International Journal of Science and Research (IJSR)
ISSN: 2319-7064
Index Copernicus Value (2016): 79.57 | Impact Factor (2017): 7.296

Stability Control of TORA Based on LQR Controller

HE Tongzhao

Tianjin Key Laboratory of Information Sensing and Intelligent Control, Tianjin University of Technology and Education, Tianjin, China

Abstract: ***An under-actuated translational oscillator with rotating actuator is a typical under-actuated system. Based on Lagrange equation for mechanical control system, the dynamic model is derived for TORA. And then the characteristics of controllability and observability of the system are analyzed. A LQR controller is designed for stabilizing the system. The system simulation experiments verify that the controller can make the system to reach steady state and effectively decrease vibration of the system in a short time.***

Keywords: TORA, LQR, Under-actuated system, Controller

1. Introduction

It is a common method in control theory and engineering to idealize and linearize the system and then find an approximate linear model instead of control and research. However, the actual control system is mostly nonlinear, and the complexity of the controlled object and the continuous improvement of the performance requirements of the control system make the linear control difficult to meet the actual needs. The translation oscillator with rotating actuator (TORA) is a typical under-actuated nonlinear control system consisting of an translation oscillator and a driven rotating ball. The TORA system consists of a car in the horizontal plane and a rotating ball in the vertical plane. The car is connected to the wall by a spring and moves horizontally. The ball is driven by a vertical disk on the car, moving both horizontally and vertically. This mechanical system is a simplified model of the dual-spin spacecraft to study the resonance capture phenomenon, so the study of its stability has practical significance.

At the end of 20th century, some scholars began to study the stability of TORA system. Wan et al. transformed the TORA system into a partially feedback linear cascade nonlinear system, and implemented global progressive stability control through Backstepping[1]. Avis et al. achieved simultaneous stability control of multiple TORA systems through energy mixing control scheme[2].In recent years, research on stability control of TORA system mainly includes energy - based control methods[3], state feedback control method based on Backstepping[4], sliding mode control method[5[,]6], and adaptive neural network output feedback control method[7],[8].

In this paper, the Lagrange dynamic model of the TORA system is established, and the controllability and observability of the system are analyzed. LQR feedback controller is designed, and the rationality and stability of the controller are verified by simulation experiment.

2. Dynamic Model

2.1 Establishment of dynamic modle

The physical modeling of TORA system is shown in figure 1. In the figure,xis the motion displacement of the car,θ is the angle of rotation of the ball, M is the mass of the car, m is the mass of the ball, r is the rotation radius of the ball, J is the moment of inertia of the ball, k is spring's elastic coefficient, τ is the input torque acting on the ball.

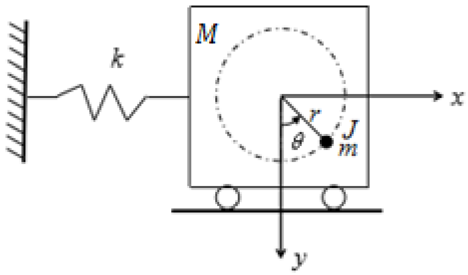


Figure 1: Physical modeling of TORA system

Rectangular coordinate system is established for TORA. The free end of the spring is used as the origin, and the direction of the motion of the car is used as the horizontal axis to establish the coordinate system (The torque acting on the ball is the control input, the ball rotation Angle and the car displacement are the two control outputs of the system). Taking car displacement x and ball angle θ as generalized coordinates,the parameter equation of the ball is

$$\begin{cases} x_1 = x + r\sin\theta \\ y_1 = r\cos\theta \end{cases} \quad (1)$$

Take the derivative of the equations (1), we can get the equations:

$$\begin{cases} \dot{x}_1 = \dot{x} + \dot{\theta} r\cos\theta \\ \dot{y}_1 = -\dot{\theta} r\sin\theta \end{cases} \quad (2)$$

According to the superposition principle of kinetic energy, the total kinetic energy of the system is the sum of the kinetic energy of the car and the ball:

$$K = K_1 + K_2 = \frac{1}{2}(M+m)\dot{x}^2 + mr\dot{x}\dot{\theta}\cos\theta + \frac{1}{2}(mr^2 + J)\dot{\theta}^2 \quad (3)$$

In the TORA system,there is an elastic potential energy in the spring as an external force, andthe zero of its potential energy is defined as the original state of the spring. As the motion in a straight plane is affected by gravity, the ball has gravitational potential energy. The total potential energy of the system is:

$$P = \frac{1}{2}kx^2 = mgr\cos\theta \quad (4)$$

Based on the energy balance equation, the Lagrange factor is obtained from equations (3) and (4)

$$L = K - P \tag{5}$$
$$= \frac{1}{2}(M+m)\dot{x}^2 + mr\dot{x}\dot{\theta}\cos\theta + \frac{1}{2}(mr^2+J)\dot{\theta}^2 + mgr\cos\theta - \frac{1}{2}kx^2$$

According to the Lagrange equation, the system dynamics equation is derived:

$$\begin{cases}(M+m)\ddot{x} + mr\ddot{\theta}\cos\theta - mr\sin\theta\,\dot{\theta}^2 + kx = 0\\ mr\ddot{x}\cos\theta + (mr+J)\ddot{\theta} + mgr\sin\theta = \tau\end{cases} \tag{6}$$

The standard dynamic equation matrix form of the system is

$$M(q)\ddot{q} + C(q,\ddot{q}) + G(q) = U$$

$$q = \begin{bmatrix}x\\ \theta\end{bmatrix},\ M(q) = \begin{bmatrix}M+m & mr\cos\theta\\ mr\cos\theta & mr^2+J\end{bmatrix},$$
$$C(q,\dot{q}) = \begin{bmatrix}0 & -mr\dot{\theta}\sin\theta\\ 0 & 0\end{bmatrix},$$
$$G(q) = \begin{bmatrix}kx\\ mgr\sin\theta\end{bmatrix},\ U = \begin{bmatrix}0\\ \tau\end{bmatrix}$$

2.2 Analyze the stability of the system

Controllability of the TORA system mainly depends on whether the control input of the system can affect the internal state of the system. In the limited time domain, if a control input can make the system in the unbalanced state return to the equilibrium state, the system is controllable.

The equation of state for the system is:

$$\begin{cases}\dot{x} = Ax + Bu\\ y = Cx\end{cases} \tag{7}$$

Among them, $x = [x \quad \dot{x} \quad \theta \quad \dot{\theta}]^T$,

According to equation (7), the coefficient matrixes of the equation of state are:

$$A = \begin{bmatrix}0 & 1 & 0 & 0\\ a_{21} & 0 & 0 & 0\\ 0 & 0 & 0 & 1\\ a_{41} & 0 & 0 & 0\end{bmatrix}, B = \begin{bmatrix}0\\ b_{21}\\ 0\\ b_{41}\end{bmatrix}, C = \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0\end{bmatrix}$$

Among them,

$$\begin{cases}a_{21} = \frac{(mr^2+J)\cdot k}{(mr\cos\theta)^2-(mr^2+J)(M+m)}\\ a_{41} = \frac{mr\cos\theta\cdot k}{(mr^2+J)-(M+m)(mr\cos\theta)^2}\\ b_{21} = \frac{mr\cos\theta}{(mr\cos\theta)^2-(mr^2+J)(M+m)}\\ b_{41} = \frac{M+m}{(mr^2+J)-(M+m)(mr\cos\theta)^2}\end{cases} \tag{8}$$

The controllability discriminant matrix of the TORA system is:

$$Q_C = [B \quad AB \quad A^2B \quad A^3B] \tag{9}$$
$$= \begin{bmatrix}b_{21} & a_{21}\cdot b_{21} & & 0\\ b_{41} & a_{41}\cdot b_{21} & & \\ & 0 & b_{21} & a_{21}\cdot b_{21}\\ & & b_{41} & a_{41}\cdot b_{21}\end{bmatrix}$$

Substituting equation (7) into equation (9) can be used to determine whether the matrixQ_C is full rank: if $\cos\theta = 0$, $rankQ_C = 0$,the TORA system was not controllable; if $\cos\theta \neq 0$. $rankQ_C = 4$,the TORA system was controllable. Therefore, the TIRA system is balanced at the equilibrium state$(x \quad \dot{x} \quad \theta \quad \dot{\theta}) = (0 \quad 0 \quad \theta \quad 0), \theta \neq \frac{k\pi}{2}$(k=±1, ±3 ...).

3. Design of Controller

When the rotation angle of the ball is very small, the cosine value of the angle in the equation (8) is approximately equal to zero, so we can obtain the linear model of the system. The state space of the system is described as:

$$\begin{cases}\dot{x}_1 = x_2\\ \dot{x}_2 = f_1(x) + g_1(x)\tau\\ \dot{x}_3 = x_4\\ \dot{x}_4 = f_2(x) + g_2(x)\tau\end{cases} \tag{10}$$

Among them,

$$\begin{cases}f_1(x) = \frac{(mr^2+J)\cdot kx}{(mr)^2-(mr^2+J)(M+m)}\\ g_1(x) = \frac{mr}{(mr)^2-(mr^2+J)(M+m)}\\ f_2(x) = \frac{mrkx}{(mr^2+J)-(M+m)(mr)^2}\\ g_2(x) = \frac{M+m}{(mr^2+J)-(M+m)(mr)^2}\end{cases} \tag{11}$$

For a system, it is assumed that there is a feedback coefficient matrix K, making u$(u = -R^{-1}B^TP = -Kx)$ the optimal control, so the control quantity u(t) needs to satisfy the condition that: the energy value of the performance index $J = \int_0^\infty (x^TQx + u^TRu)\,dt$ is the minimum..The factor q_i in the matrix Q weighs the weights of each state variable, while the factor r_i in the matrix R is a weight of the control variableu_i.

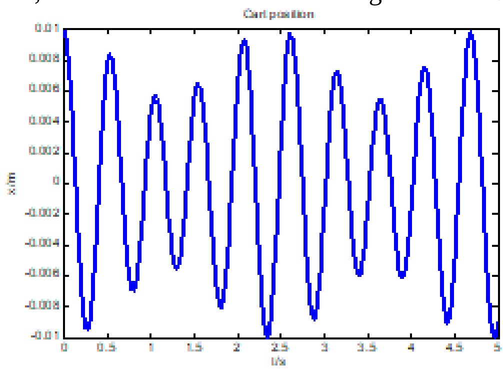
4. Simulation

According to the dynamic equation (7) of TORA system, we used MATLAB for simulation to verify the validity of the above control scheme. The selection of system physical parameters is shown in table 1

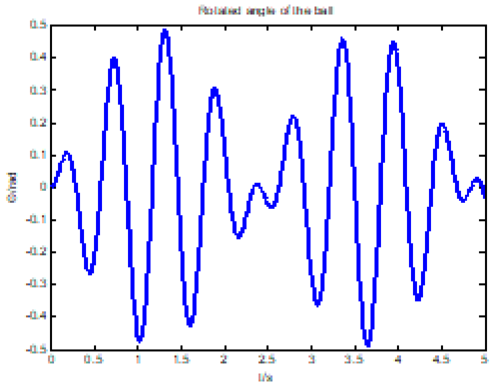
Table 1: Margin specifications

Parameters of the symbol	Physical definition	Value of parameter
M	Mass of car	$1.3608kg$
m	Mass of ball	$0.096kg$
k	spring's elastic coefficient	$186.3N/m$
r	Rotation radius of ball	$0.0592m$
J	rotary inertia of ball	$0.0002175kg\cdot m^2$

Set the control input as zero, the initial angle of the ball as zero, and the initial position of the car as $0.01m$. The displacement of the car and the angle of the ball are respectively shown in figure. 2 (a) and (b).It can be seen from figure 2 that both the displacement of the car and the rotation Angle of the ball continue to oscillate and the system is unstable. According to the structural analysis of TORA system, if the angle of the ball and the displacement of the car are fixed respectively, the system will be transformed into a classical spring mass system and a simple pendulum system. If the angle of the ball in the system is fixed, that is to say, both θ and $\dot{\theta}$ are identical to zero, and the initial state of the car is $x = 0.01m$, then the system is a simple harmonic motion that the car moving without friction on the horizontal surface. The differential equation of the system is$(M+m)\ddot{x} + kx = 0$.The simulation result is shown in figure 3 (a).If the car in the system and the spring fixed, that is to say both x and $\dot{x}$ are equal to zero, initial angle of ball is 30°, then the system is a simple pendulum motion. The simulation result is shown in figure 3 (b).The differential equation of the system is $(mr + J)\ddot{\theta} + mgrsin\theta = 0$. Therefore, the correctness of the modeling is verified.

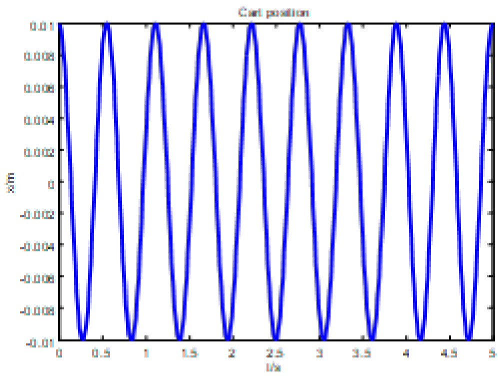


(a) Displacement of the car

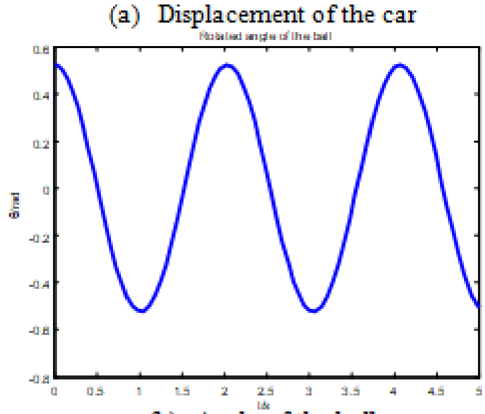


(b) Angle of the ball

Figure 2: The state of the system without control



(a) Displacement of the car



(b) Angle of the ball

Figure 3: The state of the system under the action of single variables

We add the above LQR controller to the TORA system and conduct simulation experiments. Through the method of setting the individual parameters one by one, observe the contrast control effect, and finally determine the parameter values of the controller are Q=diag[100 100 1 1],R = 100. Figure 4 shows the car displacement and ball rotation Angle of the TORA system under the action of the controller. According to the change curve of the two states of the system with time in figure 4, we can see that the free end of the car is stable when the spring is expanded after about 0.8 seconds from the initial position 0.01m; the ball also final steady at 0 °. The system stabilizes from the initial state to the origin state $[x \quad \dot{x} \quad \theta \quad \dot{\theta}]^T = [0 \quad 0 \quad 0 \quad 0]^T$. LQR control method is used to reduce the oscillation period of the system. During the control process, the swing Angle of the ball was controlled within 0.1rad. This control method can make the TORA system reach stable state in a short time, and the control function is stronger.

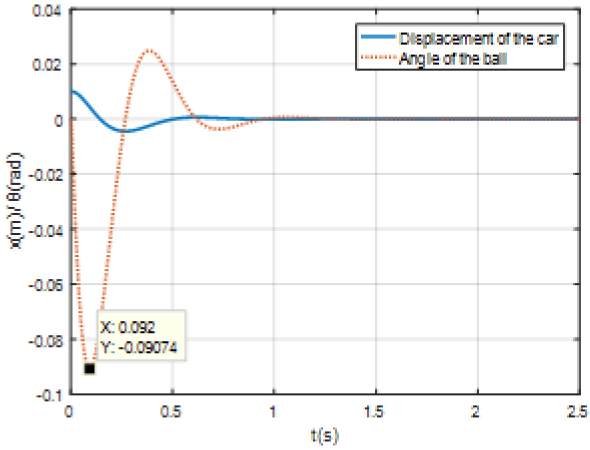


Figure 4: The system state under the action of the controller

5. Conclusion

The dynamic model of TORA system is established using Lagrange method. Based on the model, the controllability of the system is analyzed, and LQR regulator is designed based on the optimal control. The stability simulation experiment verifies the effectiveness of the above control scheme. LQR controller is used to control the system effectively, and the control task is completed in a short time, and the system jitter within the stable interval is reduced. This control method requires fast response of control input and high demand for controller. Adjusting the parameters of the controller can effectively regulate the stability time of the system to meet the requirements of the controller with different sensitivity.

References

[1] WAN C J, DENNIS S B, VINCENT T C. “Global stabilization of the oscillating eccentric rotor.” In Proceedings of the 33rd IEEE Control Decision and Control, pp. 4024-4029, 1994.

[2] AVIS J M, NERESESOV S G, NATHAN R. “Decentralized energy-based hybrid control for the multi-TORA system.” In 2009 IEEE American Control Conference, pp. 895-900, 2009.

[3] GAO BingtuanM, “Dynamical Modeling and Energy-based Control Design for TORA” Acta Automatica Sinica,34(9), pp. 1221-1224, 2008.

[4] GAO Bingtuan, JIA Zhiyong, CHEN Hongjun, et al. "Dynamical modeling and backstepping control of TORA ," Control and Decision, 22(11), pp. 1284-1288, 2007.
[5] SUN Shizhuo, "Research on motion control of the underacturated TORA system," Harbin: Harbin Institute of Technology, 2013.
[6] ZHANG Yu,"Motion control of translational oscillator with rotating actuator,"Dalian: Dalian University of Technology, 2016.
[7] GUAN Tingting, "Output feedback of the nonlinear system based on adaptive neural network,"Harbin: Harbin University of Science and Technology, 2011.
[8] ZHANG Jing, SUN Qiang, GUAN Tingting, et al, "Output feedback control of the TORA system based on adaptive neural network," Journal of Harbin University of Science and Technology,20(6), pp. 83-87, 2015.

Author Profile

HE Tongzhao is a postgraduate student in Tianjin University of Technology and Education. The main research areas are system modeling and intelligent control.