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# Hybrid Multiple Attribute Two-Sided Matching Decision Making Based on Grey Relational Fractional Programming Method

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**Abstract:** ***In this paper, for a multiple attributes two-sided decision making problems, takes into account different formats of evaluation information of matching objects and proposes a method for dealing with such two-sided matching decision making problems. A methodology for determining the comprehensive weights of attributes of matching objects under the hybrid attribute environment based on some method for determining the subject and objective attribute weights is presented. Based on the grey relational degree and the technique for order preference by similarity to ideal solution (TOPSIS) principle, optimal degrees of membership for matching objects by fractional programming is proposed and is defined matching satisfactory degree by it, and gets the best matching for the matching objects by solving a binary integer programming problem. The illustrative examples show the feasibility and effectiveness of the proposed method.***

**Keywords:** decision making, two-sided matching, optimization model, grey relational analysis, fractional programming method

## 1. Introduction

Two-sided matching decision making problems are of great significance in practical, there are many these problems in socio-economic and management areas. In recent years, due to the wide existence in our actual life, two-sided matching problems have received more and more attention from many researchers. Thus, in recent years, the subject of research for two-sided matching decision making has been attracted to much decision makers. The two-sided matching problem initially originates from college admissions and the matching of marriage [1]. The two-sided matching problems are suggested in various fields, such as matching between employees and job positions [2-7] and exchange matching between buyers and sellers [8-11], and there is a need to develop analysis methods to deal with such problems. In [7] have been considered different formats of evaluation information for the employees and positions and proposed a decision making method based on binary integer programming. In [8] have been established a multi-objective assignment model and designed a multi-nutrient colony location algorithm to solve the model. In order to solve military personnel assignment problems, a two-sided matching based decision support system to assist the decision makers have been established in [4].The system could generate positions' preferences from position requirement profiles and personnel competence profiles by using analytic hierarchy process and match personnel to positions by using two-sided matching. For personnel assignment problem, in [6] have been proposed a bi-objective binary integer programming-based approach with a feedback mechanism, in which the interdependencies among positions and the differences among the selected employees were considered simultaneously. An expert system for the matching between an unemployed with an offered job based on the analysis of a corporate database of unemployed and enterprises profile data using Neuro-Fuzzy techniques presented in [2]. An approach to optimize the matching of one-shot multi-attribute exchanges with quantity discounts in E-brokerage based on the conception and definition of matching degree and quantity discount. In order to solve matching problem between ventures and venture capitalists through an intermediary presented in [9]. A multi-objective optimization model and gave the corresponding solution to the model established in [12]. A new method for two-sided matching decision making of PPP projects based on intuitionistic fuzzy Choquet integral presented in [13]. A two-sided matching decision method for supply and demand of technological knowledge studied in [14]. A fuzzy multi-criteria decision making approach for solving a bi-objective personnel assignment problem presented in [15]. This paper proposes a systematic approach with a feedback mechanism in which the interdependences among positions and the differences among the selected employees are considered simultaneously. An analytic hierarchy process and two-sided matching based decision support system for military personnel assignment presented in [16]. A matching decision model for self- adaptability of knowledge manufacturing system based on the fuzzy neural network model presented in [17]. The proposed fuzzy neural network model is employed to evaluate the matching degree. A problem of marriage matching based on fuzzy duality decision. Prior studies have significantly advanced the area of two-sided matching studied in [18].

Due to the lack of knowledge, for actual two-sided matching problems, there may be cases that the attributes weight information for matching objects is not given accurately. For instance, for a matching problem between employees and positions, the employees may consider that the attribute "developing space" is twice as important as the attribute "salary level". However, most of the existing approaches aim to deal with two-sided matching problems with perfect weight information of evaluation attributes, and studies for two-sided matching problems with incomplete weight information have seldom been addressed. Although, an analysis method for trade matching problem with incomplete attribute weight in consumer to consumer e-commerce environment, the method couldn't deal with two-sided matching problems with different attribute sets for the two sides presented in [11].

In addition, the evaluation information given by the matching objects is usually diverse, which is also an issue that needs to be considered for actual two-sided matching problems.

In the process of practical matching decision making, the decision information is sometimes expressed in the form of heterogeneous data. In classical matching decision making, the ratings and the attribute weight are known precisely. During the matching decision process, due to the complexity of decision-making environment and the ambiguity and uncertainty of human thinking and judgments, preferences decision makers cannot be estimated with a precise numerical data and the estimated values of attributes are often not so deterministic. The assessment information of alternatives under each attribute given by the decision makers (or experts) over the matching decision making may not be all described by exact numbers, sometimes they may take the heterogeneous form, such as precise numerical values, interval numbers, triangular fuzzy numbers, linguistic labels, or even intuitionistic fuzzy numbers. Matching decision making problem with heterogeneous form of attribute information is called the hybrid (or heterogeneous) matching decision making problem, which is very complex and interesting problem in real applications [19, 20]. Due to the wide existence of such problems in the real world, it is necessary to develop new analysis method to deal with two-sided matching problems with incomplete weight information and different formats of evaluation information, which is also the motivation of this paper. This method has not been discussed yet in all of matching decision making papers mentioned above, which is also else motivation of this paper. The rest of this paper is organized as follows. In section 2, we give the formal description of two-sided matching problem with different evaluation attributes for the two sides and normalizing method of decision matrix. After that, we present a method for determine the comprehensive weights of attribute on matching object in section 3. After that, we present a decision analysis method to deal with the two-sided matching decision making problem through establishing some optimization models in section 4. In section 5, two illustrative examples are given to illustrate the feasibility and effectiveness of the proposed method.

## 2. Description of the Two-Sided Matching Decision Making Problem

For the convenience of analysis, in this section, we give the formal description of the two-sided matching decision making problem considered in this paper.

Let $A=\{A_1,A_2,\cdots,A_m\}$ and $B=\{B_1,B_2,\cdots,B_n\}$ be the set of matching objects. Here $A_i$ and $B_j$ denote the $i$ th matching object in $A$ and the $j$ th matching object in $B$, respectively. The matching object $A_i$ $(i=1,2,\cdots,m)$ evaluates the matching object in $B$ according to attribute set $G=\{G_1,G_2,\cdots,G_q\}$, and the weight vector is denoted as $(w_{i1},w_{i2},\cdots,w_{iq})$. Here $0\le w_{ik}\le 1$ $(i=1,2,\cdots,m)$, $\sum_{k=1}^{q} w_{ik}=1$.

The matching object $B_j$ $(j=1,2,\cdots,n)$ evaluates the matching object in $A$ according to attribute set $I=\{I_1,I_2,\cdots,I_p\}$, and the weight vector is denoted as $\mu_j=(\mu_{j1},\mu_{j2},\cdots,\mu_{jp})$. Here $0\le \mu_{il}\le 1$ $(j=1,2,\cdots,n)$, $\sum_{l=1}^{p}\mu_{jl}=1$.

In this paper, the two-sided matching decision making problem which we are going to solve is to obtain the optimal matching for the matching objects by considering all the information of the attribute weight of matching object.

First of all, we denote the evaluation value given by the $i$ $i$th matching object in $A$ on $B_j$ with regard to the $k$ th attribute by $a_{ijk}$ ( $i=1,2,\cdots,m$ , $j=1,2,\cdots,n$ , $k=1,2,\cdots,q$ ) and an evaluation matrix given by $A_i$ can be obtained as $X_i=(a_{ijk})_{n\times q}$ ( $i=1,2,\cdots,m$, $j=1,2,\cdots,n$, $k=1,2,\cdots,q$ ).

For different types of evaluation attributes, the matching objects may express their assessment using different formats of information.

In this paper, we consider the following four formats of evaluation information, namely real number, interval number, linguistic value and uncertain linguistic value

Suppose that $S=(s_0,s_1,\cdots,s_{l-1})$ is a finite and fully ordered discrete term set. Here $l$ an odd number is. In real situations, $l$ would be equal to 3, 5, 7, 9, etc. For example, when $l=7$, a set $S$ can be given as follows: $S=(s_0,s_1,s_2,s_3,s_4,s_5,s_7)$ = {very poor, poor, slightly poor, fair, slightly good, good, and very good}.

**[Definition 1]**([21]). suppose $\tilde{s}=[s_a,s_b],s_a,s_b\in S$ ( $a\le b$ ) are the lower limit and the upper limit of $\tilde{s}$, respectively. Then, $\tilde{s}$ is called an uncertain linguistic variable.

Each linguistic value can be represented by triangle fuzzy number $s=[a^L,a^M,a^U]$ , $a^L\le a^M\le a^U$ which has a membership function defined by

$$\mu_s(x)=\begin{cases}(x-a^L)/(a^M-a^L), & a^L\le x\le a^M\\ (a^U-x)/(a^U-a^M), & a^M\le x\le a^U\\ 0, & \text{otherwise}\end{cases} \tag{1}$$

The expression forms of triangle fuzzy number corresponding to $S$ are as follows:' absolutely low(AL)' = [0, 0, 0.1], 'very low(VL)'= [0, 0.1, 0.2], 'rather low(RL)'= [0.1, 0.2, 0.3], 'low(L)'= [0.2, 0.3, 0.4], 'slightly low(SL)'= [0.3, 0.4, 0.5], 'middle(M)' = [0.4, 0.5, 0.6], 'slightly high(SH)' = [0.5, 0.6, 0.7], ' high(H)'=[0.6, 0.7, 0.8], 'rather high(RH)' = [0.7, 0.8, 0.9], 'very high(VH)'= [0.8, 0.9, 1.0], 'absolutely high(AH)' = [0.9, 1.0, 1.0].

Let $\tilde{A}=[\alpha,\beta,\gamma,\delta]$ be trapezoid fuzzy number. Then, its membership function is defined by

$$\mu_{\tilde{A}}(x)=\begin{cases}(x-\alpha)/(\beta-\alpha), & \alpha\le x\le\beta\\ 1, & \beta<x<\gamma\\ (\delta-x)/(\delta-\gamma), & \gamma\le x\le\delta\\ 0, & \text{otherwise}\end{cases} \tag{2}$$

Let $s_a = [a^L, a^M, a^U]$ and $s_b = [b^L, b^M, b^U]$ are two triangle fuzzy numbers. Then, we can define a trapezoid fuzzy number $\tilde{s} = (a^L, a^M, b^M, b^U)$ by the membership function as follows:

$$\tilde{s}(x) = \begin{cases} (x - a^L)/(a^M - a^L), & a^L \le x \le a^M \\ 1 & , a^M \le x \le b^M \\ (b^U - x)/(b^U - b^M), & b^M \le x \le b^U \\ 0 & , \text{otherwise} \end{cases} \qquad (3)$$

Then, $\tilde{s} = [s_a, s_b]$ can be regards as an uncertain linguistic value which has the membership function by Eq. (3)

**[Definition 2]** Let $a_{ijk} = (a_{ijk}^{(1)}, a_{ijk}^{(2)}, a_{ijk}^{(3)} a_{ijk}^{(4)})$ ( $a_{ijk}^{(1)} \le a_{ijk}^{(2)} \le a_{ijk}^{(3)} \le a_{ijk}^{(4)}$ ). Then $a_{ijk}$ ( $i = 1,2,\cdots,m$ , $j = 1, 2,\cdots,n$ , $k = 1,2,\cdots,q$ )is called a hybrid attribute value given by the $i$th matching object in $A$ with regard to the $k$ th attribute on matching object $B_j$

The concrete types of $a_{ijk} = (a_{ijk}^{(1)}, a_{ijk}^{(2)}, a_{ijk}^{(3)}, a_{ijk}^{(4)})$ are such as;

(i) Real number type : $a_{ijk}^{(1)} = a_{ijk}^{(2)} = a_{ijk}^{(3)} = a_{ijk}^{(4)}$ ;

(ii) Interval real number type : $a_{ijk}^{(1)} = a_{ijk}^{(2)} < a_{ijk}^{(3)} = a_{ijk}^{(4)}$ ;

(iii) Linguistic value type : $a_{ijk}^{(1)} = a_{ijk}^{(2)} < a_{ijk}^{(3)} = a_{ijk}^{(4)}$

(iv) Uncertain linguistic value type : $a_{ijk}^{(1)} < a_{ijk}^{(2)} < a_{ijk}^{(3)} < a_{ijk}^{(4)}$

Thus, the hybrid attribute values treated in this paper can be dealt uniformly with values $a_{ijk} = (a_{ijk}^{(1)}, a_{ijk}^{(2)}, a_{ijk}^{(3)}, a_{ijk}^{(4)})$ ( $a_{ijk}^{(1)} \le a_{ijk}^{(2)} \le a_{ijk}^{(3)} \le a_{ijk}^{(4)}$ ) in four dimensional Euclidean space

**[Definition 3]** Let $a_{ijk} = (a_{ijk}^{(1)}, a_{ijk}^{(2)}, a_{ijk}^{(3)}, a_{ijk}^{(4)})$ and $b_{ijk} = (b_{ijk}^{(1)}, b_{ijk}^{(2)}, b_{ijk}^{(3)}, b_{ijk}^{(4)})$ be the hybrid attribute values, respectively. The Euclidean distance of between $a_{ijk}$ and $b_{ijk}$ is defined by

$$d(a_{ijk}, b_{ijk}) = \sqrt{(b_{ijk}^{(1)} - a_{ijk}^{(1)})^2 + (b_{ijk}^{(2)} - a_{ijk}^{(2)})^2 + (b_{ijk}^{(3)} - b_{ijk}^{(3)})^2 + (b_{ijk}^{(4)} - a_{ijk}^{(4)})^2}\ .$$

For matching object $A_i$ , the normalized decision matrix $X = \{x_{ij}\}_{m \times n}$ of given decision matrix $R_i = (a_{ijk})_{n \times q}$ ( $i = 1,2,\cdots,m$ , $j = 1,2,\cdots,n$ , $k = 1,2,\cdots,q$ ) is obtained by the following method.

(i) Normalization in the case which $a_{ijk}$ is real number.

In this case, we can obtain $x_{ijk} = \bar{x}_{ijk}$ because of $a_{ijk}^{(1)} = a_{ijk}^{(2)} = a_{ijk}^{(3)} = a_{ijk}^{(4)} = \bar{a}_{ijk}$ .

Therefore, we can obtain $x_{ijk}^{(1)} = x_{ijk}^{(2)} = x_{ijk}^{(3)} = x_{ijk}^{(4)} = \bar{x}_{ijk}$ . So if $a_{ijk}$ is the effect -type estimated value, then $\bar{x}_{ijk}$ is obtained by the normalization as follows: $\bar{x}_{ijk} = \bar{a}_{ijk} / \sqrt{\sum_{j=1}^{n} (\bar{a}_{ijk})^2}$ , and if $a_{ijk}$ is the cost-type estimated value, then $\bar{x}_{ijk}$ is obtained by the normalization as follows : $\bar{x}_{ijk} = (1/\bar{a}_{ijk}) / \sqrt{\sum_{j=1}^{n} (1/\bar{a}_{ijk})^2}$ .

(ii) Normalization in the case which $a_{ijk}$ is interval number

In this case, we can obtain $x_{ijk} = [\underline{x}_{ijk}, \bar{x}_{ijk}]$ because of $\underline{a}_{ijk} = a_{ijk}^{(1)} = a_{ijk}^{(2)} < a_{ijk}^{(3)} = a_{ijk}^{(4)} = \bar{a}_{ijk}$ . Therefore, we can obtain $\underline{x}_{ijk} = x_{ijk}^{(1)} = x_{ijk}^{(2)} < x_{ijk}^{(3)} = x_{ijk}^{(4)} = \bar{x}_{ijk}$ . So if $a_{ijk}$ is the effect -type estimated value, then $\underline{x}_{ijk}$ and $\bar{x}_{ijk}$ is obtained by the normalization as follows: $\underline{x}_{ijk} = \underline{a}_{ijk} / \sqrt{\sum_{j=1}^{n} (\bar{a}_{ijk})^2}$

$\bar{x}_{ijk} = \bar{a}_{ijk} / \sqrt{\sum_{j=1}^{n} (\underline{a}_{ijk})^2}$ , and if $a_{ijk}$ is the cost-type estimated value, then $\underline{x}_{ijk}$ and $\bar{x}_{ijk}$ are obtained by the normalization as follows: $\underline{x}_{ijk} = (1/\bar{a}_{ijk}) / \sqrt{\sum_{j=1}^{n} (1/\underline{a}_{ijk})^2}$ , $\bar{x}_{ijk} = (1/\underline{a}_{ijk}) / \sqrt{\sum_{j=1}^{n} (1/\bar{a}_{ijk})^2}$ .

(iii) Normalization in the case which $a_{ijk}$ is linguistic value type

In this case, we can obtain $x_{ijk} = [x_{ijk}^L, x_{ijk}^*, x_{ijk}^U]$ because of $a_{ijk}^{(1)} = a_{ijk}^{(2)} < a_{ijk}^{(3)} = a_{ijk}^{(4)} = [a_{ijk}^L, a_{ijk}^*, a_{ijk}^U]$ . So if $a_{ijk}$ is the effect -type estimated value, then $x_{ijk}^L, x_{ijk}^*$ and $x_{ijk}^U$ is obtained by the normalization as follows:

$$x_{ijk}^L = \frac{a_{ijk}^L}{\sqrt{\sum_{j=1}^{n} (a_{ijk}^U)^2}},\ x_{ijk}^* = \frac{a_{ijk}^*}{\sqrt{\sum_{j=1}^{n} (a_{ijk}^*)^2}},\ x_{ijk}^U = \frac{a_{ijk}^U}{\sqrt{\sum_{j=1}^{n} (a_{ijk}^L)^2}}$$

and if $a_{ijk}$ is the cost-type estimated value, then

$$x_{ijk}^L = \frac{1/a_{ijk}^U}{\sqrt{\sum_{j=1}^{n} (1/a_{ijk}^L)^2}},\ x_{ijk}^* = \frac{1/a_{ijk}^*}{\sqrt{\sum_{j=1}^{n} (1/a_{ijk}^*)^2}},\ x_{ijk}^U = \frac{1/a_{ijk}^L}{\sqrt{\sum_{j=1}^{n} (1/a_{ijk}^U)^2}}$$

(iv) Normalization in the case which $a_{ijk}$ is uncertain linguistic value type

In this case, we can obtain $x_{ijk} = [x_{ijk}^L, x_{ijk}^*, x_{ijk}^{**}, x_{ijk}^U]$ because of $a_{ijk}^{(1)} < a_{ijk}^{(2)} < a_{ijk}^{(3)} < a_{ijk}^{(4)} = [a_{ijk}^L, a_{ijk}^*, a_{ijk}^{**}, a_{ijk}^U]$.

Therefore, we take its normalization as follows: if $a_{ijk}$ is the effect -type estimated value, then

$$x_{ijk}^L = \frac{a_{ijk}^L}{\sqrt{\sum_{j=1}^{n} (a_{ijk}^*)^2}},\ x_{ijk}^* = \frac{a_{ijk}^*}{\sqrt{\sum_{j=1}^{n} (a_{ijk}^{**})^2}},\ x_{ijk}^{**} = \frac{a_{ijk}^{**}}{\sqrt{\sum_{j=1}^{n} (a_{ijk}^*)^2}},$$

$$x_{ijk}^U = \frac{a_{ijk}^U}{\sqrt{\sum_{j=1}^{n} (a_{ijk}^L)^2}}$$ and if $a_{ijk}$ is the cost-type estimated

value, then $x_{ijk}^{L}=\frac{1/a_{ijk}^{L}}{\sqrt{\sum_{j=1}^{n}(1/a_{ijk}^{L})^2}}$, $x_{ijk}^{*}=\frac{1/a_{ijk}^{**}}{\sqrt{\sum_{j=1}^{n}(a_{ijk}^{L})^2}}$, $x_{ijk}^{**}=\frac{1/a_{ijk}^{*}}{\sqrt{\sum_{j=1}^{n}(1/a_{ijk}^{**})^2}}$, $x_{ijk}^{U}=\frac{1/a_{ijk}^{L}}{\sqrt{\sum_{j=1}^{n}(a_{ijk}^{U})^2}}$

## 3. Determination of the Comprehensive Weights of Attribute on Matching Object

### 3.1 Determination of the subjective weights of attribute on matching object

Analytic Hierarchy Process(AHP), developed by [22], addresses how to determine the relative importance of a set of activities in a multi-criteria decision problem. Therefore, AHP method is usually used in subjective weight determination of attribute.

The subjective weight of matching object is determined using group AHP method by a decision-making group consisting of $L$ decision-experts.

Let $\alpha_l=[\alpha_l^1,\cdots,\alpha_l^j,\cdots,\alpha_l^n]$ $(l=1,2,\cdots,L)$ be the subjective weights of matching object determined by AHP from decision-makers.

By using the weights $\alpha_1,\cdots,\alpha_L$ determined by $L$ decision-makers, the weight of attribute $G_j$ for all matching object $A_i$ $(i=1,2,\cdots,m)$ is determined by interval grey number $\alpha_j(\otimes)$ $(j=1,2,\cdots,n)$ such as $\alpha_j(\otimes)\in[\underline{\alpha}_j,\overline{\alpha}_j]$, $0\le\underline{\alpha}_j\le\overline{\alpha}_j$.
Here $\underline{\alpha}_j=\min_{1\le l\le L}\{\alpha_l^j\}$, $\overline{\alpha}_j=\max_{1\le l\le L}\{\alpha_l^j\}$ $(j=1,2,\cdots,n)$.

Final subjective weight of attribute $G$ for all matching object $A_i$ $(i=1,2,\cdots,m)$ is given as follows: $\alpha(\otimes)=(\alpha_1(\otimes),\alpha_2(\otimes),\cdots,\alpha_n(\otimes))$. $\alpha_k(\otimes)\in[\underline{\alpha}_k,\overline{\alpha}_k]$, $0\le\underline{\alpha}_k\le\overline{\alpha}_k, k=1,2,\cdots,q$.

### 3.2 Determination of the objective weights of attribute on matching object

(1) Determination of the objective weight of attribute based on maximizing variation

First of all, we denote the normalized evaluation value given by the $i$th matching object in $A_i$ over $B_j$ with regard to the $k$th attribute by $x_{ijk}$ $(i=1,2,\cdots,m,\ j=1,2,\cdots,n,k=1,2,\cdots,q)$
Let $X=(x_{ijk})_{m\times n}$ $(k=1,2,\cdots,q)$ is the normalized decision matrix of matching object $A_i$ on matching object $B_j$

We determine the attribute weight vector given by matching object $A_i$ $(i=1,2,\cdots,m)$.

We define the weighted deviation of matching object $A_i$ from all other matching object for attribute $G_k$ in normalized decision matrix $X=(x_{ijk})_{m\times n}$ for all matching object $B_j$ $(j=1,2,\cdots,n)$ as follows.

$$D_{ijk}(\beta^{opt})=\sum_{l=1}^{m}d(x_{ijk},x_{ljk})\beta_{ijk}^{opt}$$
$$=\sum_{l=1}^{m}\sqrt{(x_{ijk}^{(1)}-x_{ljk}^{(1)})^2+(x_{ijk}^{(2)}-x_{ljk}^{(2)})^2+(x_{ijk}^{(3)}-x_{ljk}^{(3)})^2+(x_{ijk}^{(4)}-x_{ljk}^{(4)})^2}\,\beta_{ijk}^{opt}$$

Here $\beta_{ijk}^{opt}$ is the weight to be funded by maximizing variation method of for attribute $G_k$ on matching object $A_i$ $(i=1,2,\cdots,m)$ over matching object $B_j$.

In order to find weight $\beta_{ijk}^{opt}$ such that sum of overall deviation attains maximum for all matching object $B_j$ $(j=1,2,\cdots,n)$, we define a deviation function

$$D_j(\beta)=\sum_{k=1}^{q}\sum_{i=1}^{m}\sum_{l=1}^{m}d(x_{ijk},x_{ljk})\beta_{ijk}\quad(j=1,2,\cdots,n)$$

And solve the following nonlinear programming problem.

$$[\text{P1}]\qquad \max D_j(\beta)=\sum_{k=1}^{q}\sum_{i=1}^{m}\sum_{l=1}^{m}d(x_{ijk},x_{ljk})\beta_{ijk}$$
$$s.t.\ \sum_{k=1}^{q}\beta_{ijk}^2=1,\ \beta_{ijk}\ge0,\ i=1,2,\cdots,m,\ j=1,2,\cdots,n$$

**[Theorem 1]** The solution of problem P1 for all matching object $B_j$ $(j=1,2,\cdots,n)$ is given by

$$\beta_{ijk}^{*}=\frac{\sum_{i=1}^{m}\sum_{l=1}^{m}d(x_{ijk},x_{ljk})}{\sqrt{\sum_{k=1}^{q}\left[\sum_{i=1}^{m}\sum_{l=1}^{m}d(x_{ijk},x_{ljk})\right]^2}}$$

The above Eq. is obtained from P1 by Lagrange's method. That is, to solve this model, we construct the Lagrange function:

$$L(\beta_{ijk},\lambda)=\sum_{k=1}^{q}\sum_{i=1}^{m}\sum_{l=1}^{m}d(x_{ijk},x_{ilk})\beta_{ijk}+\lambda(\sum_{k=1}^{q}\beta_{ijk}^2-1)$$

Here $\lambda$ is the Lagrange multiplier.
Differentiating Lagrange function $L(\beta_{ijk},\lambda)$ with respect to $\beta_{ijk}$ $(i=1,2,\cdots,m,\ j=1,2,\cdots,n,k=1,2,\cdots,q)$ and $\lambda$, and setting these partial derivatives equal to zero, we can get the above Eq.
By the normalization of $\beta_{ijk}^{*}$ $(i=1,2,\cdots,m,j=1,2,\cdots n,$ $k=1,2,\cdots,q)$ for all matching object $B_j$ $(j=1,2,\cdots,n)$, we obtain: $\beta_{ijk}^{opt}=\frac{\sum_{i=1}^{m}\sum_{l=1}^{m}d(x_{ijk},x_{ilk})}{\sum_{k=1}^{q}\sum_{i=1}^{m}\sum_{l=1}^{m}d(x_{ijk},x_{ilk})}$

(2) Determination of the objective weight by Entropy method.

The entropy weight $\beta_{ijk}^{ent}$ of attribute $G_k$ on matching object $A_i$ $(i=1,2,\cdots,m)$ for all matching object $B_j$ $(j=1,2,\cdots,n)$ is obtained as follows: $\beta_{ijk}^{ent}=\dfrac{1-E_{ijk}}{q-\sum_{k=1}^{q}E_{ijk}}$

Here $E_{ijk}=-\dfrac{1}{\ln m}\sum_{k=1}^{q}\dfrac{D_{ijk}}{D_{ijk}}\ln\dfrac{D_{ijk}}{D_{ijk}}$ $(i=1,2,\cdots,m,j=1,2,\cdots m,$ $k=1,2,\cdots,q)$.

$$D_{ijk}=\sum_{l=1}^{m}d(x_{ijk},x_{ljk}),\ D_{ijk}=\sum_{k=1}^{q}D_{ijk}=\sum_{i=1}^{m}\sum_{l=1}^{m}d(x_{ijk},x_{ljk})$$

In the above Eq., if $\dfrac{D_{ijk}}{D_{ijk}}=0$, then we regard that $\dfrac{D_{ijk}}{D_{ijk}}\ln\dfrac{D_{ijk}}{D_{ijk}}=0$ $(i=1,2,\cdots,m,j=1,2,\ \cdots,n,k=1,2,\cdots,q)$.

(3) Determination of the objective weight by weighted average deviation method with least membership degree.

Here $J^+$ is set of profit type attributes and $J^-$ is set of cost profit type attributes of $G_k$ $(k=1,2,\cdots,q)$ on matching object $A_i$ $(i=1,2,\cdots,m)$ for all matching object $B_j$ $(j=1,2,\cdots,n)$.

Let $x_j^+=\{x_{j1}^+,x_{j2}^+,\cdots,x_{jq}^+\}$ is the positive ideal object over matching object $A_i$ $(i=1,2,\cdots,m)$ for all matching object $B_j$ $(j=1,2,\cdots,n)$ ([24]).

Here $x_{jk}^+=\{x_{jk}^{+(1)},x_{jk}^{+(2)},x_{jk}^{+(3)},x_{jk}^{+(4)}\}$ and

$$x_{jk}^{+(t)}=\{\max_{1\le i\le m}x_{ijk}^{(t)}\big|j\in J^+,\min_{1\le i\le m}x_{ijk}^{(t)}\big|j\in J^-,t=1,2,3,4\}$$
$(j=1,2,\cdots,n,k=1,2,\cdots,q)$

The deviation between estimated value $x_{ijk}$ and the positive ideal object $x_j^+$ by considering object weight $\beta_{ik}^{ave}$ over all matching object $B_j$ $(j=1,2,\cdots,n)$ is defined as

$$f_j(\beta_{ijk}^{ave})=\sum_{i=1}^{m}\sum_{k=1}^{q}\left[(x_{jk}^{+(1)}-x_{ijk}^{(1)})+(x_{jk}^{+(2)}-x_{ijk}^{(2)})+(x_{jk}^{+(3)}-x_{ijk}^{(3)})+(x_{jk}^{+(4)}-x_{ijk}^{(4)})\right]\cdot\beta_{ijk}^{ave}$$

Evidently, the smaller $f_i(\beta_{ijk}^{ave})$ is, the better it. Therefore, we can construct an optimization problem as following.

[P2] $\min Z=$

$$\sum_{i=1}^{m}\sum_{k=1}^{q}\left[(x_{jk}^{+(1)}-x_{ijk}^{(1)})+(x_{jk}^{+(2)}-x_{ijk}^{(2)})+(x_{jk}^{+(3)}-x_{ijk}^{(3)})+(x_{jk}^{+(4)}-x_{ijk}^{(4)})\right]\cdot\beta_{ijk}^{ave}$$

$$s.t.\begin{cases}\sum_{k=1}^{q}(\beta_{ijk}^{ave})^2=1 & (i=1,2,\cdots,m,j=1,2,\cdots,n)\\ \beta_{ijk}^{ave}\ge 0\end{cases}$$

**[Theorem 2]** The solution of problem P2 is given by

$$\beta_{ijk}^{ave}=\frac{\sum_{k=1}^{q}\left[(x_{jk}^{+(1)}-x_{ijk}^{(1)})+(x_{jk}^{+(2)}-x_{ijk}^{(2)})+(x_{jk}^{+(3)}-x_{ijk}^{(3)})+(x_{jk}^{+(4)}-x_{ijk}^{(4)})\right]}{\sqrt{\sum_{i=1}^{m}\left[\sum_{k=1}^{q}\left[(x_{jk}^{+(1)}-x_{ijk}^{(1)})+(x_{jk}^{+(2)}-x_{ijk}^{(2)})+(x_{jk}^{+(3)}-x_{ijk}^{(3)})+(x_{jk}^{+(4)}-x_{ijk}^{(4)})\right]\right]^2}}$$

$(i=1,2,\cdots,m,\ j=1,2,\cdots,n,\ k=1,2,\cdots,q)$

The above Eq. is obtained from P2 by Lagrange's method. That is, to solve this model, we construct the Lagrange function:

$$L(\beta_{ijk}^{ave},\lambda)=\sum_{i=1}^{m}\sum_{k=1}^{q}\beta_{ik}^{ave}\cdot\left[(x_{jk}^{+(1)}-x_{ijk}^{(1)})+(x_{jk}^{+(2)}-x_{ijk}^{(2)})+(x_{jk}^{+(3)}-x_{ijk}^{(3)})+(x_{jk}^{+(4)}-x_{ijk}^{(4)})\right]+\lambda(\sum_{k=1}^{q}(\beta_{ijk}^{ave})^2-1)$$

Here $\lambda$ is the Lagrange multiplier.

Differentiating Lagrange function $L(\beta_{ijk}^{ave},\lambda)$ with respect to $\beta_{ijk}^{ave}$ $(i=1,2,\cdots,\ m=1,2,\cdots,q)$ and $\lambda$, and setting these partial derivatives equal to zero, we can get the above Eq. for determining the attribute weights of matching object.

By the normalization of $\beta_{ijk}^{ave}$ $(i=1,2,\cdots,m,j=1,2,\cdots m,$ $k=1,2,\cdots,q)$, we obtain attribute weight $\beta_{ik}^{ave}$ on matching object $A_i$ $(i=1,2,\cdots,m)$ for all matching object $B_j$ $(j=1,2,\cdots,n)$ as

$$\beta_{ijk}^{ave}=\frac{\sum_{i=1}^{m}\left[(x_{jk}^{+(1)}-x_{ijk}^{(1)})+(x_{jk}^{+(2)}-x_{ijk}^{(2)})+(x_{jk}^{+(3)}-x_{ijk}^{(3)})+(x_{jk}^{+(4)}-x_{ijk}^{(4)})\right]}{\sum_{k=1}^{q}\sum_{i=1}^{m}\left[(x_{jk}^{+(1)}-x_{ijk}^{(1)})+(x_{jk}^{+(2)}-x_{ijk}^{(2)})+(x_{jk}^{+(3)}-x_{ijk}^{(3)})+(x_{jk}^{+(4)}-x_{ijk}^{(4)})\right]}$$

$(i=1,2,\cdots,m,\ j=1,2,\cdots,n,\ k=1,2,\cdots,q)$

(4) Determination of the objective weight by mean squared deviation method

The weight of mean-squared deviation $\beta_{ijk}^{dev}$ of attribute $G_k$ on matching object $A_i$ $(i=1,2,\cdots,m)$ for all matching object $B_j$ $(j=1,2,\cdots,n)$ is obtained as follows:

$$\beta_{jik}^{dev}=\frac{\sigma(S_{ik})}{\sum_{k=1}^{q}\sigma(S_{ik})}\quad(i=1,2,\cdots,m,j=1,2,\cdots m,\ \ k=1,2,\cdots,q).$$

Here $\sigma(S_{ijk})=\sqrt{\sum_{t=1}^{4}\sum_{i=1}^{m}(x_{ijk}^{(t)}-E(S_{jk}^{(t)}))^2}$, $E(S_{jk}^{(t)})=\dfrac{1}{m}\sum_{i=1}^{m}x_{ijk}^{(t)}$

(5) Determination of the objective weight by grey relational degree method

We can compute the weighted grey relational coefficient between positive object $x_j^+=\{x_{j1}^+,x_{j2}^+,\cdots,x_{jq}^+\}$ and attribute value $x_{ijk}$ of matching object $A_i$ $(i=1,2,\cdots,m)$ over matching object $B_j$ $(j=1,2,\cdots,n)$ by the following formulae ( [23])

$$\gamma_{ijk}^{+}=\frac{\min_i\min_k d(x_{ijk},x_{jk}^{+})+\rho\max_i\max_k d(x_{ijk},x_{jk}^{+})}{d(x_{ijk},x_{jk}^{+})+\rho\max_i\max_k d(x_{ijk},x_{jk}^{+})}$$

( $i=1,2,\cdots,m$ , $j=1,2,\cdots,n$ , $k=1,2,\cdots,q$ )

Here $\rho\in[0,1]$ is a discriminative coefficient or resolving factor. Commonly, it is taken as $\rho=0.5$ , and

$$d(x_{ijk},x_{jk}^{+})=$$

$$\sum_{k=1}^{q}\sqrt{(x_{ijk}^{(1)}-x_{jk}^{+(1)})^2+(x_{ijk}^{(2)}-x_{jk}^{+(2)})^2+(x_{ijk}^{(3)}-x_{jk}^{+(3)})^2+(x_{ijk}^{(4)}-x_{jk}^{+(4)})^2}$$

The grey relational coefficient between positive ideal matching object and itself is $(1,1,\cdots,1)$ , so the comprehensive gray relational coefficient deviation sum is

$$d_i(\beta_{ijk}^{grey})=\sum_{k=1}^{q}(1-\gamma_{ijk}^{+})\beta_{ijk}^{grey}$$

Here $\beta_{ik}^{grey}$ is weight of attribute of $G_k$ on matching object $A_i$ $(i=1,2,\cdots,m)$.

So, we can establish the following multiple objective optimization problem to obtain the weight information:

[P3]

$$\min d_i(\beta_{ijk}^{grey})=\sum_{k=1}^{q}(1-\gamma_{ijk}^{+})\beta_{ijk}^{grey}\ ,(i=1,2,\cdots,m,\ j=1,2,\cdots,n)$$

$$s.t.:\ \beta_{ijk}^{grey}\in H$$

Here $H$ is a set of the given weight information. Since each matching object $A_i$ $(i=1,2,\cdots,m)$ is non-inferior, so there exists no preference relation on the all the matching objects. Then, we may regard the above multiple objective optimization model as problem to find the weights in the following single objective optimization model:

[P4] $$\min d(\beta_{ijk}^{grey})=\sum_{i=1}^{m}\sum_{k=1}^{q}(1-\gamma_{ijk}^{+})\beta_{ijk}^{grey}\ (\ j=1,2,\cdots,n\ )$$

$$s.t.\ \ \beta_{ijk}^{grey}\in H\ (\ i=1,2,\cdots,m\ ,\ k=1,2,\cdots,q\ )$$

By solving the model [P4] we get the optimal solution $\beta_{ij}^{grey}=(\beta_{ij1}^{grey},\beta_{ij2}^{grey},\cdots,\beta_{ijq}^{grey})$ , which can be used as the weight vector of attributes $G$ on matching object $A_i$ over matching object $B_j$ .

If the information about attribute weights is completely unknown, we can establish another multiple objective programming model as follows:

[P5] $$\min d_i(\beta_{ijk}^{grey})=\sum_{k=1}^{q}(1-\gamma_{ijk}^{+})\beta_{ijk}^{grey}\ ,$$

$$s.t.\ \sum_{k=1}^{q}\beta_{ijk}^{grey}=1,\beta_{ijk}^{grey}\geq 0\,(i=1,2,\cdots,m,\ j=1,2,\cdots,n)$$

Similarly, may regard the above multiple objective optimization model as problem to find the weights in the following single objective optimization model:

[P6] $$\min d(\beta_{ijk}^{grey})=\sum_{i=1}^{m}\sum_{k=1}^{q}[(1-\gamma_{ijk}^{+})\beta_{ijk}^{grey}]^2$$

$$s.t.:\sum_{k=1}^{q}\beta_{ijk}^{grey}=1,\beta_{ijk}^{grey}\geq 0\ \ (i=1,2,\cdots,m,\ j=1,2,\cdots,n)$$

To solve this model, we construct the Lagrange function:

$$L(\beta_{ijk}^{grey},\lambda)=\sum_{i=1}^{m}\sum_{k=1}^{q}[(1-\gamma_{ijk}^{+})\beta_{ijk}^{grey}]^2+2\lambda(\sum_{k=1}^{q}\beta_{ijk}^{grey}-1)$$

Here λ is the Lagrange multiplier.

Differentiating Lagrange function $L(\beta_{ijk}^{grey},\lambda)$ with respect to $\beta_{ijk}^{grey}$ $(i=1,2,\cdots,m,\ \ j=1,2,\cdots,n,k=1,2,\cdots,q)$ and $\lambda$ , and setting these partial derivatives equal to zero, we get a simple and exact formula for determining the weight of attribute $G_k$ on matching object $A_i$ over all matching object $B_j$ as follows:

$$\beta_{ijk}^{grey}=[\sum_{i=1}^{m}(\sum_{k=1}^{q}(1-\gamma_{ijk}^{+})^2)^{-1}]^{-1}/\sum_{i=1}^{m}(1-\gamma_{ijk}^{+})^2\quad (\ i=1,2,\cdots,m\ ,$$
$j=1,2,\cdots,n$ , $k=1,2,\cdots,q$ )

(ⅵ) Determination of the comprehensive objective weights

The comprehensive objective weight of attribute $G_k$ on matching object $A_i$ $(i=1,2,\cdots,\ m)$ over all matching object $B_j$ is determined by the interval grey number $\beta_{ij}(\otimes)=(\beta_{ij1}(\otimes),\beta_{ij2}(\otimes),\cdots,\beta_{ijk}(\otimes),\cdots,\ \beta_{ijq}(\otimes))$ .

Here $\beta_{ijk}(\otimes)\in[\underline{\beta}_{ijk},\overline{\beta}_{ijk}]$ and $\underline{\beta}_{ijk}(\otimes)=\min\{\beta_{ijk}^{opt},\ \beta_{ijk}^{ent},\ \beta_{ijk}^{avee},\beta_{ijk}^{dev},\beta_{ijk}^{grey}\}$ , $\overline{\beta}_{ijk}(\otimes)=\max\{\beta_{ijk}^{opt},\ \beta_{ijk}^{ent},\ \beta_{ijk}^{avee},\beta_{ijk}^{dev},\ \beta_{ijk}^{grey}\}$ .

### 3.3 Determination of the final comprehensive weight

When the subjective comprehensive weight $\alpha(\otimes)=(\alpha_1(\otimes),\alpha_2(\otimes),\cdots,\alpha_q(\otimes))$ ( $\alpha_k(\otimes)\in[\underline{\alpha}_k,\overline{\alpha}_k]$ ) and objective comprehensive weight $\beta_{ij}(\otimes)=(\beta_{ij1}(\otimes),\beta_{ij2}(\otimes),\cdots,\beta_{ijk}(\otimes),\cdots,\beta_{ijq}(\otimes))$ ( $\beta_{ijk}(\otimes)\in[\underline{\beta}_{ijk},\overline{\beta}_{ijk}]$ ) of attribute $G_k$ on matching object $A_i$ $(i=1,2,\cdots,m)$ over all matching object $B_j$ are given, in order to consider the overall importance degree among the attributes, are given , then we assume that the comprehensive weights $w_{ijk}(\otimes)\in[\underline{w}_{ijk},\overline{w}_{ijk}]$ $(i=1,2,\cdots,m,\ j=1,2,\cdots n,k=1,2,\cdots q)$ are determined by the geometric mean as follows: $w_{ijk}(\otimes)=\dfrac{\alpha_k(\otimes)\times\beta_{ijk}(\otimes)}{\sum_{k=1}^{q}\alpha_k(\otimes)\times\beta_{ijk}(\otimes)}$ .

Then, we can obtain the final comprehensive weight $w_{ijk}(\otimes)\in[\underline{w}_{ijk},\overline{w}_{ijk}]$ $(i=1,2,\cdots,m,\ \ j=1,2,\cdots n,k=1,2,\cdots q)$ .

## 4. Fractional programming method for two-sided matching decision making problems with hybrid attribute values based on grey relational analysis

By considering weight $w_{ijk}$ of attribute of $G_k$ for matching object $A_i$ $(i=1,2,\cdots,m)$ over each matching object $B_j$ $(j=1,2,\cdots,n)$ the degree of grey relational degree between positive ideal object $x_j^+=\{x_{j1}^+,x_{j2}^+,\cdots,x_{jq}^+\}$ and estimated value $x_{ijk}$ is defined as follows;

$$\gamma_{ij}^+=\sum_{k=1}^{q}\xi_{ijk}^+w_{ijk}\ ,\ i=1,2,\cdots,m\ ,\ j=1,2,\cdots,n$$

Let $x_j^-=\{x_{j1}^-,x_{j2}^-,\cdots,x_{jq}^-\}$ be negative ideal object over each matching object $A_i$ $(i=1,2,\cdots,m)$ over all matching object $B_j$.

Here $x_{jk}^-=\{x_{jk}^{-(1)},x_{jk}^{-(2)}x_{jk}^{-(3)}x_{jk}^{-(4)}\}$ and

$$x_{jk}^{-(t)}=\{\min_{1\le i\le m}x_{ijk}^{(t)}\Big|j\in J^+,\max_{1\le i\le m}x_{ijk}^{(t)}\Big|j\in J^-,t=1,2,3,4\}$$
$$(j=1,2,\cdots,n,k=1,2,\cdots,q)$$

We can compute the grey relational coefficient between negative ideal object $x_{jk}^-=\{x_{jk}^{-(1)},x_{jk}^{-(2)},x_{jk}^{-(3)},x_{jk}^{-(4)}\}$ and evaluation value $x_{ijk}$ $(i=1,2,\cdots,m,j=1,2,\cdots,n,k=1,2,\cdots,q)$ of each matching object $A_i$ $(i=1,2,\cdots,m)$ over all matching object $B_j$ $(j=1,2,\cdots,n)$ by the following formulae:

$$\xi_{ijk}^-=\frac{\min_i\min_k d(x_{ijk},x_{jk}^-)+\rho\max_i\max_k d(x_{ijk},x_{jk}^-)}{d(x_{ijk},x_{jk}^-)+\rho\max_i\max_k d(x_{ijk},x_{jk}^-)}$$

Here $\rho\in[0,1]$ is a discriminative coefficient or resolving factor. Commonly, it is taken as $\rho=0.5$ and $d(x_{ijk},x_{ik}^-)=$

$$\sum_{k=1}^{q}\sqrt{(x_{ijk}^{(1)}-x_{ik}^{-(1)})^2+(x_{ijk}^{(2)}-x_{ik}^{-(2)})^2+(x_{ijk}^{(3)}-x_{ik}^{-(3)})^2+(x_{ijk}^{(4)}-x_{ik}^{-(4)})^2}$$

Similarly, by considering weight $w_{ik}$ of attribute of $G_k$ on each matching object $A_i$ $(i=1,2,\cdots,m)$ the degree of grey relational degree between negative ideal object $x_i^-=\{x_{i1}^-,x_{i2}^-,\cdots,x_{iq}^-\}$ and estimated value $x_{ijk}$ is defined as follows

$$\gamma_{ij}^-=\sum_{k=1}^{q}\xi_{ijk}^-w_{ijk}\ ,\ i=1,2,\cdots,m\ ,\ j=1,2,\cdots,n$$

Using Eq. (6) and (8), the relative closeness coefficients of positive ideal object $x_j^+=\{x_{j1}^+,x_{j2}^+,\cdots,x_{jq}^+\}$ for matching object $A_i$ $(i=1,2,\cdots,m)$ over matching object $B_j$ are defined as follows

$$C_{ij}=\frac{\gamma_{ij}^-}{\gamma_{ij}^++\gamma_{ij}^-}=\frac{\sum_{k=1}^{q}\xi_{ijk}^-w_{ik}}{\sum_{k=1}^{q}(\xi_{ijk}^++\xi_{ijk}^-)w_{ik}} \tag{4}$$

respectively.

Obviously, $C_{ij}$ is different for different value $w_{ijk}\in$ . $[\underline{w}_{ijk},\overline{w}_{ijk}]$. Values of $C_{ij}$ should be in some range when $w_{ijk}$ take all values in the interval $[\underline{w}_{ijk},\overline{w}_{ijk}]$.

In other words, $C_{ij}$ is an interval, denoted by $[\underline{C}_{ij},\overline{C}_{ij}]$. The lower and upper bounds $\underline{C}_{ij}$ and $\overline{C}_{ij}$ of $[\underline{C}_{ij},\overline{C}_{ij}]$ can be captured solving the following pair of nonlinear fractional programming models

[P7]

$$\underline{C}_{ij}=\min\left\{\frac{\sum_{k=1}^{q}\xi_{ijk}^-w_{ijk}}{\sum_{k=1}^{q}(\xi_{ijk}^++\xi_{ijk}^-)w_{ijk}}\right\}\ (i=1,2,\cdots,m;\ j=1,2\cdots,n) \tag{4.1}$$

$$\text{s.t.}\quad S=\{\underline{w}_{ijk}\le w_{ijk}\le\overline{w}_{ijk},w_{ijk}\ge 0,\sum_{k=1}^{q}w_{ijk}=1\}$$

$$\text{[P8]}\quad \overline{C}_{ij}=\max\left\{\frac{\sum_{k=1}^{q}\xi_{ijk}^-w_{ijk}}{\sum_{k=1}^{q}(\xi_{ijk}^++\xi_{ijk}^-)w_{ijk}}\right\}\ (i=1,2,\cdots,m;\ j=1,2\cdots,n)$$

$$\text{s.t.}\quad S=\{\underline{w}_{ijk}\le w_{ijk}\le\overline{w}_{ijk},w_{ijk}\ge 0,\sum_{k=1}^{q}w_{ijk}=1\}$$

Solving these two fractional programming models [P7] and [P8], we can obtain the interval of relative approach degree $[\underline{C}_{ij},\overline{C}_{ij}]$ $(i=1,2,\cdots,m;\ \ j=1,\ 2,\cdots,n)$.

(4.2)

Denote $B_{ij}\succeq B_{il}$ $(i=1,2,\cdots,m)$, which means ‘‘matching object $B_j$ being not inferior to matching object $B_l$”.

The likelihood of $B_{ij}\succeq B_{il}$ is characterized by $C_{ij}\succeq C_{il}$, which $C_{ij}$ and $C_{il}$ are interval numbers corresponding to $B_{ij}$ and $B_{il}$ $(j,l=1,2,\cdots,n)$, respectively.

Then, for any two matching objects $B_j$, $B_l$, the likelihood of $B_{ij}\succeq B_{il}$ each on matching object $A_i$ $(i=1,2,\cdots,m)$ is defined as follows( [24]):

$$p(B_{ij}\succeq B_{il})=p(C_{ij}\ge C_{il})=\max\left\{1-\max\left\{\frac{\overline{C}_{il}-\underline{C}_{ij}}{L(C_{ij})+L(C_{il})},0\right\},0\right\}$$

Here $C_{ij}=[\underline{C}_{ij},\overline{C}_{ij}]$ , $C_{il}=[\underline{C}_{il},\overline{C}_{il}]$ , $L(C_{ij})=\overline{C}_{ij}-\underline{C}_{ij}$ , $L(C_{il})=\overline{C}_{il}-\underline{C}_{il}$ . (4.3)

Thus likelihood matrix can be obtained and expressed as follows

$$P=(p_{ilj})_{n\times n}=\begin{array}{c}\\ B_1\\ B_2\\ \vdots\\ B_n\end{array}\begin{array}{c}\begin{array}{cccc}B_1 & B_2 & \cdots & B_n\end{array}\\ \begin{bmatrix}p_{i11} & p_{i12} & \cdots & p_{i1n}\\ p_{i21} & p_{i22} & \cdots & p_{i2n}\\ \vdots & \vdots & \ddots & \vdots\\ p_{in1} & p_{in2} & \cdots & p_{inn}\end{bmatrix}\end{array} \tag{5}$$

Here $p_{ilj}=p(B_{ij}\succeq B_{il})$ $(i=1,2,\cdots,m,j,l=1,2,\cdots,n)$.

Optimal degrees of membership for matching objects $B_j$ $(j=1,2,\cdots,n)$ on matching object $A_i$ $(i=1,2,\cdots,m)$ are defined as follows ([24]): (4.5)

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$$\theta_{ij}=\frac{1}{n(n-1)}\left(\sum_{l=1}^{n}p_{ilj}+\frac{n}{2}-1\right) \tag{6}$$

Based on the optimal degrees of membership calculated in the previous, the matching satisfaction degree (MSD) of the two sides can be obtained.

Specifically, the MSD for matching object $B_j$ $(j=1,2,\cdots,n)$ on $A_i$ $(i=1,2,\cdots,m)$ can be denoted as

$\alpha_{ij}=1-\theta_{ij}$ $(\forall\, i=1,2,\cdots,m,\ j=1,2,\cdots,n)$

And the MSD matrix for matching objects in $B$ on matching objects in $A$ can be obtained as $(\alpha_{ij})_{m\times n}$.

In the same way, the MSD matrix for matching objects in $B$ on matching objects in $A$ can also be denoted as $E=(\beta_{ji})_{n\times m}$ $(i=1,2,\cdots,m,\ \ j=1,2,\cdots,n)$

Here $\beta_{ji}$ denotes the MSD for matching object $B_j$ $(j=1,2,\cdots,m)$ on matching object $A_i$ $(i=1,2,\cdots,n)$

Let $x_{ij}$ $(i=1,2,\cdots,m,\ j=1,2,\cdots,n)$ be a binary decision variable that denotes whether $A_i$ $(i=1,2,\cdots,m)$ matches $B_j$ $(j=1,2,\cdots,n)$ or not, i.e. if $A_i$ $(i=1,2,\cdots,m)$ matches $B_j$ $(j=1,2,\cdots,n)$, then $x_{ij}=1$, else $x_{ij}=0$.

It is obvious that the matching is optimal when the matching satisfaction degree of the two sides is the maximum. Thus, based on the MSD matrices, a multi-objective optimization model which aims to maximize the matching satisfaction degree of all the matching objects can be established as

[P8] $\quad \max Z_1=\sum_{i=1}^{m}\sum_{j=1}^{n}\alpha_{ij}x_{ij}$, $\max Z_2=\sum_{i=1}^{m}\sum_{j=1}^{n}\beta_{ji}x_{ij}$

$$s.t.\begin{cases}\sum_{j=1}^{n}x_{ij}\le \mu_i, i=1,2,\cdots,m\\ \sum_{i=1}^{m}x_{ij}\le \theta_j, j=1,2,\cdots,n\\ x_{ij}\in\{0,1\}, i=1,2,\cdots,m,\ j=1,2,\cdots,n\end{cases}$$

Here $\mu_i$ and $\theta_j$ denote the maximum number of matching objects for $A_i$ $(i=1,2,\cdots,m)$ and $B_j$ $(j=1,2,\cdots,n)$, respectively.

Given a weight coefficient $\omega$, model [P8] can be transformed into a single objective model by the linear weighted method, i.e.

[P9] $\quad \max Z=\omega\sum_{i=1}^{m}\sum_{j=1}^{n}\alpha_{ij}x_{ij}+(1-\omega)\sum_{i=1}^{m}\sum_{j=1}^{n}\beta_{ij}x_{ij}$

$$s.t.\begin{cases}\sum_{j=1}^{n}x_{ij}\le \mu_i, i=1,2,\cdots,m\\ \sum_{i=1}^{m}x_{ij}\le \theta_j, j=1,2,\cdots,n\\ x_{ij}\in\{0,1\}, i=1,2,\cdots,m,\ j=1,2,\cdots,n\end{cases}$$

Model [P9] is a simple linear binary programming problem which can be solved using optimization software packages.

According to the value of the decision variables, the optimal matching can be obtained.

## 5. Illustrative Examples

In this section, we take the matching problem between employees and job positions as an example to illustrate the feasibility and effectiveness of the proposed method.

(4.9)

BS company intends to recruit staff for four job positions $\{B_1,B_2,B_3,\ B_4\}$ and eight candidates $\{A_1,A_2,\cdots,A_8\}$ apply for the four positions (adapted from [7]). The eight candidates give their evaluation on the four positions and the criteria are developing space $(G_1)$, working environment $(G_2)$, industry potentiality $(G_3)$, and salary and welfare level $(G_4)$. All the evaluation information is given by linguistic terms. The company also assesses the candidates with regard to eight criteria, i.e. work experience $(I_1)$, whether the employee masters two foreign languages $(I_2)$, expected salary $(I_3)$, professional knowledge $(I_4)$, English proficiency $(I_5)$, computer skills $(I_6)$, cooperation skills $(I_7)$ and honesty $(I_8)$. Among the eight criteria, $I_1$ and $I_2$ are binary value criteria, $I_3$ is a cost criterion with interval utility value, $I_4$ and $I_5$ are ordinal value and ordinal interval value criteria, respectively, and $I_6$, $I_7$ and $I_8$ are linguistic value criteria. Here the linguistic terms set for the company and the candidates are the same, i.e. $S=\{s_0=AL$: Absolutely Low, $s_1$ = $V\ L$: Very Low, $s_2=L$ : Low, $s_3=M$ : Middle, $s_4=H$:: High, $s_5=VH$:: Very High, $s_6=VH$:: Absolutely High}.

The evaluation information provided by the candidates and the company is shown in Table 1. We also assume that a position can only recruit one candidate, and a candidate can only be selected for one position.

We solve this matching problem between employees and job positions according to the optimization model [P9].

Let $\omega=0.5$. By solving the optimization model [P9] we can get $x_{51}=x_{42}=x_{33}=x_{74}$ $=1$ and the values of other decision variables are 0. According to the solution of the optimization model, we can conclude that the optimal matching for the candidates and the positions are $(A_5,B_1)$, $(A_4,B_2)$, $(A_3,B_3)$, $(A_7,B_4)$ and there are no position that can match $A_1$, $A_2$, $A_6$ and $A_8$ This is slightly different to the result obtained by the method in [7].

However, our method takes into account the case when the attribute weight information of the matching objects is incomplete, which can describe the actual situation.

Besides that, the proposed method incorporates other two types of evaluation information, i.e. ordinal value and ordinal interval value, which may appear in actual two-sided matching decision making problems.

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**Table 1:** Candidates' evaluation information on the positions

| $A_i$ | $B_1$ | | | | $B_2$ | | | | $B_3$ | | | | $B_4$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $G_1$ | $G_2$ | $G_3$ | $G_4$ | $G_1$ | $G_2$ | $G_3$ | $G_4$ | $G_1$ | $G_2$ | $G_3$ | $G_4$ | $G_1$ | $G_2$ | $G_3$ | $G_4$ |
| $A_1$ | VH | H | VH | H | AH | H | H | H | H | M | H | M | M | L | H | H |
| $A_2$ | H | H | H | VH | AH | H | VH | VH | VH | H | M | H | H | M | M | H |
| $A_3$ | H | H | H | M | H | M | H | H | VH | M | M | M | H | M | H | M |
| $A_4$ | M | VH | H | H | M | H | H | VH | H | H | M | H | H | H | H | H |
| $A_5$ | M | M | H | VH | H | M | M | VH | H | M | M | H | H | M | M | H |
| $A_6$ | H | M | H | M | H | M | M | H | H | M | L | M | M | L | L | M |
| $A_7$ | M | H | M | AH | M | H | M | VH | M | M | M | H | L | L | L | H |
| $A_8$ | H | H | H | VH | VH | H | M | VH | H | M | M | H | H | L | M | M |

## References

[1] Gale, D., Shapley, L.S. "College admissions and the stability of marriage", The American Mathematical Monthly, 69(1), pp. 9-15, 1962.

[2] Drigas, A., Kouremenos, S., Vrettos, S., Vrettaros, J., Kouremenos, D., "An Expert system for job matching of the unemployed", Expert Systems with Applications, 26(2), pp.217– 224 ,2004.

[3] Golec, A., Kahya, E., "A fuzzy model for competency-based employee evaluation and selection", Computer and Industrial Engineering, 52(1), pp. 143-161, 2007.

[4] Korkmaz, I., Gken, H., Etinyokus, T., "An analytic hierarchy process and Two-sided matching based decision support system for military personnel assignment", Information Sciences, 178(14), pp. 2915–2927, 2008.

[5] Lin, H.T., "A job placement intervention using fuzzy approach for two-way choice", Expert Systems with Applications, 36(2), pp. 2543–2553, 2009.

[6] Huang, D.K., Chiu, H.N., Yeh, R.H., Chang, J.H., "A fuzzy multi-criteria Decision making approach for solving a bi-objective personnel assignment problem", Computer and Industrial Engineering, 56(1), pp.1-10, 2009.

[7] Chen, X., Fan, Z.P., "Research on two-sided matching problem between Employees and positions based on multiple format information", Operations Research: Man. and Science, 18(6), pp. 103–109 2009.

[8] Wang, D.W., "Multi-objective trade matching problem and optimization method of E- Brokerage", China Journal of Information Systems, 1(1), pp. 102-109, 2007.

[9] Jiang, Z.Z., Ip, W.H., Lau, H.C.W., Fan, Z.P., "Multi-objective optimization Matching for one-shot multi-attribute exchanges with quantity discounts in e-brokerage", Expert Systems with Applications, 38(4), pp. 4169–4180, 2011.

[10] Kameshwaran, S., Narahari, Y., Rosa, C.H., Kulkarni, D.M., Tew, J.D., "Multi attribute electronic procurement using goal programming", European Journal Operational Research, 179(2), pp. 518–536, 2007.

[11] Jiang, Z.Z., Sheng, Y., Fan, Z.P., Yuan, Y., "Research on multi-objective decision model for bipartite matching with incomplete information on attribute weights", Operations Research: Man. and Science, 17(4), pp. 138-142, 2008.

[12] Li, Y.H., Fan, Z.P., Chen, X., Kang, F., "A multi-objective optimization model for matching ventures and venture capitalists", In: 4th International Conference on Wireless Communications, Networking and Mobile Computing, New York: IEEE Press, 2008.

[13] Wanh Pinyue, Shen Jiang, Zhang Bin, "A new method for two-sided matching decision making of PPP projects based on intuitionistic fuzzy Choquet integral", Journal of Intelligent & Fuzzy Systems. 31(4), pp. 221-2230, 2016.

[14] Yong Liu, Kevin W.Li, "A two-sided matching decision method for supply and demand of technological knowledge", Journal of knowledge Management., 21(3), pp. 592-606, 2017.

[15] Deng Kui Huang, Huan Neng Chiu, Ruey Huei Yeh , Jen Huei Chang," A fuzzy multi-criteria decision making approach for solving a bi-objective personnel assignment problem" , Computer and Industrial Engineering, 56(1), pp.1–10 , 2009.

[16] Ibrahim Korkmaz, Hadi Gökçen, Tahsin Çetinyokus, "An analytic hierarchy process and two-sided matching based decision support system for military personnel assignment", Information. Sciences, 178, pp. 2915–2927, 2008.

[17] Yufang Wang, Hongsen Yan and Xiangang Meng, "Matching decision model for self- adaptability of knowledge manufacturing system", International Conference on Information Science and Technology March 26-28, Nanjing, Jiangsu, China, pp. 891-895,2011.

[18] Li Jinhong, Zhao Kangpei, "Problem of marriage matching based on fuzzy duality decision", 2012 International Conference on Intelligent Systems Design and Engineering Application, pp. 883-885, 2012.

[19] Zhen Zhang, Chong-Hui Guo, "A hybrid multiple attributes two-sided matching decision making method with incomplete weight information", B. Hu et al. (Eds.): BI 2011, LNAI 6889, pp. 272–283, Springer-Verlag Berlin Heidelberg, 2011.

[20] Xiao Liu, Huiman Ma, “A two-sided matching decision model based on uncertain preference sequences”, Math. Probl. Eng. Volume 2015, Aricale ID 241379, pp. 10-22, 2015.
[21] Z.S. Xu, “A note on linguistic hybrid arithmetic averaging operator in multiple attribute group decision making with linguistic information”, Group Dec. Negot. 15, pp. 593–604, 2006.
[22] Saaty TL, “The analytic hierarchy process”. New York: McGraw-Hill, 1980.
[23] Gol Kim, Yunchol Jong, Sifeng Liu, “Generalized hybrid grey relation method for multiple attribute mixed type decision making”, The Journal of Grey Systems. 26(2), pp. 142-152, 2014.
[24] Xu ZS, “Deviation measures of linguistic preference relations in group decision making”, Omega, 33, pp. 249–254, 2005.