An Application of Power Domination of Zero Forcing

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Abstract: In this paper we give all graphs G on n vertices with girth atleast 7 and sum of the power propagation time of a graphs and We also study a generalization of power propagation time, known as k-power propagation time, by characterizing all simple graphs on n vertices whose k-power propagation time is n −1 or n −2 (for k ≥ 1) and n −3 (for k ≥ 2).

Keywords: power propagation time, zero forcing, Girth, minimum power dominating set

1. Introduction

The study of the power domination number of a graph arose from the question of how to monitor electric power networks at minimum cost, see Haynes et al. [9]. Intuitively, the power domination problem consists of finding a set of vertices in a graph that can observe the entire graph according to certain observation rules. The formal definition is given below immediately after some graph theory terminology. A graph G = (V, E) is an ordered pair formed by a finite nonempty set of vertices V = V (G) and a set of edges E = E(G) containing unordered pairs of distinct vertices (that is, all graphs are simple and undirected). The complement of G = (V, E) is the graph G = (V, E), where E consists of all two element subsets of V that are not in E. For any vertex v ∈ V, the neighborhood of v is the set N(v) = {u ∈ V : {u, v} ∈ E} and the closed neighborhood of v is the set N[v] = N(v) ∪ {v}. Similarly, for any set of vertices S, N(S) = Uv∈S N(v) and N[S] = Uv∈S N[v]. For a set S of vertices in a graph G, define PD(S) ⊆ V (G) recursively as follows:
2) While there exists a ∈ P D(S) such that |N(a) \ P D(S)| = 1: P D(S) := P D(S) \ N(a).

A set S ⊆ V (G) is called a power dominating set of a graph G if, at the end of the process above, P D(S) = V (G)[4]. A minimum power dominating set is a power dominating set of minimum cardinality[10]. The power domination number of G, denoted by γp(G), is the cardinality of a minimum power dominating set. Power domination is naturally related to domination and to zero forcing. A set S ⊆ V (G) is called a dominating set of a graph G if N[S] = V (G). A minimum dominating set is a dominating set of minimum cardinality. The domination number of G, denoted by γ(G), is the cardinality of a minimum dominating set. Clearly γp(G) ≤ γ(G). Zero forcing was introduced independently in combinatorial matrix theory [1] and control of quantum systems [5]. From a graph theory point of view, zero forcing is a coloring game on a graph played according to the color change rule: If u is a blue vertex and exactly one neighbor w of u is white, then change the color of w to blue. We say u forces w. A zero forcing set for G is a subset of vertices B such that when the vertices in B are colored blue and the remaining vertices are colored white initially, repeated application of the color change rule can color all vertices of G blue. A minimum zero forcing set is a zero forcing set of minimum cardinality [5]. The zero forcing number of G, denoted by Z(G), is the cardinality of a minimum zero forcing set. Power domination can be seen as a domination step followed by a zero forcing process, and we will use the terminology “v forces w” to refer to Step 2 of power domination. Clearly γ P (G) ≤ Z(G). The rest of the system is observed according to the following propagation rules:
1) Any vertex that is incident to an observed edge is observed.
2) Any edge joining two observed vertices is observed.
3) If a vertex is incident to a total of t > 1 edges and if t − 1 of these edges are observed, then all t of these edges are observed.

We remark that there are mathematical connections between the power domination number and the zero forcing number defined in AIM Minimum Rank – Special Graphs Work Group et al. (2008) and Burgarth and Giovannetti (called graph infection in the latter), and between the power propagation time discussed here and the propagation time defined in Hogben et al.. We refer the reader to Benson et al. for a discussion of the relationship between the power domination number and the zero forcing number. Power domination is closely related to the well known domination problem in graph theory. A set S ⊆ V (G) is a dominating set if N[S] = V (G). The domination number of a graph G, denoted γ(G), is the minimum cardinality over all dominating sets of G. Note that each dominating set is a power dominating set. A set S ⊆ V (G) is called a girth dominating set of G if every vertex in V − S is adjacent to at least one vertex in the girth cycle of G. The minimum cardinality of a girth dominating set of G is called its girth domination number of G denoted by (G) [6].

1.1 Zero Forcing

The concept of zero forcing can be explained via coloring game on the vertices of G[9]. The color change rule is: If u is a blue vertex and exactly one neighbor w of u is white, then change the color of w to blue. We say u forces w and denote this by u → w. A zero forcing set for G is a subset of
Forcing set of G
A set S is a power dominating set of G if and only if N[S] is a zero forcing set of G. It follows that N(S) \ S is a zero forcing set of G \ S.

The authors of [2] introduced the propagation time of a zero forcing set of a graph. Due to the close relationship between zero forcing and power domination, many of the questions studied in this paper were motivated by results of the propagation time of a zero forcing set.

Notes
The graph L(s, t) is the lollipop graph consisting of a complete graph Kn and a path on t vertices where one endpoint of the path is connected to one vertex of Kn via a bridge.

1.4 A.K THEOREM
Let G be a graph on n ≤ 7 vertices that has girth at least 7, then ppt(G) ≤ 5.

Proof
Let s be an efficient power domination for G. We will show that |N[s^*]| ≥ n - 2, then its follow Lemma 1 ppt(G) ≤ 5.

Assume that |N[s^*]| ≤ n - 4 so that V \ N[s^*] ≥ 4.

Let u be in v/N[s^*] such that u is forced by some v \ N[s^*]. Since any edge joining two observed vertices is observed Recall that in order for v to force u.

u must be the only neighbor of v in N[s^*]. Let x and y be two vertices in v/N[s^*] such that x≠u and y≠w.

we first show that x and y be two vertices must be adjacent. Choose w ∈ S' and w ∈ S such that v ∈ N(S) [this s and w guaranteed since v ∈ N[S^*] / S'.

Note that x,y\N[s],so x and y must be adjacent, then the graph induced by {x,y,s,w,v} is k_2 U k_3 =C_5. This is contradiction of the hypothesis, that the Girth of G is at least 7.

So|N[s^*]| ≥ n - 2 and ppt(G) ≤ 5.

1.5 Proposition
Let G be a graph on n vertices with girth at least 7, then ppt(G) + ppt(G) ≤ n

Proof:
It follows from inspection that the claim hold for n ≤ 6

Assume that n ≥ 7 By theorem ppt(G) ≤ 5

Suppose first Δ(G) ≥ 5 and let G_t be connected component of G that has a vertex of degree at least 5.then there exist a minimum power Result 1

Therefore |N[s]| ≥ 6.

For any minimum power domination set S of G with s_t ⊆ s

So,ppt(G) ≤ ppt(G,s) ≤ n-Lemma 1

Since n ≥ 7,ppt(G) + ppt(G) ≤ n

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2. Conclusion

In this paper, we have discussed on the propagation time of a graph in power domination of zero forcing. In future we propose to extend this work with many graphs.

References


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