Design and Optimization of Trapezium in Curved Beam

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Abstract: ***We come across many applications of curved beam principles in engineering, the most common being the frames of machines such as punches, presses, planners etc. Another common example of curved beam application is the crane hook. In case of straight beams, the neutral axis of the section coincides with the centroidal axis, which is not true for the sections of the curved beam. Although exact stress analysis of curved beam is available, but it is limited in use due to cumbersome calculations involved. These calculations are eased to greater extent by the use of computer. In modern competitive environment, the cost reduction is the major factor for which various industries are aspiring for many technology innovations are used in this direction. Curved beam deign is generally done with considering many parameters and better design can be done. In the present work we have tried to optimum the cross sectional area of curved beam, by properly selecting the parameters with the help of computer.***

Keywords: Eccentricity, curved beam, bending equation, stresses and width ratio

1. List of Symbols

Symbols Unit
A = Area of Cross-Section cm^2
E = Modulus of Elasticity kg/cm^2
G = Modulus of Rigidity kg/cm^2
I = Moment of Inertia cm^4
M = Bending Moment $kg\text{-}cm^2$
P = Applied Force kg_f
R = Radius of Neutral Axis cm
R_i = Radius of Inner Fiber cm
R_m =Radius of Centroidal Axis cm
R_o = Radius of Outer Fiber cm
b_i = Width of Inner Fiber cm
b_o = Width of outer Fiber cm
α = Ratio of Inner Width to Outer Width -
∂ = Deflection cm
σ = Bending Stress kg/cm^2
σ_t = Total Stress kg/cm^2
τ = Shear Stress kg/cm^2

2. Introduction

We come across many applications of curved beam principles in engineering, the most common being the frames of machines such as punches, presses, planners etc. Another common example of curved beam application is the crane hook.

In case of straight beams, the neutral axis of the section coincides with the centroidal axis, which is not true for the sections of the curved beam.

Although exact stress analysis of curved beam is available, but it is limited in use due to cumbersome calculations involved. These calculations are eased to greater extent by the use of computer. A software for the stress distribution analysis of curved beams has been developed in the present work.

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In the present work we have tried to optimum the cross sectional area of curved beam, by properly selecting the parameters with the help of computer.

A curved beam is defined as a beam in which the neutral axis in the unloaded condition is curved instead of straight.

The distribution of stresses in a curved beam is [1]

$$\sigma = \frac{My}{Ae(R-y)} \quad \text{.......................................(1)}$$

The bending stress in this straight beam varies linearly with the distance from neutral axis, however in curved beam the stress distribution is hyperbolic.

In symmetrical cross-section such as the circle or rectangle, the maximum bending stresses always occur at the inner fiber. In unsymmetrical section, it is necessary to calculate the stresses at inner as well as outer fibers to determine the maximum stress. Bending stress is zero at the neutral axis and centroid is that point at which the total area of a plane figure (like rectangle, triangle circle etc) is assumed to be concentrated in that area.

The bending moment is due to forces acting to one side of cross section under consideration. In this case the bending moment is completed about the centroidal axis not neutral axis. Also an additional tensile or compressive stress must be added to the bending stress and to obtain total stress acting on this section.

An optimum section of a curved beam can be designed by relating the inner and outer fiber stresses in the same ratio as the tensile and compressive strength of beam material is steel, the and compressive strength are nearly equal.

Expressions of 'e' from above equation are different for different sections. In most of the engineering problems, the magnitude of e is very small and it should be calculated precisely to avoid large percentage error in the final results.

Design of Curved Beam:

Design of Beam

A satisfactory design of beam will be that where the beams, carry the applied loads safely and economically. For safety, the permissible stresses should not be exceeded anywhere and for economy of material the actual stresses should approach the permissible stresses closely so that the material could be utilized to the maximum extent. Design of beam should satisfy following criteria.

1) The permissible stress in bending and shear should not exceed. The actual stresses should be as close to the allowable one as possible.
2) The maximum deflection should be within permissible limits because machine member may cease to function properly if an excessive amount of elastic deflection is allowed.
3) When a curved bar is bent by transverse forces acting in its plane of symmetry, the forces acting upon the portion of the bar to one side of any cross-section may be reduced to a couple and a force applied at the center of cross-section. The moment of this force couple equals that of external forces with respect to the centroidal axis of the cross-section. The force is resolved into two components, a longitudinal force in the direction of the tangent to the centerline of the bar and the shear force in the cross section. The longitudinal force produces tensile or compressive stresses uniformly distributed over the cross section.

(i) Bending Stress

As discussed above in case of curved beam loaded with load P shown in figure an additional axial tensile or the compressive stress is added to the bending stress given by equation 1 to obtain resultant stress. The maximum resultant stress must be below the safe stress for the beam to be safe.

(ii) Shear Stress

The sheer force produces shearing stress and distribution of these stresses over the cross section can be taken same as for a straight bar given by equation:

$$\tau = \frac{VQ}{Ib}$$

The maximum shear stress must be well below the safe stress for the beam to be safe.

Optimize Section of Curved Beam

The equation 1

$$\sigma = \frac{My}{Ae(R-y)}$$

is valid for pure bending. In usual and more general cases such as crane hook, U frame, of a press or the frame of a clamp, the bending moment is due to forces acting to one side of the cross section under consideration to one side of cross section under consideration. In this case bending moment is computed about the centroidal axis, not the neutral axis. Also, the additional axial tensile or compressive stress is added to bending stress given by the equation 1, to obtain resultant stress acting on section.

We have taken general configuration of a curved beam as shown in figure.

Stresses on inner fiber and outer fiber are given:

$$\sigma_i = \frac{P(X+R_m)(R-R_i)}{A_e R_i} + \frac{P}{A} \ldots\ldots\ldots\ldots\ldots(2)$$

$$\sigma_o = \frac{P(X+R_m)(R_o-R_i)}{A_e R_o} + \frac{P}{A} \ldots\ldots\ldots\ldots(3)$$

Six sections circular, rectangular, triangular, trapezoidal T and I - section have been studied. It is found that the trapezoidal section is the most suitable section because maximum stress on the section is least with this section for the same P, A, X, Ri.

Results establishing the fact that trapezoidal section is the most suitable section for curved beam used in crane hook, clamps etc. are given in appendix B.

Design of Trapezoidal Section

Use of curved beam with trapezoidal section is common in industry, lifting hooks and machine frames are most common among them. Design optimization of such a versatile component is important. Very often thickness h of the section is a constraint from the point of view of overall size. Ratio of width α has been defined as

$$\alpha = \frac{b_o}{b_i}$$

Best criteria of design would be to arrive at a section for which greater of the two stresses σ_i, σ_o is the lowest.

We start with area of cross section A equal to 0.25 cm^2. For steel allowable tensile and compressive tresses are practically same. We, therefore, try to arrive at a value of α which should give lowest possible stresses. Suppose for a given value of α, σ_i is greater than σ_o. In this case only σ_o is of interest as the other stress is smaller. Similarly for another value of α, it may be that σ_o is higher than σ_i. Obviously then the interest will be in σ_o only , since σ_i is lower . We have print out of σ_i, σ_o for every value of α ($0 \le \alpha \le 1$) small enough steps of α, the optimum of α for which the higher of the two stresses σ_i, σ_o is the lowest can be selected. If the higher stress for optimum α happens to be σ_i, The corresponding σ_o must be less than (or in the extreme case, equal to) σ_o, and vice versa. This ensures that α so selected is the optimum.

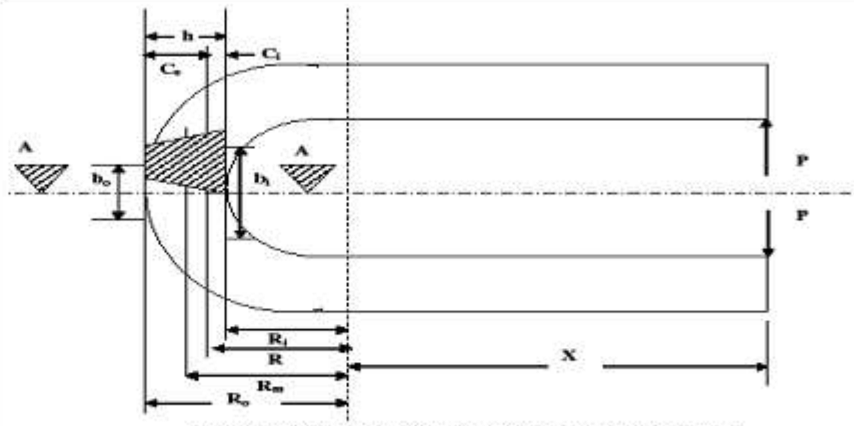


FIG.: CURVED BEAM WITH TRAPEZOIDAL SECTION

For selected value of α maximum stress on the section is compared with allowable stress in bending. If maximum stress on the section is greater than the allowable stress, beam is not safe and area of cross-section has to be increased. ‘A’ is increased in the steps of 0.25cm^2. This procedure is repeated till we arrive at ‘A’ for which maximum stress on the section for selected value of α is less than the safe stress. With this ‘A’ and α beam will be safe in bending.

The maximum shear stress on the section of maximum shear force is calculated and is compared with the allowable shear stress. If maximum shear stress on the section is greater than the allowable shear stress, A has to be increased. A is increased in the steps of the 0.25cm^2 till maximum shear stress on the section is less than allowable shear stress.

At least increase in distance between the points m and n on beam where the load P is applied is calculated. It is ensured that this δ is within permissible limits.

This computes the design of curved beam with trapezoidal section.

Data for Analysis:

Data of sample problems foretasting the program for eccentricity of curved beam and stress distribution analysis of curved beam used in crane hook are given with results obtained by the computer.

Following data is taken for testing the program for design optimization of curved beam with trapezoidal section shown in figure and taken material is alloy steel [8].

(i) Load on beam = 1000 kg
(ii) Inner radius of curved beam = 1 cm
(iii) Depth of section = 2 cm
(iv) Length of straight portion of beam = 4 cm
(v) Allowable bending stress = 4410 kg/cm^2 [9]
(vi) Allowable shear stress = 4310 kg/cm^2
(vii) Modulus of elasticity = 2.07*10 ^{6} kg /cm^2
(vii) Modulus of rigidity = 0.89 * 10 ^{6} kg /cm^2

Flow Chart and Computer Programs:

Program Development

The flow chart for software for design optimization of curved beam with trapezoidal the program developed is shown in fig.3. It shows various steps involved in solving the problems. The programs have written in CPP. All variables are defined with the help of comment statements in the program.

In Put Data

Following data is required as input for design of the curved beam with trapezoidal section

1) Load on beams
2) Length of straight portion of beam
3) Inner radius of curved beam
4) Depth of section
5) Allowable bending stress
6) Allowable shear stress
7) Modulus of elasticity
8) Modulus of rigidity

Post Processor

Following output parameters will be obtained for optimal design of curved beam with trapezoidal section.

1) Area of cross section
2) Inner and outer width of the trapezoidal section
3) Distance between neutral and centroidal axis
4) Maximum bending stress
5) Maximum shear stress
6) Distance of fiber at which shear stress is maximum from top fiber.

3. Results and Discussion

Results obtained from the software for stress distribution analysis of curved beam show that stress can be easily calculated on any fiber of beam section if all the dimensions of section, load and distance from centroidal axis on it are known to us.

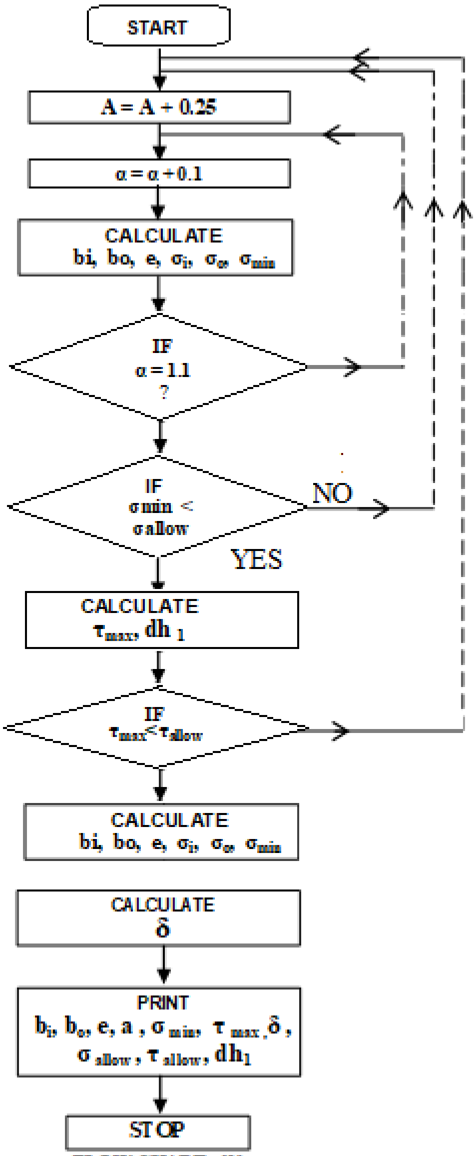
P = 1000, 2000 kg.
A = 19.64, 30, 15, 22.5, 18, 15. respectively
Ixx = 30.68, 62.5, 20.83, 45.1, 53.50, 51.52. respectively Y = distance top and bottom fiber form centroidal axis is different for different sections.

Eccentricity has been calculated with the help of graph of circular, rectangular, triangular, trapezoidal, T and I – sections with maximum possible load is
I - 0.65, -0.8, +0.5, -0.25, +0.5, -1.3.
II - 0.60, -0.80, +0.6, -0.3, +0.4, -1.4. cm
For curved beam used in crane hook with load P.

Taking:
Load, Area and Y is same as eccentricity
Applied load angle 60^0 from vertical axis.

Total stress has been calculated at the outermost fibers of circular, rectangular, trapezoidal T and I - sections is
I – 504.56, 257.88, 322.60, 181.11, 272.46, 368.98,
II – 1009.12, 515.76, 645.20, 362.22, 544.92, 737.96 kg/cm^2 respectively.

It can be concluded from above results that trapezoidal section is the most suitable section for the curved beam considered because minimum eccentricity and maximum stress is the lowest with this section.



FLOW CHART [1]

Same conclusions are made on changing the magnitudes P, A, and Ixx, as it is evident from the results.
Software for design optimization or curved beam with trapezoidal section has been run for sample problem.
Results obtained are;
A =1.75 cm^2
b_i = 1.46 cm
b_o = 0.29 cm
Maximum bending stress = 3898.51 kg/cm^2
Maximum shear stress = 236.97 kg/cm^2
Maximum deflection = 0.016
Thus beam is safe in both bending and shear.

Table: Final table obtained

no	a_1	bi	bo	e	sti	sto	Stm
1	0	1.75	0	0.12	4082.82	3350.44	4082.82
2	0.1	1.59	0.16	0.14	3934.37	2871.22	3934.37
3	0.2	1.46	0.29	0.15	3898.51	2580.85	3898.51
4	0.3	1.35	0.4	0.16	3918.4	2388.27	3918.4
5	0.4	1.25	0.5	0.17	3968.69	3968.69	3968.69
6	0.5	1.17	0.58	0.17	4036.56	2153.33	4036.56
7	0.6	1.09	0.66	0.17	4114.79	2078.18	4114.79
8	0.7	1.03	0.72	0.18	4196.5	2019.93	4196.5
9	0.8	0.97	0.78	0.18	4286.92	1970.06	4286.92
10	0.9	0.92	0.83	0.18	4178.35	1937.4	4178.35
11	1	0.92	0.88	0.18	4466.33	1907.84	4466.33

smin = 3898.51
ast = 4410.00
bi = 1.46
bo = 0.29
e = 0.25
a = 1.75
dh_1 = 0.90
taumax = 86.17
atau = 4310
delta = 0.016
Software for design optimization or curved beam with trapezoidal section has been run for sample problem.

Results obtained are;
A =1.75 cm^2
b_i = 1.46 cm
b_o = 0.29 cm
Maximum bending stress = 3898.51 kg/cm^2
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Thus beam is safe in both bending and shear.

4. Conclusions

Stress distribution on inner and outer of curved beams with manually and with the help of software is approximately same. In these calculations we have seen trapezoidal beam bear maximum loads with lighter weight instead of circular, rectangular, triangular, T- and I- sections with applied similar load, area, and radius of neutral axis and distance of the fiber from centroidal axis. Eccentricity is minimum of trapezoidal section among other sections. Continuously increases the value of width ratio from 0.25 to best criteria of design would be to arrive at a section for which greater of the two stresses σ_i, σ_o is the lowest. On the basis of these analyses Trapezoidal is the best section. So, best optimum section is trapezoidal for design criteria. For curved beam with section of optimization is to arrive at a section for which will stand for higher bending moment.

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