

Constructions of Balanced Incomplete Block Design Generation (127, 2667, 63, 3, 1)

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Abstract: The aim of this article is to construct a balanced incomplete block design by using the Greedy algorithm. Block designs play an important role in the area of statistical experiment design under some conditions in order to examine optimal design. Additionally, the main advantages of applying experimental design is to decrease systematic variation and enhance the accuracy of impact prediction. Balanced incomplete blocks designs (BIBD) as a type of block designs can be considered for the situations in which same number of every block and application level exist. Also, BIBD generation is a NP-hard problem that provides an excellent benchmark since it is scalable and has a wide variety of problem instances.

Keywords: Balanced incomplete block design, greedy algorithm, design theory

1. Introduction

Block design theory is one of the most important fields in Combinatorial Mathematics. The basic principles of Combinatorial block design theory is closely related to the statistical theory of experimental design, as well as to the 19th century's recreational mathematics and geometry. The applications of Combinatorial block design is widely used in graph theory, coding theory, theoretical computer science, statistics, and algebra, among others. The main problems in design theory are, generally speaking, those of the existence, enumeration and classification, structural properties, and applications [7].

Balanced incomplete block designs (BIBD) is developed by Yates in 1936. The topic of balanced incomplete block designs and its combinatorial problems have been attractive to combinatorists and algebraists as well as to statisticians. There are a number of methods for the construction of balanced incomplete block designs which have been investigated. The simplest examples are all the sets of k out of v varieties. These are called the unreduced designs [3].

BIBD generation is a NP-hard problem that provides an excellent benchmark since it is scalable and has a wide variety of problem instances, ranging from easy instances to very difficult ones. The scalability of the problem as well as its difficulty make it an adequate setting to test the behavior of different techniques/algorithms [8]

2. Balanced incomplete block designs

A Balanced Incomplete Block Design (BIBD) with parameters (v, b, r, k, λ) is a pair (X, B) , where X is a set, B is a collection of subsets of X and the five parameters are defined as follows:

v is elements of X , b (block number) is the number of elements of B , r (replication number) is the number of blocks to which every point belongs, k (block size) is the common size of each block, λ is the number of blocks to which every pair of distinct points belongs

The five parameters are integers and are not independent.

$vr = bk$ and $\lambda(v-1) = r(k-1)$
 $r = \frac{\lambda(v-1)}{k-1}$ and $b = \frac{\lambda v(v-1)}{k(k-1)}$ must be integers. the necessary conditions for the existence of a BIBD but is not sufficient. For example, The parameters (22, 33, 12, 8, 4) provide the above conditions. However, the existence of such a BIBD design has not yet been demonstrated. [5, 6]

Consider a balanced incomplete block design with parameters v, b, r, k, λ . Let $n_{ij} = 1$ or 0 according to the i th variety does or does not occur in the j th block. That is, n_{ij} denotes the number of times that the i th variety appears in the j th block. It is easy to see that $N = (n_{ij})$ is the incidence matrix of the design. It has v rows and b columns.

$$N = \begin{bmatrix} n_{11} & \cdots & n_{1b} \\ \vdots & \ddots & \vdots \\ n_{v1} & \cdots & n_{vb} \end{bmatrix}$$

Let $B_1, B_2, \dots, B_b$ blocks and $P_1, P_2, \dots, P_v$
BIBD (7,7,3,3,1)
 $P_1 = 1, P_2 = 2, P_3 = 3, P_4 = 1, P_5 = 5$

$P_6 = 6, P_7 = 7$

$B_1 = \{1, 2, 3\}, B_2 = \{1, 4, 5\}, B_3 = \{1, 6, 7\}$

$B_4 = \{2, 4, 6\}, B_5 = \{2, 5, 7\}, B_6 = \{3, 4, 7\}$

$B_7 = \{3, 5, 6\}$

$$N = \begin{matrix} & \begin{matrix} B_1 & B_2 & B_3 & B_4 & B_5 & B_6 & B_7 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{matrix} & \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \end{bmatrix} \end{matrix}$$

2.1 Statistical analysis

Balanced incomplete block designs are typically used when all comparisons are equally important for the experiment, but the researcher is not able to run all possible combinations. In such cases, the treatments that are used in each block should be selected in *balanced manner*, i.e. any pair occurs together in the same number of times as any other pair [4]

Consider a BIBD (v, b, r, k, λ) . The statistical model for this design is given by:

$$Y_{ij} = \mu + \beta_i + \alpha_j + \varepsilon_{ij} \quad \begin{matrix} i = 1, 2, \dots, v \\ j = 1, 2, \dots, b \end{matrix}$$

y_{ij} : the i^{th} observation in the j^{th} block

μ : General mean effect

β_i : The effect of i^{th} treatment

α_j : The effect of j^{th} block

$\varepsilon_{ij} \sim IND(0, \sigma_\varepsilon^2)$: i.i.d random error component

3. Construction of BIBD(127,2667,63,3,1)

The BIBD problem has been tackled by a number of different techniques in the literature with different success. In this article, a balanced block design was created with the Greedy algorithm.

The k-element subsets and the m-element subsets of the v-element X-set are listed as lexicographic. After selecting the first block from the k-element subsets, the associated block is added to the design after determining the ones associated with the number of t from the m-element subsets. The subsets m are common with the k-element subsets and the number t are subtracted from the order. Then the block in the new sequence is taken and this process is continued sequentially. In m-element subsets, the algorithm is stopped when no elements are compared.

When applying the algorithm, the first block in the sequence was taken and the blocks represented by this block were removed from the order. Then, the first block in the remaining sequence was taken and similar operations were repeated. The algorithm is stopped when no sequence is left [2]

4. Conclusion

The construction of BIBD for some particular parameters combinations still remain unsolved. There are a number of methods for the construction of balanced incomplete block designs. In this article (127,2667,63,3,1) design can be obtained by the Greedy algorithm. In general, for $n \geq 2$ parameters $v = 2^n - 1$, $b = \frac{v(v-1)}{2}$, $r = 2^{n-1} - 1$,

$k = 3$ and $\lambda = 1$ designs can be obtained by the Greedy algorithm. Current "best" covers have been collected by Center for Communications Research, www.ccrwest.org.

Table 1: Blocks of BIBD (127,2667,63,3,1)

1	2	3	2	4	6	3	5	6
1	4	5	2	5	7	3	8	11
1	6	7	2	8	10	3	9	10
1	8	9	2	9	11	3	12	15
1	10	11	2	12	14	3	13	14
1	12	13	2	13	15	3	16	19
1	14	15	2	16	18	3	17	18
1	16	17	2	17	19	3	20	23
1	18	19	2	20	22	3	21	22
1	20	21	2	21	23	3	24	27
1	22	23	2	24	26	3	25	26
1	24	25	2	25	27	3	28	31
1	26	27	2	28	30	3	29	30
1	28	29	2	29	31	3	32	35
1	30	31	2	32	34	3	33	34
1	32	33	2	33	35	3	36	39
1	34	35	2	36	38	3	37	38
1	36	37	2	37	39	3	40	43
1	38	39	2	40	42	3	41	42
1	40	41	2	41	43	3	44	47
1	42	43	2	44	46	3	45	46
1	44	45	2	45	47	3	48	51
1	46	47	2	48	50	3	49	50
1	48	49	2	49	51	3	52	55
1	50	51	2	52	54	3	53	54
1	52	53	2	53	55	3	56	59
1	54	55	2	56	58	3	57	58
1	56	57	2	57	59	3	60	63
1	58	59	2	60	62	3	61	62
1	60	61	2	61	63	3	64	67
1	62	63	2	64	66	3	65	66
1	64	65	2	65	67	3	68	71
1	66	67	2	68	70	3	69	70
1	68	69	2	69	71	3	72	75
1	70	71	2	72	74	3	73	74
1	72	73	2	73	75	3	76	79
1	74	75	2	76	78	3	77	78
1	76	77	2	77	79	3	80	83
1	78	79	2	80	82	3	81	82
1	80	81	2	81	83	3	84	87
1	82	83	2	84	86	3	85	86
1	84	85	2	85	87	3	88	91
1	86	87	2	88	90	3	89	90
1	88	89	2	89	91	3	92	95
1	90	91	2	92	94	3	93	94
1	92	93	2	93	95	3	96	99
1	94	95	2	96	98	3	97	98
1	96	97	2	97	99	3	100	103
1	98	99	2	100	102	3	101	102
1	100	101	2	101	103	3	104	107
1	102	103	2	104	106	3	105	106
1	104	105	2	105	107	3	108	111
1	106	107	2	108	110	3	109	110
1	108	109	2	109	111	3	112	115
1	110	111	2	112	114	3	113	114
1	112	113	2	113	115	3	116	119
1	114	115	2	116	118	3	117	118
1	116	117	2	117	119	3	120	123
1	118	119	2	120	122	3	121	122
1	120	121	2	121	123	3	124	127
1	122	123	2	124	126	3	125	126
1	124	125	2	125	127	4	8	12
1	126	127	3	4	7	4	9	13

4	10	14	5	17	20	6	24	30
4	11	15	5	18	23	6	25	31
4	16	20	5	19	22	6	26	28

4	17	21	5	24	29	6	27	29	7	48	55	8	55	63	9	70	79
4	18	22	5	25	28	6	32	38	7	49	54	8	64	72	9	71	78
4	19	23	5	26	31	6	33	39	7	50	53	8	65	73	9	80	89
4	24	28	5	27	30	6	34	36	7	51	52	8	66	74	9	81	88
4	25	29	5	32	37	6	35	37	7	56	63	8	67	75	9	82	91
4	26	30	5	33	36	6	40	46	7	57	62	8	68	76	9	83	90
4	27	31	5	34	39	6	41	47	7	58	61	8	69	77	9	84	93
4	32	36	5	35	38	6	42	44	7	59	60	8	70	78	9	85	92
4	33	37	5	40	45	6	43	45	7	64	71	8	71	79	9	86	95
4	34	38	5	41	44	6	48	54	7	65	70	8	80	88	9	87	94
4	35	39	5	42	47	6	49	55	7	66	69	8	81	89	9	96	105
4	40	44	5	43	46	6	50	52	7	67	68	8	82	90	9	97	104
4	41	45	5	48	53	6	51	53	7	72	79	8	83	91	9	98	107
4	42	46	5	49	52	6	56	62	7	73	78	8	84	92	9	99	106
4	43	47	5	50	55	6	57	63	7	74	77	8	85	93	9	100	109
4	48	52	5	51	54	6	58	60	7	75	76	8	86	94	9	101	108
4	49	53	5	56	61	6	59	61	7	80	87	8	87	95	9	102	111
4	50	54	5	57	60	6	64	70	7	81	86	8	96	104	9	103	110
4	51	55	5	58	63	6	65	71	7	82	85	8	97	105	9	112	121
4	56	60	5	59	62	6	66	68	7	83	84	8	98	106	9	113	120
4	57	61	5	64	69	6	67	69	7	88	95	8	99	107	9	114	123
4	58	62	5	65	68	6	72	78	7	89	94	8	100	108	9	115	122
4	59	63	5	66	71	6	73	79	7	90	93	8	101	109	9	116	125
4	64	68	5	67	70	6	74	76	7	91	92	8	102	110	9	117	124
4	65	69	5	72	77	6	75	77	7	96	103	8	103	111	9	118	127
4	66	70	5	73	76	6	80	86	7	97	102	8	112	120	9	119	126
4	67	71	5	74	79	6	81	87	7	98	101	8	113	121	10	16	26
4	72	76	5	75	78	6	82	84	7	99	100	8	114	122	10	17	27
4	73	77	5	80	85	6	83	85	7	104	111	8	115	123	10	18	24
4	74	78	5	81	84	6	88	94	7	105	110	8	116	124	10	19	25
4	75	79	5	82	87	6	89	95	7	106	109	8	117	125	10	20	30
4	80	84	5	83	86	6	90	92	7	107	108	8	118	126	10	21	31
4	81	85	5	88	93	6	91	93	7	112	119	8	119	127	10	22	28
4	82	86	5	89	92	6	96	102	7	113	118	9	16	25	10	23	29
4	83	87	5	90	95	6	97	103	7	114	117	9	17	24	10	32	42
4	88	92	5	91	94	6	98	100	7	115	116	9	18	27	10	33	43
4	89	93	5	96	101	6	99	101	7	120	127	9	19	26	10	34	40
4	90	94	5	97	100	6	104	110	7	121	126	9	20	29	10	35	41
4	91	95	5	98	103	6	105	111	7	122	125	9	21	28	10	36	46
4	96	100	5	99	102	6	106	108	7	123	124	9	22	31	10	37	47
4	97	101	5	104	109	6	107	109	8	16	24	9	23	30	10	38	44
4	98	102	5	105	108	6	112	118	8	17	25	9	32	41	10	39	45
4	99	103	5	106	111	6	113	119	8	18	26	9	33	40	10	48	58
4	104	108	5	107	110	6	114	116	8	19	27	9	34	43	10	49	59
4	105	109	5	112	117	6	115	117	8	20	28	9	35	42	10	50	56
4	106	110	5	113	116	6	120	126	8	21	29	9	36	45	10	51	57
4	107	111	5	114	119	6	121	127	8	22	30	9	37	44	10	52	62
4	112	116	5	115	118	6	122	124	8	23	31	9	38	47	10	53	63
4	113	117	5	120	125	6	123	125	8	32	40	9	39	46	10	54	60
4	114	118	5	121	124	7	8	15	8	33	41	9	48	57	10	55	61
4	115	119	5	122	127	7	9	14	8	34	42	9	49	56	10	64	74
4	120	124	5	123	126	7	10	13	8	35	43	9	50	59	10	65	75
4	121	125	6	8	14	7	11	12	8	36	44	9	51	58	10	66	72
4	122	126	6	9	15	7	16	23	8	37	45	9	52	61	10	67	73
4	123	127	6	10	12	7	17	22									
5	8	13	6	11	13	7	18	21	10	68	78	11	83	88	12	98	110
5	9	12	6	16	22	7	19	20	10	69	79	11	84	95	12	99	111
5	10	15	6	17	23	7	24	31	10	70	76	11	85	94	12	100	104
5	11	14	6	18	20	7	25	30	10	71	77	11	86	93	12	101	105
5	16	21	6	19	21	7	26	29	10	80	90	11	87	92	12	102	106
									10	81	91	11	96	107	12	103	107
7	27	28	8	38	46	9	53	60	10	82	88	11	97	106	12	112	124
7	32	39	8	39	47	9	54	63	10	83	89	11	98	105	12	113	125
7	33	38	8	48	56	9	64	73	10	84	94	11	99	104	12	114	126
7	34	37	8	49	57	9	55	62	10	85	95	11	100	111	12	115	127
7	35	36	8	50	58	9	64	73	10	86	92	11	101	110	12	116	120
7	40	47	8	51	59	9	65	72	10	87	93	11	102	109	12	117	121
7	41	46	8	52	60	9	67	74	10	96	106	11	103	108	12	118	122
7	42	45	8	53	61	9	68	77	10	97	107	11	112	123	12	119	123
7	43	44	8	54	62	9	69	76	10	98	104	11	113	122	13	16	29

10	99	105	11	114	121	13	17	28	14	38	40	15	53	58	16	76	92
10	100	110	11	115	120	13	18	31	14	39	41	15	54	57	16	77	93
10	101	111	11	116	127	13	19	30	14	48	62	15	55	56	16	78	94
10	102	108	11	117	126	13	20	25	14	49	63	15	64	79	16	79	95
10	103	109	11	118	125	13	21	24	14	50	60	15	65	78	16	96	112
10	112	122	11	119	124	13	22	27	14	51	61	15	66	77	16	97	113
10	113	123	12	16	28	13	23	26	14	52	58	15	67	76	16	98	114
10	114	120	12	17	29	13	32	45	14	53	59	15	68	75	16	99	115
10	115	121	12	18	30	13	33	44	14	54	56	15	69	74	16	100	116
10	116	126	12	19	31	13	34	47	14	55	57	15	70	73	16	101	117
10	117	127	12	20	24	13	35	46	14	64	78	15	71	72	16	102	118
10	118	124	12	21	25	13	36	41	14	65	79	15	80	95	16	103	119
10	119	125	12	22	26	13	37	40	14	66	76	15	81	94	16	104	120
11	16	27	12	23	27	13	38	43	14	67	77	15	82	93	16	105	121
11	17	26	12	32	44	13	39	42	14	68	74	15	83	92	16	106	122
11	18	25	12	33	45	13	48	61	14	69	75	15	84	91	16	107	123
11	19	24	12	34	46	13	49	60	14	70	72	15	85	90	16	108	124
11	20	31	12	35	47	13	50	63	14	71	73	15	86	89	16	109	125
11	21	30	12	36	40	13	51	62	14	80	94	15	87	88	16	110	126
11	22	29	12	37	41	13	52	57	14	81	95	15	96	111	16	111	127
11	23	28	12	38	42	13	53	56	14	82	92	15	97	110	17	32	49
11	32	43	12	39	43	13	54	59	14	83	93	15	98	109	17	33	48
11	33	42	12	48	60	13	55	58	14	84	90	15	99	108	17	34	51
11	34	41	12	49	61	13	64	77	14	85	91	15	100	107	17	35	50
11	35	40	12	50	62	13	65	76	14	86	88	15	101	106	17	36	53
11	36	47	12	51	63	13	66	79	14	87	89	15	102	105	17	37	52
11	37	46	12	52	56	13	67	78	14	96	110	15	103	104	17	38	55
11	38	45	12	53	57	13	68	73	14	97	111	15	112	127	17	39	54
11	39	44	12	54	58	13	69	72	14	98	108	15	113	126	17	40	57
11	48	59	12	55	59	13	70	75	14	99	109	15	114	125	17	41	56
11	49	58	12	64	76	13	71	74	14	100	106	15	115	124	17	42	59
11	50	57	12	65	77	13	80	93	14	101	107	15	116	123	17	43	58
11	51	56	12	66	78	13	81	92	14	102	104	15	117	122	17	44	61
11	52	63	12	67	79	13	82	95	14	103	105	15	118	121	17	45	60
11	53	62	12	68	72	13	83	94	14	112	126	15	119	120	17	46	63
11	54	61	12	69	73	13	84	89	14	113	127	16	32	48	17	47	62
11	55	60	12	70	74	13	85	88	14	114	124	16	33	49	17	64	81
11	64	75	12	71	75	13	86	91	14	115	125	16	34	50	17	65	80
11	65	74	12	80	92	13	87	90	14	116	122	16	35	51	17	66	83
11	66	73	12	81	93	13	96	109	14	117	123	16	36	52	17	67	82
11	67	72	12	82	94	13	97	108	14	118	120	16	37	53	17	68	85
11	68	79	12	83	95	13	98	111	14	119	121	16	38	54	17	69	84
11	69	78	12	84	88	13	99	110									
11	70	77	12	85	89	13	100	105	17	70	87	18	101	119	20	36	48
11	71	76	12	86	90	13	101	104	17	71	86	18	102	116	20	37	49
11	80	91	12	87	91	13	102	107	17	72	89	18	103	117	20	38	50
11	81	90	12	96	108	13	103	106	17	73	88	18	104	122	20	39	51
11	82	89	12	97	109	13	112	125	17	74	91	18	105	123	20	40	60
									17	75	90	18	106	120	20	41	61
13	113	124	15	16	31	16	39	55	17	76	93	18	107	121	20	42	62
13	114	127	15	17	30	16	40	56	17	77	92	18	108	126	20	43	63
13	115	126	15	18	29	16	41	57	17	78	95	18	109	127	20	44	56
13	116	121	15	19	28	16	42	58	17	79	94	18	110	124	20	45	58
13	117	120	15	20	27	16	43	59	17	96	113	18	111	125	20	46	58
13	118	123	15	21	26	16	44	60	17	97	112	19	32	51	20	47	59
13	119	122	15	22	25	16	45	61	17	98	115	19	33	50	20	64	84
14	16	30	15	23	24	16	46	62	17	99	114	19	34	49	20	65	85
14	17	31	15	32	47	16	47	63	17	100	117	19	35	48	20	66	86
14	18	28	15	33	46	16	64	80	17	101	116	19	36	55	20	67	87
14	19	29	15	34	45	16	65	81	17	102	119	19	37	54	20	68	80
14	20	26	15	35	44	16	66	82	17	103	118	19	38	53	20	69	81
14	21	27	15	36	43	16	67	83	17	104	121	19	39	52	20	70	82
14	22	24	15	37	42	16	68	84	17	105	120	19	40	59	20	71	83
14	23	25	15	38	41	16	69	85	17	106	123	19	41	58	20	72	92
14	32	46	15	39	40	16	70	86	17	107	122	19	42	57	20	73	93
14	33	47	15	48	63	16	71	87	17	108	125	19	43	56	20	74	94
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57	82	107	59	81	106	61	80	109	63	85	106						

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