International Journal of Science and Research (IJSR)
ISSN: 2319-7064
Index Copernicus Value (2016): 79.57 | Impact Factor (2017): 7.296

Measuring Extreme Risks in the Rwanda Stock Market

Bizumutimajean Claude[1], Dr. Marcel NDENGO[2], Dr. Joseph K. Mung'atu[3]

Faculty of Applied Sciences, Department of statistics and Actuarial Sciences, Jomo Kenyatta University of Agriculture and Technology, Kigali, Rwanda

Abstract: ***The stability of stock prices is very important for investors. Investors are attracted by shares which exhibit less volatility, less risks and hence stable and predictable prices overtime. Despite the importance of stock volatility in attracting investors, there are few studies talking about measuring extreme risk in Rwanda stock Exchange. This research project aims to measure extreme risks of shares of selected companies by EVT approach from Rwanda stock exchange using data for the period 2013 to 2016. The source of data was including monthly average share price of Bank of Kigali (B.K) companies and bralirwa. The study was utilizing maximum likelihood method to estimate the model parameter and the model's goodness of fit was assessed by using Q-Q, P-P and density Plot. The size the extreme monthly Rwanda stock market movement was then computed using expected shortfall risk measures at some high quintiles, based on the GPD model. This paper provides a step by step guideline for extreme risk analysis in the R software with several examples.***

Keywords: Extreme Value Theory (EVT), Peak-Over-Threshold(POT),Expected shortfall, Generalized Pareto Distribution (GDP)

1. Introduction

Measuring the impact of extreme risks in Rwanda stock exchange is the key area of the study. Risk is negative deviation of actual outcomes from the expected outcome. This arises from uncertainty that exists in the market due to changes in market activities. There are various types of extremes risks include: Credit-Risk due to uncertainty in a counter party's ability to perform on an obligation, Liquidity- Risk due to uncertainty in the ability to unwind a position especially because the market cannot fully absorb it, Market –Risk arising from uncertainty in the market value of the portfolio due to changes in market condition. As the measuring of this rare event involve the measuring of extreme quantiles. One of the methods that were used in quantifying these risks is expected shortfall (ES). ES have been found capture the quantile risk in the tails of the Distribution (Harlow1991).

The main motivation behind this study was the need to examine the performance of the EVT method in the analysis of the Rwanda stock market. This work thus contributes to empirical evidence of the research into the behavior of the extreme returns of financial time series in Africa and especially in Rwanda. We then considered the various estimation issues and question that had come up in the context of measuring extreme risks and how analysts and researchers have tried to deals with them. We evaluate how EVT fitted into and contracts with the other risks assessment measures. The expected shortfall or mean excess function over threshold plays a fundamental role in many fields for instance, $e(u)$ is called residuals lifetime in medical statistics, $e(u)$ is excess of loss in the context of financial risk management is referred to expected shortfall.

This project focused on the measuring extreme losses. one method of extracting upper extremes from a set of data is to take the exceedances over a predetermined high threshold. This involves the use of Peak-over-threshold (POT) distributions such as The Generalized Pareto Distribution (GDP) The article explores the usefulness of EVT in measuring extreme events in stock market.

2. Literature Review

Extreme Value Theory is a well-developed theory in the field of probability studies the distribution of extreme risks of a given distribution function, or of a stochastic process, satisfying suitable assumption measure of portfolio risk. EVT also is used to analyze the value of random variables that exceed fall below a given threshold value. Hence observations that exceed a given threshold u are called exceedances over a threshold value. And observation that fall below a given threshold u is shortfalls. So exceedances and shortfalls play an important role in extremes.

EVT method have two features which make them attractive for tail estimation: they are based on a sound statistical theory, and they offer a parameter form for the ail of the distribution. Two main approaches are proposed in the literature to calibrate extreme value theory: The Block Maxima (BM) based on the Generalized Extreme Value (GEV) and the Peak Over Threshold (POV) based on the Generalized Pareto Distribution (GPD).

Da,Silva etal.(2003) study ten Asian stock market and showed the accuracy of GEV. VaR calculated over more one-month horizon. McNeil et al. (2000) first combined the generalized Pareto distribution and the GARCH model to estimate conditional VaR. Fernandez (2003) and Gençay et al. (2004) also combine GPD and GARCH model to measure conditional VaR on several emerging market. Measuring the extremes of stock market such as exceedances and shortfall of Rwanda stock market was discussed in chapter three.

2.1. Value at Risk

Value at Risk is generally defined as the capital sufficient to cover, in most instances, losses from a portfolio over a holding period of a fixed number of days. It is a high quantiles (typically the 95th 99th percentile) of the distribution of negative returns and provides an upper bound on the losses with a specified probability. The VaR of an asset represents in single number, the largest possible loss over a given time horizon with a given confidence interval.

Suppose a random variable X with continuous distribution function F models losses or negative returns on a certain financial instrument over a certain horizon. VaR_p can then be defined as the *p*-th quantile of the Distribution F. $\mathrm{VaR}_p = F^{-1}(1 - p)$;where F^{-1} is the so called quantile function defined as the inverse of the distribution function F.

2.2 Expected shortfall

Another informative measure of risk is the expected shortfall (ES) or the tail condtional expectation which estimates the potential size of the loss exceeding VaRp. Artzner et al. propose use of expected shortfall instesd of VaRp, the expected shortfall is the expected size of loss that exceed VaR and is coherent according to their definition.

Let X be a random variable with function F and right endpoint xf . for a fixed threshold $u<xf$ we define the expected shortfall as follow:
$e(u) = Pr((X - u < x|X > u)x > 0$

Artzner et al. (1999) argue that expected shortfall, as opposed to Value at Risk, is a coherent risk measure. Excesses over threshold play an important role in many field. Such as medical statistics, In the insurance context $e(u)$ referred to us excess of loss of function and in the context of financial risk management is referred to expected shortfall.

2.3 Research Gap

Most studies have been carried out on developed countries and less on developing countries as the evidenced from theoretical literature. The few studies that have been done in developing countries have not looked at Rwanda in isolation. This study therefore seeks to add knowledge about measuring extreme risk in stocks market by looking at Rwanda this is a gap that exist and need to be filled.

3. Research Methodology

The model and methodology used to examine the effect of extreme risks on monthly stock prices indices of Rwanda Stock market. It is followed by an explanation of variables used, source of data and diagnostics tests employed in the study. This study was utilized quantitative data because it involves systematic investigation of observation phenomena via statistical or numerical data. This study aims at establishing the effect of extreme risks on monthly stocks prices indices of Rwanda stocks market.

3.1 Generalized Autoregressive Conditional Heteroskedasticity Model (GARCH)

The GARCH model takes into consideration both previous observation in the time series and previous volatilities for prediction the coming A GARCH (p,q), is defined as:

Let y_t is a daily stock return series

$$y_t = \mu + \epsilon_t \epsilon_t \sim N(0, \sigma_t^2) \qquad (1)$$

Where ϵ_t is sequence of independent and identically distributed random variable with 0 mean and variance 1 then y_t follows the GARCHP, (p, q)

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha i \, \epsilon_{t-i}^2 + \sum_{j=1}^{q} \beta j \, \sigma_{t-j}^2 \qquad (2)$$

Where σ_t^2 conditional variance or volatility at time t,$\alpha_0$is intercept, $\alpha_i \epsilon_{t-i}^2$ is the information about volatility during the previous period and $\beta_j \sigma_{t-j}^2$ is the fitted variance from the model during the previous period

Or $\sigma_t^2 = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \cdots + \alpha_q \epsilon_{t-q}^2 + \beta_1 \sigma_{t-1}^2 + \cdots + \beta_p \sigma_{t-p}^2$(3)

3.2 Generalized Pareto Distribution (GDP)

The Generalized Pareto Distribution (GDP) arises when you consider excesses of random variables above or below a given threshold. It has two- parameters with distribution function.

$$G_{\xi,\sigma(u)}(y) = \begin{cases} 1 - (1 + \frac{\xi y}{\sigma})^{\frac{-1}{\xi}} & \xi \neq 0 \\ 1 - \exp\left(-\frac{y}{\sigma}\right) & \xi = 0 \end{cases} \qquad (4)$$

Where$\sigma > 0$, and $y \geq 0$ when $\xi \geq 0$ and$0 \leq y \leq -\sigma/\xi$when $\xi < 0$. The parameters ξ and σ are referred to respectively as the *shape* and *scale* parameters.

3.3 Peak-Over-Threshold Method

The POV model are generally considered to be appropriate for practical applications. The main advantages of POV to GDP approach are that it focuses attention directly on the tail distribution.

For a set of observations $X1, X2, X3, \ldots, Xn$ with cumulative distribution function$F(x)$, and a predetermined threshold u, the interest here is in that of the distribution of the exceedances or the values of$x$ above the threshold$u$, given that u is in fact exceeded.

Thus an exceedance occurs when $Xi > u$, for any i = 1, 2, …, n. Hence we can define $y = Xi - u$and its corresponding distribution function is known as the conditional excess distribution function $Fu(y)$ defined as $Fu(y) = P(x - u \leq y/x > u), 0 \leq y \leq xF - u$, where $xF < \infty$ is the right endpoint of F.

$$Fu(y) = P(x \leq y + u/x > u) = \frac{Pr(x \leq y + u)}{Pr(x > u)} x > u$$

$$= \frac{Pr(x \leq y + u) - Pr(x \leq u)}{Pr(x > u)} \qquad (5)$$

Hence, since $x = u + y forX > u$, expressing F in terms of Fu gives

$$Fu(y) = \frac{F(u + y) - F(u)}{1 - F(u)} = \frac{F(x) - F(u)}{1 - F(u)} \qquad (6)$$

3.4 Parameter Estimation

The maxmum likelihood method for estimating parameters for statistical model was used Let $Y1, Y2, Y2 \ldots Yk(u)$be a sequence of iid random variables from an unknown distribution F, $Yi>u$. Shape $\xi$and scale $\sigma > 0$ parameters are then defined on the threshold . These GPD parameters can be determined by maximum likelihood (ML) methods. The log likelihood function of the GPD for $\xi \neq 0$

$$L(\xi,\sigma) = -k(u)\log(\sigma) - \left(1+\frac{1}{\xi}\right)\sum_{i=1}^{n}\log\left(1+\xi^{\,yi}/_{\sigma}\right)(7)$$

whereyisatisfies the constraints specified for yi. If $\xi = 0$, the log likelihood function is:

$$l(\sigma) = -k(u)\ln(\sigma) - \sigma^{-1}\sum_{i=1}^{n} yi \qquad (8)$$

ML estimates are then found by maximizing the log-likelihood function using numerical optimization methods. We can get these $\hat{\xi}$ and $\hat{\sigma}$estimates where ($\xi \neq 0$).

The extreme Value Theory (EVT) was used in measuring the extreme risks of Rwanda stock Exchanges. The results of this work was supported by GAECH model. In particular the results was showed that the participant in RSE market can rely on EVT – based model such as GPD when modeling the conditional Heteroskedasticity of extreme events.

The study also had important implications for investors and risk managers. Baytron(2005) indicated that expected shortfall (ES) performed under a GARCH EVT Framework is superior to a number of parametric approaches.

4. Data Analysis and Results

In our study we have introduced a progressive methodology for measuring risk in Rwanda stock market and the finding of the study showed the following results the data used are monthly closing prices indices from Rwanda stock market mentioned.

4.1. Descriptive Statistics

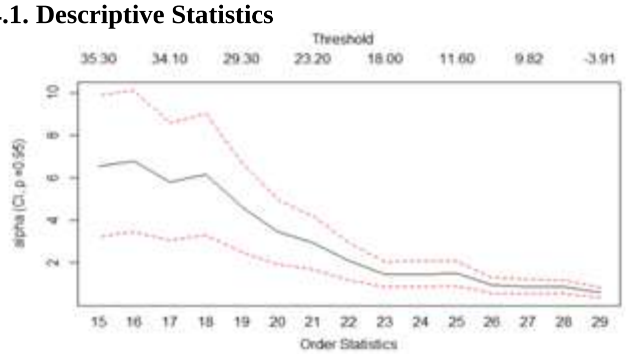


Figure 4.1: order statistics for Bank of Kigali resulting for fitting the Generalized Pareto Distribution to the monthly data.

4.2. Mean excess function

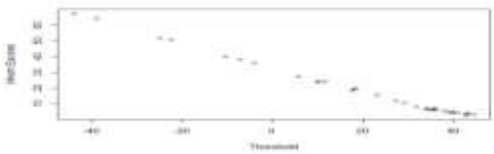
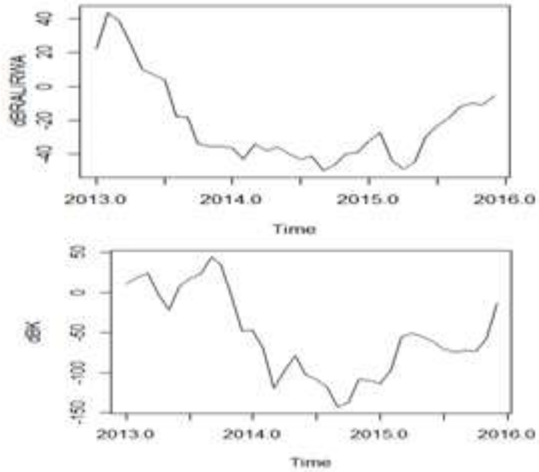


Figure 4.2: Mean excess Function

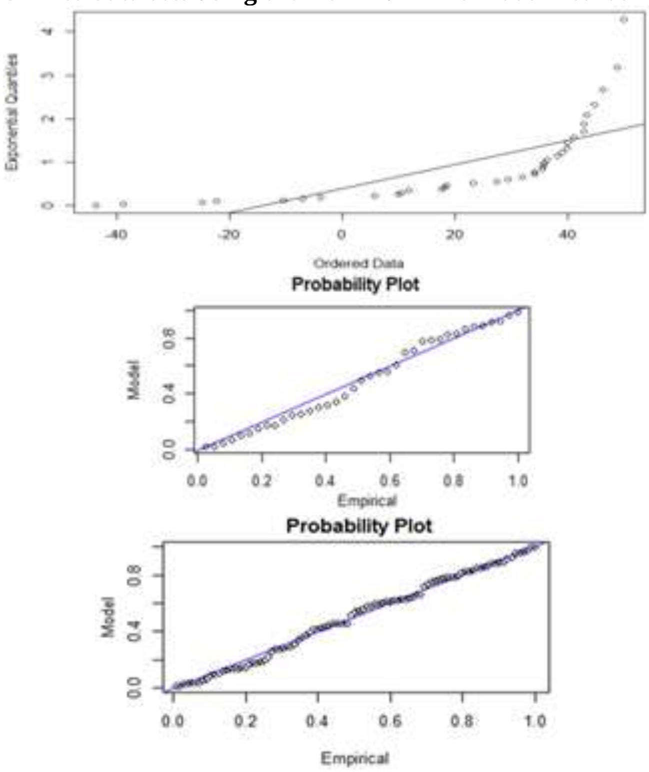
One significant observation is that at the chosen threshold, the mean excess plot is broadly speaking, flat. At a higher threshold, the mean excess plot decisively slopes downward.

4.3 Hill Plots for Bank of Kigali and Bralirwa companies



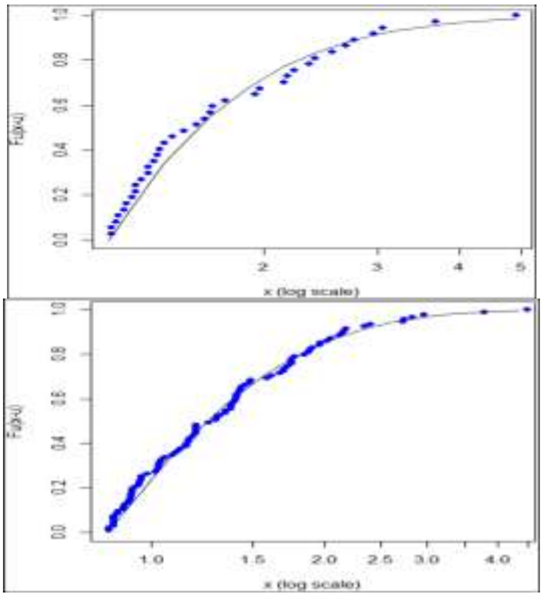
The hill plot are constructed such that the estimated shape parameter is plotted as a function either of k upper- order statistics or the threshold. It is the conditional maximum likelihood estimators for the heavy – tailed distributions. The hill plot was used because it only depends on the shape the distribution tail, not on the entire distribution, simplicity of the formula. And the hill plot is effectively when the distribution pareto or closed to pareto.

4.4 Q-Q And P-P Plots obtained from fitting the maximal GPD to data sets using the maximum likelihood method



These plots the data values against the equivalent percentiles for the parameterized GPD. The Q-Q plot is to this line measure of goodness of fit. Drift away from this line in one direction indicate that the underlying distribution may have a heavier (or lighter tail) than the fitted distribution. Note that if the model fits the data well the pattern of points on Q-Q plot was exhibit a 45- degree straight line.

4.5 CDF Plot to fit the maximal GPD



This plot the empirical claims distributions (CDF) against the GDP. The same comments on the values apply as for the p-p plot, in that both the empirical and fitted. The plots suggest that good fit has not been obtained, then the threshold needs to be increased until, either an accepted plot or it becomes clear that a GDP is not a good of fit.

4.6. Fitting a Generalized Pareto Distribution

Number of observation above the selected threshold left for measuring extremes risks was 195 for Bank of Kigali and 345 for Bralirwa. This were more than 10% of the total number of the observations in each company.

Table 1: Shape and scale parameters

Estimates	MLE	
Scale	B.K	BRALIRWA
	1.15	0.84
Shape Parameter	0.204217	0.04817
Std Error	0.256639	0.113611
Scale Parameter	0.578913	0.578579
Std Error	0.175336	0.088531

This is the reports of the shape $(\hat{\xi})$ and the scale $(\hat{\sigma})$ parameters from the fitted GPD for Bank of Kigali and Bralirwa data

Table 2: Risk Measure and computation VaR and ES

Probability	VaR	ES
Measure of Risk (Bank of Kigali)		
0.99	3.485851	4.812761
0.995	4.272103	5.800784
0.999	6.590082	8.713611
0.9995	7.848366	10.294801
0.9999	11.55796	14.956364
(Bralirwa)		
0.99	2.902482	3.614718
0.995	3.380319	4.116737
0.999	4.553328	5.349109
0.9995	5.087214	5.910013
0.9999	6.397817	7.286942

The result indicate that with probability of 0.1 hence a 99%level of confidence, the expected market return would not gain by more than 3.49% and if it does by any chance increase by more than 3.49% then the average gain of 4.81% is expected within a one day period. If for any reason a loss is experienced then it was not exceed 2.90% with probability 0.01 and the loss if it does exceed 2.9% then the expected loss was 3.61%.

With probability 0.005 thus a 99.5% level of confidence. The monthly market was not more than 4.27% but if it does go beyond then it will not exceed 5.8%. In the case of losses with probability 0.005 the loss will not exceed 3.38% but if it does exceed this then the los is expected to be 4012%. From these results indicate that for an investors in Rwanda Stock Exchange, the probability of losses is lower compared to the probability of gains.

5. Conclusion

We have considered in this paper Extreme value Theory as a tool for calculating VaR for a set of monthly stock price indices of Rwanda Sock Market, we have described Extreme Value Theory, (EVT), Generalized Pareto Distribution (GPD), we would like to refer interested reader to all cited publications, where the extensive real case studies can be found. we saw that according to our diagnostics, we would be well modeled by a GPD as expected, the EVT VaR model did not properly estimate VaR for the Rwanda index.

However, mean excess function was founded and plotted over the threshold value u. It tend toinfinite typical heavy-tailed distribution and provides adequate estimation for VaR and ES. The value at Risk and expected shortfall and the result showed that for the investors in RSE, the probability of losses is lower compared to the probability of gains.

References

[1] Andjelic G, Milosevic I, Djakovic V (2010) “Extreme Value in emerging market” Benimadhu S (2003) Challenges facing African capital markets. Presentation at UNECA Forum. Johannesburg.
[2] Huber, P.J. (1967), “The behaviors of maximum likelihood estimates under Non- Standard Assumption,” *Proceedings of the Fifth Berkeley Symposium on* Mathematical Statistics and Probability 1(1), pp. 221-23.
[3] Grimshaw, S. (1993). “Computing Maximum likelihood Estimate for Generalized Pareto Distribution to Data”. *Technometrics*, 35(2):185-191.
[4] Dupuis, D. J. (1998). “Exceedances over high thresholds: A guide to threshold selection. Extremes,” 1(3):251-261.
[5] Castillo, E. and Hadi, A. (1997). “Fitting the generalized Pareto Distribution Data Jounal of American statistical Association,”92(440):1609-1620.
[6] Smith, R. (1985), “Maximum likelihood Estimation in a class of nonregular cases” Biometrika72, 67-92.
[7] McNeil, A.J. amdfrey, R. (1998), “Estimation of Tail-related risk measures for heteroscedastic financial time series: an extreme value approach’’, preprint, ETH Z¨urich.

[8] Reiss, R.D., Thomas, M. (2001), "Statistical Analysis of Extreme Values" from Insurance,Finance, Hydrology and Other Fields, BirkhauserVerlag, Basel.

[9] Coles, S. (2001), An Introduction to Statistical Modeling of Extreme Values, Spring series instatistics, Springer-Verlag London Limited.

[10] Artzner, P. (2001), Application of coherent risk measures to capital requirements in Insurance, North American Actuarial Journal, N° 3, 11-25

[11] Gencay R, Selcuk F. Extreme value theory and value-at-risk: relative performance in emerging markets. Int J Forecast.

[12] F. Ren and D. E. Giles, "Extreme Values Theory of daily Canadian crude oil prices" *EconometricsWorking Paper*, *EWP0708*. University of Victoria, 2007.

[13] J. Pickands, "statistical inference using extremes order statistics", *Annals of Statistics*, 3, 1975, pp. 119-131.

[14] Djakovic, G. Andjelic and J. Borocki, "Performance of Extreme Value Theory in emerging market : An empirical treatment" *African Journal of Business Management*, Vol 5 (2), 2011, pp. 340-369.

[15] P. I. Louangrath, ''Stock price analysis under extreme value theory'', *Bangkok University International College, Thailand*, 2016.

Author Profile

Bizumutima Jean Claude, Student in Masters of Applied Statistics, Faculty of Applied Sciences, Department of statistics and Actuarial Sciences, Jomo Kenyatta University of Agriculture and Technology, Kigali, Rwanda.