3614
α=0.05									
1	0.2268	0.1852	0.1605	0.1439	0.1314	0.1219	0.1133	0.1069	0.1021
2	0.4535	0.3605	0.3083	0.2738	0.2489	0.2295	0.2143	0.2020	0.1911
3	0.5767	0.4537	0.3863	0.3423	0.3106	0.2861	0.2669	0.2510	0.2379
4	0.6549	0.5121	0.4351	0.3849	0.3491	0.3216	0.2996	0.2817	0.2666
5	0.7095	0.5528	0.4689	0.4145	0.3756	0.3460	0.3223	0.3031	0.2868
6	0.7503	0.5832	0.4943	0.4366	0.3954	0.3640	0.3392	0.3190	0.3017
7	0.7823	0.6069	0.5139	0.4539	0.4110	0.3784	0.3524	0.3313	0.3133
8	0.8083	0.6261	0.5299	0.4679	0.4236	0.3899	0.3631	0.3414	0.3229
9	0.8299	0.6421	0.5431	0.4795	0.4338	0.3992	0.3719	0.3497	0.3307
10	0.848	0.6556	0.5543	0.4893	0.4428	0.4076	0.3796	0.3566	0.3373
α=0.01									
1	0.1011	0.0825	0.0722	0.0648	0.0583	0.0548	0.051	0.049	0.0469
2	0.2929	0.2331	0.1995	0.1773	0.1612	0.1487	0.1389	0.1307	0.1235
3	0.421	0.3313	0.2821	0.2501	0.2268	0.209	0.1948	0.1835	0.1739
4	0.5079	0.3973	0.3376	0.2989	0.2708	0.2497	0.2327	0.2186	0.207
5	0.5712	0.4453	0.3779	0.3342	0.3028	0.2787	0.2599	0.2443	0.2313
6	0.6197	0.4818	0.4084	0.3611	0.3268	0.301	0.2806	0.2634	0.2493
7	0.6583	0.5110	0.4328	0.3824	0.3460	0.3186	0.2968	0.2791	0.2642

8	0.6899	0.5349	0.4527	0.3998	0.3619	0.3332	0.3103	0.2915	0.2757
9	0.7163	0.5547	0.4694	0.4145	0.3750	0.3451	0.3216	0.3021	0.2857
10	0.7391	0.5718	0.4837	0.4269	0.3863	0.3554	0.3310	0.3110	0.2944

Table 3: Life test termination time in units of scale parameter t/σ_0 in OELLD for $\hat{\lambda} = 2.7932$ and $\hat{\theta}$=20.4032

r	*n*=2*r*	3*r*	4*r*	5*r*	6*r*	7*r*	8*r*	9*r*	10*r*
					α=0.25				
1	1.4704	1.2719	1.1472	1.0597	0.9928	0.9392	0.8958	0.8585	0.8268
2	1.8627	1.5797	1.4126	1.2977	1.2112	1.1435	1.0884	1.0420	1.0028
3	2.0264	1.7053	1.5195	1.3932	1.2992	1.2257	1.166	1.1158	1.0728
4	2.1188	1.7755	1.5793	1.4465	1.3484	1.2714	1.209	1.1570	1.1120
5	2.1794	1.8214	1.6186	1.4817	1.3803	1.3012	1.2373	1.1837	1.1379
6	2.2228	1.8541	1.6466	1.5065	1.4032	1.3226	1.2572	1.2027	1.1557
7	2.2555	1.8788	1.6675	1.5252	1.4206	1.3387	1.2725	1.2168	1.1702
8	2.2811	1.8984	1.6842	1.5403	1.4343	1.3517	1.2844	1.2285	1.1808
9	2.3020	1.9142	1.6976	1.5522	1.4452	1.3617	1.2941	1.2378	1.1895
10	2.3196	1.9274	1.7088	1.5623	1.4546	1.3706	1.3022	1.2454	1.1970
					α=0.10				
1	1.0268	0.8884	0.8007	0.7398	0.6938	0.6563	0.625	0.6001	0.5771
2	1.5061	1.2777	1.1429	1.0499	0.9803	0.9254	0.8808	0.8434	0.8118
3	1.7240	1.4512	1.2936	1.1860	1.1062	1.0435	0.9928	0.9502	0.9139
4	1.8519	1.5526	1.3817	1.2656	1.1796	1.1120	1.0576	1.0119	0.9731
5	1.9381	1.6205	1.4404	1.3187	1.2285	1.1582	1.1009	1.0534	1.0127
6	2.0005	1.6697	1.4830	1.3569	1.2641	1.1913	1.1323	1.0837	1.0413
7	2.0485	1.7074	1.5159	1.3868	1.2916	1.2168	1.157	1.1068	1.0639
8	2.0869	1.7377	1.5419	1.4104	1.3132	1.2373	1.1761	1.1248	1.0810
9	2.1182	1.7624	1.5631	1.4295	1.3310	1.2540	1.1918	1.1398	1.0957
10	2.1445	1.7831	1.5813	1.4456	1.346	1.2683	1.2050	1.1527	1.1081
					α=0.05				
1	0.794	0.6866	0.6198	0.5731	0.5371	0.5089	0.4830	0.4634	0.4483
2	1.3037	1.1062	0.9889	0.9086	0.8485	0.8007	0.7623	0.7308	0.7023
3	1.5486	1.3042	1.1624	1.0660	0.9944	0.9375	0.8921	0.8535	0.8215
4	1.6962	1.4224	1.2656	1.1594	1.0810	1.0194	0.9691	0.9271	0.8912
5	1.7963	1.5024	1.3353	1.2224	1.1392	1.0742	1.0209	0.9771	0.9392
6	1.8696	1.5612	1.3868	1.2688	1.182	1.1139	1.0590	1.0134	0.9739
7	1.9265	1.6063	1.4259	1.3047	1.2152	1.1454	1.0884	1.0413	1.0005
8	1.9722	1.6426	1.4576	1.3334	1.2416	1.1702	1.1120	1.0639	1.0224
9	2.0096	1.6725	1.4834	1.3569	1.2630	1.1901	1.1311	1.0823	1.0399
10	2.041	1.6976	1.5053	1.3766	1.2818	1.2078	1.1478	1.0976	1.0548
					α=0.01				
1	0.4452	0.3849	0.3496	0.3239	0.3003	0.2871	0.2727	0.265	0.2569
2	0.9535	0.8096	0.7242	0.6656	0.6215	0.5869	0.5588	0.5349	0.5138
3	1.2362	1.0413	0.928	0.8515	0.7940	0.7487	0.712	0.6822	0.6563
4	1.4140	1.186	1.0555	0.9674	0.9013	0.8505	0.8085	0.7731	0.7436
5	1.5380	1.287	1.1441	1.0477	0.9763	0.9201	0.8751	0.8372	0.8052
6	1.6305	1.3617	1.2095	1.1075	1.0312	0.9723	0.9245	0.8837	0.8495
7	1.7025	1.4202	1.2609	1.1539	1.0742	1.0127	0.9626	0.921	0.8856
8	1.7608	1.4674	1.3022	1.1913	1.1094	1.0456	0.9936	0.9502	0.913
9	1.8087	1.5061	1.3363	1.2224	1.1379	1.0721	1.0194	0.9747	0.9366
10	1.8497	1.5392	1.3654	1.2486	1.1624	1.095	1.0406	0.9952	0.9568

Table 4: Operating characteristic (O.C) values of sampling plans (n, r, t/σ_0) for $\lambda = 2, \theta = 2$

r	n=2*r*	3*r*	4*r*	5*r*	6*r*	7*r*	8*r*	9*r*	10*r*
					α=0.25				
1	0.7500	0.7498	0.7500	0.7497	0.7495	0.7498	0.7495	0.7497	0.7496
2	0.7498	0.7500	0.7498	0.7496	0.7499	0.7499	0.7498	0.7499	0.7495
3	0.7499	0.7497	0.7499	0.7497	0.7497	0.7497	0.7494	0.7494	0.7496
4	0.7499	0.7499	0.7500	0.7499	0.7497	0.7498	0.7496	0.7492	0.7498
5	0.7499	0.7498	0.7498	0.7496	0.7498	0.7499	0.7494	0.7494	0.7492
6	0.7499	0.7498	0.7495	0.7498	0.7498	0.7496	0.7497	0.7492	0.7501
7	0.7498	0.7499	0.7499	0.7501	0.7497	0.7495	0.7497	0.7500	0.7491
8	0.7500	0.7498	0.7497	0.7495	0.7496	0.7494	0.7496	0.7496	0.7494

9	0.7500	0.7499	0.7498	0.7500	0.7499	0.7501	0.7497	0.7493	0.7496
10	0.7497	0.7498	0.7497	0.7498	0.7495	0.7492	0.7499	0.7497	0.7495
α=0.10									
1	0.8998	0.8997	0.9000	0.8998	0.8995	0.8996	0.8999	0.8995	0.8998
2	0.9000	0.9000	0.8998	0.8998	0.8998	0.8998	0.8998	0.8997	0.8994
3	0.8999	0.9000	0.8999	0.8999	0.8998	0.8998	0.8996	0.8996	0.8995
4	0.9000	0.9000	0.8997	0.8997	0.8998	0.9000	0.8997	0.8997	0.8995
5	0.8999	0.8999	0.8998	0.8998	0.8999	0.8997	0.8998	0.8996	0.8997
6	0.8999	0.9000	0.8998	0.8999	0.8998	0.8999	0.9000	0.8994	0.8998
7	0.8999	0.9000	0.8998	0.8997	0.8996	0.9000	0.8995	0.8993	0.8993
8	0.8999	0.8998	0.8998	0.8996	0.8997	0.8999	0.8997	0.8997	0.8998
9	0.8999	0.8997	0.9000	0.8998	0.8999	0.8999	0.8999	0.8998	0.8997
10	0.9000	0.8998	0.8998	0.8999	0.8998	0.8997	0.9000	0.8996	0.8992
					α=0.05				
1	0.9499	0.9499	0.9498	0.9496	0.9495	0.9493	0.9500	0.9499	0.9492
2	0.9499	0.9499	0.9500	0.9500	0.9499	0.9500	0.9499	0.9496	0.9499
3	0.9500	0.9499	0.9500	0.9499	0.9498	0.9500	0.9499	0.9499	0.9496
4	0.9499	0.9500	0.9499	0.9499	0.9497	0.9497	0.9499	0.9498	0.9499
5	0.9499	0.9500	0.9500	0.9499	0.9499	0.9498	0.9499	0.9497	0.9498
6	0.9500	0.9499	0.9498	0.9499	0.9499	0.9500	0.9499	0.9496	0.9498
7	0.9500	0.9500	0.9499	0.9499	0.9498	0.9498	0.9499	0.9497	0.9499
8	0.9499	0.9500	0.9499	0.9498	0.9498	0.9498	0.9498	0.9495	0.9497
9	0.9499	0.9499	0.9499	0.9498	0.9500	0.9500	0.9499	0.9495	0.9498
10	0.9499	0.9499	0.9499	0.9498	0.9498	0.9496	0.9496	0.9497	0.9498
α=0.01									
1	0.9898	0.9898	0.9896	0.9896	0.9899	0.9895	0.9897	0.9893	0.9891
2	0.9900	0.9900	0.9900	0.9899	0.9899	0.9899	0.9899	0.9899	0.9900
3	0.9900	0.9900	0.9900	0.9899	0.9900	0.9900	0.9900	0.9899	0.9899
4	0.9900	0.9900	0.9900	0.9899	0.9900	0.9899	0.9899	0.9900	0.9900
5	0.9900	0.9900	0.9900	0.9899	0.9899	0.9900	0.9899	0.9899	0.9899
6	0.9900	0.9900	0.9900	0.9899	0.9900	0.9900	0.9899	0.9900	0.9900
7	0.9900	0.9900	0.9900	0.9899	0.9900	0.9900	0.9900	0.9899	0.9899
8	0.9900	0.9900	0.9900	0.9899	0.9899	0.9899	0.9899	0.9900	0.9900
9	0.9900	0.9900	0.9900	0.9899	0.9900	0.9900	0.9899	0.9899	0.9900
10	0.9900	0.9900	0.9899	0.9899	0.9900	0.9900	0.9900	0.9900	0.9899

Table 5: Operating characteristic (O.C) values of sampling plans (n, r, t/σ_0) for $\hat{\lambda} = 2.7932$ and $\hat{\theta}$=20.4032

r	n=2*r*	3*r*	4*r*	5*r*	6*r*	7*r*	8*r*	9*r*	10*r*
α=0.25									
1	0.7500	0.7499	0.7500	0.7497	0.7496	0.7498	0.7495	0.7497	0.7497
2	0.7499	0.7499	0.7498	0.7496	0.7500	0.7500	0.7498	0.7499	0.7494
3	0.7498	0.7498	0.7499	0.7497	0.7498	0.7497	0.7494	0.7495	0.7497
4	0.7500	0.7499	0.7500	0.7500	0.7497	0.7497	0.7496	0.7492	0.7497
5	0.7500	0.7499	0.7497	0.7496	0.7498	0.7498	0.7493	0.7492	0.7493
6	0.7498	0.7499	0.7496	0.7498	0.7498	0.7497	0.7496	0.7494	0.7501
7	0.7498	0.7499	0.7498	0.7499	0.7496	0.7497	0.7495	0.7500	0.7489
8	1.0000	0.7498	0.7496	0.7496	0.7495	0.7493	0.7496	0.7495	0.7495
9	0.7500	0.7498	0.7498	0.7499	0.7498	0.7499	0.7498	0.7495	0.7499
10	0.7497	0.7498	0.7498	0.7498	0.7495	0.7494	0.7498	0.7498	0.7497
α=0.10									
1	0.8998	0.8997	0.9000	0.8998	0.8995	0.8996	0.8999	0.8995	0.8998
2	0.9000	0.9000	0.8998	0.8998	0.8998	0.8998	0.8998	0.8997	0.8994
3	0.8999	0.9000	0.8999	0.8999	0.8998	0.8998	0.8996	0.8996	0.8995
4	0.9000	0.9000	0.8998	0.8998	0.8998	0.9000	0.8998	0.8997	0.8996
5	0.8999	0.8999	0.8998	0.8997	0.8998	0.8998	0.8999	0.8997	0.8996
6	0.9000	0.9000	0.8998	0.8999	0.8998	0.8999	0.9000	0.8994	0.8998
7	0.8947	0.9000	0.8997	0.8997	0.8996	0.9000	0.8995	0.8994	0.8993
8	0.8999	0.8999	0.8998	0.8996	0.8997	0.8999	0.8997	0.8998	0.8999
9	0.8999	0.8998	0.8999	0.8998	0.8998	0.8999	0.8998	0.8998	0.8996
10	0.8999	0.8998	0.8997	0.8999	0.8997	0.8996	0.8999	0.8996	0.8993
α=0.05									
1	0.9498	0.9499	0.9498	0.9496	0.9495	0.9493	0.9499	0.9498	0.9492
2	0.9499	0.9499	0.9500	0.9499	0.9499	0.9500	0.9499	0.9496	0.9498
3	0.9500	0.9499	0.9499	0.9499	0.0760	0.0534	0.9498	0.9499	0.9496

4	0.9499	0.9499	0.9499	0.9499	0.9498	0.9497	0.9498	0.9498	0.9499
5	0.9500	0.9500	0.9500	0.9500	0.9499	0.9498	0.9499	0.9497	0.9497
6	0.9500	0.9499	0.9498	0.9499	0.9499	0.9500	0.9499	0.9497	0.9498
7	0.9500	0.9500	0.9500	0.9498	0.9498	0.9497	0.9498	0.9497	0.9499
8	0.9499	0.9499	0.9499	0.9498	0.9498	0.9497	0.9498	0.9496	0.9497
9	0.9499	0.9499	0.9500	0.9498	0.9500	0.9500	0.9499	0.9496	0.9498
10	0.9499	0.9499	0.9499	0.9498	0.9497	0.9497	0.9497	0.9497	0.9498
α=0.01									
1	0.9898	0.9898	0.9896	0.9895	0.9898	0.9896	0.9897	0.9893	0.9891
2	0.9900	0.9900	0.9900	0.9899	0.9899	0.9899	0.9899	0.9899	0.9900
3	0.9900	0.9900	0.9900	0.9899	0.9900	0.9900	0.9900	0.9899	0.9899
4	0.9900	0.9900	0.9900	0.9899	0.9900	0.9899	0.9899	0.9900	0.9900
5	0.9900	0.9900	0.9900	0.9899	0.9899	0.9900	0.9899	0.9899	0.9899
6	0.9900	0.9900	0.9900	0.9899	0.9900	0.9899	0.9899	0.9900	0.9900
7	0.9900	0.9900	0.9900	0.9899	0.9900	0.9900	0.9899	0.9899	0.9898
8	0.9900	0.9900	0.9900	0.9899	0.9899	0.9899	0.9899	0.9900	0.9900
9	0.9900	0.9900	0.9900	0.9899	0.9900	0.9900	0.9899	0.9900	0.9900
10	0.9900	0.9900	0.9899	0.9899	0.9899	0.9900	0.9900	0.9900	0.9899

Table 6: Comparison of life test termination times in units of scale parameter (t/σ_0) for the present sampling plans and the sampling plans of Rao et al. (2016) with producer's risk $\alpha = 0.05$ and for $\lambda = 2, \theta = 2$. (t/σ_0 values of Rao et al. (2016) are given in the parenthesis)

n r	2r	3r	4r	5r	6r	7r	8r	9r	10r
1	0.2268 (1.5)			0.1439 (1.0)	0.1314 (0.9)	0.1219 (0.8)			
2	0.4535 (1.5)		0.3083 (1.0)		0.2489 (0.8)				
3	0.5767 (1.5)								
4			0.4351 (0.9)	0.3849 (0.8)					
5									
6		0.5832 (1.0)							
7		0.6069 (1.0)							

Table 7: Comparison of life test termination times in units of scale parameter (t/σ_0) for the present sampling plans and the sampling plans of Rao et al. (2016) with producer's risk $\alpha = 0.01$ and for $\lambda = 2, \theta = 2$. (t/σ_0 values of Rao et al. (2016) are given in the parenthesis)

n r	2r	3r	4r	5r	6r	7r	8r	9r	10r
1	0.1011 (2.0)	0.0825 (1.5)				0.0548 (1.00)		0.0490 (0.9)	
2	0.2929 (2.0)						0.1389 (0.8)		
3						0.2090 (0.8)			
4				0.2989 (0.9)					
5	0.5712 (1.5)								
6									
7									
8				0.3998 (0.8)					
9			0.4694 (0.9)						