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An Application of Power Domination of Zero Forcing

Dr. S. Chandrasekaran[1], A. Sulthana[2]

[1]Head and Associate Professor, PG and Research Department of Mathematics, Khadir Mohideen College, Adirampattinam, Tamil Nadu, India

[2]Research Scholar, Department of Mathematics, KhadirMohideen College, Adirampattinam, Tamil Nadu, India

Abstract: ***In this paper we give all graphs G on n vertices with girth atleast 7 and sum of the power propagation time of a graphs ,and We also study a generalization of power propagation time, known as k−power propagation time, by characterizing all simple graphs on n vertices whose k−power propagation time is n − 1 or n − 2 (for k ≥ 1) and n − 3 (for k ≥ 2).***

Keywords: power propagation time , zero forcing, Girth, minimum power dominating set

1. Introduction

The study of the power domination number of a graph arose from the question of how to monitor electric power networks at minimum cost, see Haynes et al. [9]. Intuitively, the power domination problem consists of finding a set of vertices in a graph that can observe the entire graph according to certain observation rules. The formal definition is given below immediately after some graph theory terminology. A graph G = (V, E) is an ordered pair formed by a finite nonempty set of vertices V = V (G) and a set of edges E = E(G) containing unordered pairs of distinct vertices (that is, all graphs are simple and undirected). The complement of G = (V, E) is the graph G = (V, E), where $\bar{E}$ consists of all two element subsets of V that are not in E. For any vertex $v\in$ V, the *neighborhood* of *v* is the set N(v) = {$u\in$ V : {u, v} ∈ E} and the closed *neighborhood* of *v* is the set N[v] = N(v) ∪ {v}. Similarly, for any set of vertices S, N(S) = $U_{v\in s}N(v)$ and N[S] =$U_{v\in s}$ N[v]. For a set S of vertices in a graph G, define PD(S) ⊆ V (G) recursively as follows:

1) P D(S):= N[S] = S ∪ N(S).
2) While there exists $v\in$ P D(S) such that |N(v) \ P D(S)| = 1: P D(S) := P D(S) ∪ N(v).

A set S ⊆ V (G) is called a power dominating set of a graph G if, at the end of the process above, P D(S) = V (G)[4]. A minimum power dominating set is a power dominating set of minimum cardinality[10]. The power domination number of G, denoted by γ_p(G), is the cardinality of a minimum power dominating set. Power domination is naturally related to domination and to zero forcing. A set S ⊆ V (G) is called a dominating set of a graph G if N[S] = V (G). A minimum dominating set is a dominating set of minimum cardinality. The domination number of G, denoted by γ(G), is the cardinality of a minimum dominating set. Clearly γ_p(G) ≤γ(G). Zero forcing was introduced independently in combinatorial matrix theory [1] and control of quantum systems [5]. From a graph theory point of view, zero forcing is a coloring game on a graph played according to the color change rule: If u is a blue vertex and exactly one neighbor w of u is white, then change the color of w to blue. We say u forces w. A zero forcing set for G is a subset of vertices B such that when the vertices in B are colored blue and the remaining vertices are colored white initially, repeated application of the color change rule can color all vertices of G blue. A minimum zero forcing set is a zero forcing set of minimum cardinality [5].The zero forcing number of G, denoted by Z(G), is the cardinality of a minimum zero forcing set. Power domination can be seen as a domination step followed by a zero forcing process, and we will use the terminology "*v* forces *w*" to refer to Step 2 of power domination. Clearly γP (G) ≤Z(G).The rest of the system is observed according to the following propagation rules:

1) Any vertex that is incident to an observed edge is observed.
2) Any edge joining two observed vertices is observed.
3) If a vertex is incident to a total of t > 1 edges and if t − 1 of these edges are observed, then all t of these edges are observed.

We remark that there are mathematical connections between the power domination number and the zero forcing number defined in AIM Minimum Rank – Special Graphs Work Group et al. (2008) and Burgarth and Giovannetti (called graph infection in the latter), and between the power propagation time discussed here and the propagation time defined in Hogben et al.. We refer the reader to Benson et al. for a discussion of the relationship between the power domination number and the zero forcing number.Power domination is closely related to the well know domination problem in graph theory. A set S ⊆ V (G) is a dominating set if N[S] = V (G). The domination number of a graph G, denoted γ(G), is the minimum cardinality over all dominating sets of G. Note that each dominating set is a power dominating set, so γ_p(G) ≤ γ(G) [3]**.**A set S⊂V(G) is called a girth dominating set of G if every vertex in V-S is adjacent to at least one vertex in the girth cycle of G. The minimum cardinality of a girth dominating set of G is called its girth domination number of G denoted by (G) [6]

1.1 Zero Forcing

The concept of zero forcing can be explained viaa coloring game on the vertices of G[9].The color change rule is: If u is a blue vertex and exactly one neighbor w of u is white, then change the color of w to blue.[8] We say u forces w and denote this by u → w. A zero forcing set for G is a subset of vertices B such that when the vertices in B are colored blue and the remaining vertices are colored white initially, repeated application of the color change rule can color all vertices of G blue. A minimum zero forcing set is a zero forcing set of minimum cardinality, and the zero forcing number Z(G) of G is the cardinality of a minimum zero forcing set. The next observation is the key relationship between the two concepts.

1.2 Notes

We use Nordhaus-Gaddum sum bounds for power propagation time In Nordhaus and Gaddum gave upper and lower bounds on the sum and product of the chromatic number of a graph and its complement. we use this result to show that for all graphs on n vertices, ppt(G) + ppt(G) ≤ n + 2. We also conjecture that n is the least upper bound, and demonstrate an infinite family of graphs with ppt(G) + ppt(G) = n for each G in the family.

1.3 Basic definitions and notation

Let n be a positive integer. The path of order n is the graph P_n with V (P_n) = {x_i : 1 ≤ i ≤ n} and E(P_n) = {{x_i , x_{i+1}} : 1 ≤ i ≤ n − 1}. If n ≥ 3, the cycle of order n is the graph$C_n$with V (C_n) = x_i: 1 ≤ i ≤ n} and E(C_n)) = {{x_i , x_{i+1}}: 1 ≤ i ≤ n − 1} ∪ {{x_n, x_1}}. The complete graph of order n is the graph $K_n$with V(K_n) = {x_i : 1 ≤ i ≤ n} and E(K_n) = {{x_i,x_j} : 1 ≤ i < j ≤ n}. Let G = (V (G),E(G)) and H=(V(H), E(H)) be disjoint graphs. All of the following products of G and H have vertex set V (G) × V (H). We use P_n,C_n, and K_n to denote the path, cycle, and complete graph on vertices respectively. The notation K_n − e represents the complete graph on n vertices minus an edge and $K_{s,t}$is the complete bipartite graph with bipartition X, Y where |X| = s and |Y | = t.Let G = (V, E) be a graph and e = uv∈E(G). The graph resulting from subdividing the edge e = uvdenoted G_e, is obtained from G by adding a new vertex w such that V (G_e) = V (G) ∪ {w} and E(G_e) = (E(G) \ {uv}) ∪ {uw,wv}. To contract the edge e = uv is to identify vertices u and v as a single vertex w such that N(w) = (N(u) ∪ N(v)) \ {u, v}. The graph obtained from G by contracting the edge e is denoted by G/e.

Lemma 1
Let G be a graph on n vertices and S a power dominating set of G. Then,
PPt(G, S) ≤ n − |S| and
ppt(G, S) − 1 ≤ n − |N[S]|

This follows from the fact that at least one vertex must be forced at each step.

Result 1
Let G be a connected graph with Δ(G) ≥ 3. Then there exists a minimum power dominating set S of G such that deg(s) ≥ 3 for each s ∈ S.

Lemma2[7]
A set S is a power dominating set of G if and only if N[S] is a zero forcing set of G. It follows that N(S) \ S is a zero forcing set of G \ S.

The authors of **[2]** introduced the propagation time of a zero forcing set of a graph. Due to the close relationship between zero forcing and power domination, many of the questions studied in this paper were motivated by results of the propagation time of a zero forcing set.

Notes
The graph L(s, t) is the lollipop graph consisting of a complete graph K_s and a path on t vertices where one endpoint of the path is connected to one vertex of K_s via a bridge

1.4 A.K THEOREM

Let G be a graph on n≤ 7 vertices that has girth atleast7,then ppt($\bar{G}$) ≤5.

Proof
Let s' be an efficient power domination for $\bar{G}$. We will show that |N[s']| ≥n-2,then its follow Lemma 1ppt($\bar{G}$) ≤5.
Assume that |N[s']| ≤n-4
so that V |N[s']| ≥4.

Let u be in v/N[s'] such that u is forced by some v∈ $N[S]$/SSince Any edge joining two observed vertices is observed Recall that in order for v to force u.

u must be the only neighbor of v in$v$/N[s']Let x and y be 2 vertices in v/N[s'] such that x≠u and y≠ w.

we first show that x and y be two vertices must be adjacent.
Choose s∈ S' and w∈ S' such that v∈ $N(S)$ [this s and w guaranted since v∈ $N[S'] / S'$

Note that x,y∉N[S],so x and y must be adjacent, then the graph induced by {x,y,s,w,v} is $k_2 \cup k_3$ =$\overline{C_5}$This is contradiction of the hypothesis, that the Girth of G is atleast 7.

So|N[s']|≥n-2 and ppt($\bar{G}$)≤ 5.

1.5 Preposition1

Let G be a graph on n vertices with girth atleast 7, thenppt(G) + ppt(G) ≤ n

Proof:
It follows from inspection that the claim hold for n≤ 6
Assume that n≥ 7 By theoremppt($\bar{G}$)≤ 5

Suppose first Δ(G) ≥ 5 and let G_1 be connected component of G that has a vertex of degree atleast 5.thenthere exist a minimum power Result 1

Therefore |N[s_1| ≥ 6,

For any minimum power domination set S of G with s_1⊆s, |N[s]| ≥ 6
So,ppt(G)≤ ppt(G, S) ≤ n-Lemma 1
Since n≥7,ppt(G) + ppt(G) ≤ n .

2. Conclusion

In this paper, we have discussed on the propagation time of a graphin power domination of zero forcing, In future we propose to extend this work with many graphs.

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