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Public Tripled Coincidence Fixed Point Theorems via Contraction Mappings

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Abstract: ***The purpose of this paper is to introduce a concepts of the public tripled fixed point , public tripled coincidence fixed point and public mixed B_ monotone property. This concepts are more general than that of tripled fixed point, tripled coincidence fixed point and mixed g _monotone property. Also, we prove the existence and uniqueness of public coincidence fixed point for continuous and public commuting mappings in the partially ordered metric spaces.***

Keywords: tripled fixed point, tripled coincidence fixed point, mixed g_ monotone property

1. Introduction

The fixed point theorems in partially ordered metric space are studied by many researches, see [1-24]. In 2006, Bhaskr and Lakhikntham [4]. Introduced the concept mixed monotone property for contractive mappings they also established coupled fixed point results for mapping has mixed monotone property. On other hand, Sintuavrate et al. [25], [26] proved the existence and uniqueness-of coupled fixed point theorem for non linear contractions without the9 mixed monotone property. Also, Lakshmkatham and crict [11] introduced the concept of mixed monotone and proved some results for coupled coincidence fixed point and coupled common fixed0point for commuting mappings, this results extend of results Bhaskar and Lakshmikntham [4]. Additionally, Chodhry and Kudu[5], introduced the compatiblity of mappings in partially ordered metric space and they established a coupled coincidence point theorems. Bernde and Bocut [27], [28] introduced the concept of tripled7 fixed point and6 tripled9coincidencedofixed point, they extend the results of Bhskar and Lakshmikntham [4]and Ciric and Lakshmikanthm [11] to the 9tripled fixed point and coincidence fixed point.

Now, we recall the flowing definitions:

Definition (1.1): [29]
A *set* W with a binary operation $\preccurlyeq$ is called partially ordered set if for all $,f,g \in W$.

i. $s \preccurlyeq z$
ii. $s \preccurlyeq z$ and. $z \preccurlyeq s \Rightarrow z = s$
iii. $s \preccurlyeq z$ *and* $z \preccurlyeq e \Rightarrow s \preccurlyeq e$.

Definition (1.2): [27]
Let $A: W^3 \to W$ be a mapping then any point $(e,f,g) \in W^3$ is called tripled fixed point of A if:
$e = A_{(e,f,g)}$, $f = A_{(f,e,g)}$ and $g = A_{(g,f,e)}$.

Definition (1.3): [28]
Let $A: W^3 \to W$ *and* $B: W \to W$ be two mappings. Any point (e,f,g) is called tripled coincidence point of A *and* B: if,
$B_{(e)} = A_{((e,f,g))}$, $B_{(f)} = A_{(f,e,f)}$ and $B_{(g)} = A_{(g,f,e)}$

Definition (1.4): [28]
Let $A: W^3 \to W$ *and* $B: W \to W$ be two mapping and $(W, \preccurlyeq)$ be a parlially ordered set, then we say that A has mixed B – monotone property if A is monotone increasing in e *and* g and is monotone decreasing in f, i.e, $\forall\, e,f,g \in W$

$e_1, e_2 \in W, B_{(e_1)} \preccurlyeq B_{(e_2)} \Rightarrow A_{(e_1,f,g)} \preccurlyeq A_{(e_2,f,g)}$

$f_1, f_2 \in W, B_{(f_1)} \preccurlyeq B_{(f_2)} \Rightarrow A2_{(e,f_1,g)} \preccurlyeq A_{(e,f_2,g)}$
And, $g_1, g_2 \in W$, $B_{(g_1)} \preccurlyeq B_{(g_2)} \Rightarrow A_{(e,f,g_1)} \preccurlyeq A_{(e,f,g_2)}$.

2. Main Results

Now , we will give the new concepts.

Definition (2.1):
Let W be a nonempty set. Then we say that the mappings $A_1, A_2, \dots\dots\dots A_n: W^3 \to W$ and $B: W \to W$ are public commuting if for each $e,f,g \in W$,

$$\left(B\left(A_1\left(A_2\left(\dots\dots\left(A_{n\,(e,f,g)}\right)\dots\dots\right)\right)\right)\right)$$
$$=\left(A_1\left(A_1\left(A_2\left(\dots\dots\left(A_{n\,(e,f,g)}\right)\dots\dots\right)\right)\right)\right)$$

Definition (2.2):
Let W be a nonempty and $A, A_2, \dots, A_2: W^3 \to W$ be a mapping. Any point $(e,f,g) \in W^3$ is called a public tripled fixed point of A_1, $A_2, \dots\dots, A_n$ if

$$A\left(\left(A_2\left(\dots\dots\left(A_{n\,(e,f,g)}\right)\dots\dots\right)\right)\right)$$
$$= e, A_1\left(\left(A_2\left(\dots\dots\left(A_{n\,(f,e,g)}\right)\dots\dots\right)\right)\right)$$
$$= f$$
$$A_1\left(\left(A_2\left(\dots\dots\left(A_{n\,(g,f,e)}\right)\dots\dots\right)\right)\right) = g$$

Definition (2.3):
Let $(W, \preccurlyeq)$ be apartially ordered set and $A_1, A_2, \dots\dots, A_n: W^3 \to W$ be a mapping. We say that $A_1, A_2, \dots\dots, A_n$ are public mixed monotone if $A_1\left(.A_2\left(\dots\dots\left(A_{n\,(e,f,g)}\right)\dots\dots\right)\right)$ is monotone increasing in e *and* g and is monotone decreasing in f, i.e, $\forall\, e,f,g \in W$,

$e_1, e_2 \in W, e_1 \preccurlyeq e_2 \Rightarrow$

$A_1\left(\left(A_2\left(\ldots\left(A_{n(e_1,f,g)}\right)\ldots\right)\right)\right) \preccurlyeq$

$A_1\left(\left(A_2\left(\ldots\left(A_{n(e_2,f,g)}\right)\ldots\right)\right)\right)$

$f_1, f_2 \in W, f_1 \preccurlyeq f_2 \Rightarrow$

$A_1\left(\left(A_2\left(\ldots\left(A_{n(e,f_1,g)}\right)\ldots\right)\right)\right) \succcurlyeq A\left(\left(A_2\left(\ldots\left(A_{n(e,f_2,g)}\right)\ldots\right)\right)\right)$

and

$g_1, g_2 \in W, g_1 \preccurlyeq g_2 \Rightarrow$

$A_1\left(\left(A_2\left(\ldots\left(A_{n(e,f,g_1)}\right)\ldots\right)\right)\right) \preccurlyeq$

$A_1\left(\left(A_2\left(\ldots\left(A_{n(e,f,g_2)}\right)\ldots\right)\right)\right)$

Definition (2.4):

Let W be a nonempty set. If $A_1, A_2, \ldots\ldots, A_3 : W^3 \to W$ and $B: W \to W$ are mappings then any point $(e,f,g) \in W^3$ is called public tripled coincidence point of $A_1, A_2, \ldots, A_n$ and B if

$A_1\left(\left(A_2\left(\ldots\left(A_{n(e,f,g)}\right)\ldots\right)\right)\right) = B(e),$

$A_1\left(\left(A_2\left(\ldots\left(A_{n(f,e,f)}\right)\ldots\right)\right)\right) = B(f)$

$A_1\left(\left(A_2\left(\ldots\left(A_{n(g,f,e)}\right)\ldots\right)\right)\right) = B(g)$

Definition (2.5):

Let $(W, \preccurlyeq)$ be a partially ordered set and $A_1, A_2, \ldots\ldots, A_n : W^3 \to W$ and $B: W \to W$ are mappings, then we say that $A_1, A_2, \ldots\ldots, A_n$ are public mixed B_ monotone property, If

$A_1\left(\left(A_2\left(\ldots\left(A_{n(e,f,g)}\right)\ldots\right)\right)\right)$ is monotone increasing in e and g and is monotone decreasing in f; i.e, for all $e, f, g \in W$

$e_1, e_2 \in W, g(e_1) \preccurlyeq g(e_2) \Rightarrow$

$A_1\left(\left(A_2\left(\ldots\ldots\left(A_{n(e_1,f,g)}\right)\ldots\ldots\right)\right)\right) \preccurlyeq$

$A_1\left(\left(A_2\left(\ldots\ldots\left(A_{n(e_2,f,g)}\right)\ldots\ldots\right)\right)\right)$

$A_1, A_2 \in W, B(A_1) \preccurlyeq B(A_2) \Rightarrow$

$A_1\left(\left(A\left(\ldots\ldots\left(A_{n(e,f_1,g)}\right)\ldots\ldots\right)\right)\right) \succcurlyeq$

$A_1\left(\left(A_2\left(\ldots\ldots\left(A_{n(e,f_2,g)}\right)\ldots\ldots\right)\right)\right)$

Also, $g_1, g_2 \in W, B(g_1) \preccurlyeq B(g_2) \Rightarrow$

$A_1\left(\left(A_2\left(\ldots\ldots\left(A_{n(e,f,g_1)}\right)\ldots\ldots\right)\right)\right)$

$\succcurlyeq A_1\left(\left(A_2\left(\ldots\ldots\left(A_{n(e,f,g_2)}\right)\ldots\ldots\right)\right)\right).$

Now , Let Γ be the set of all increasing mappings such that:

$\Delta_i : [0,\infty] \to [0,\infty]$ such that $\Delta_{i(t)} = \begin{cases} t & if\ t > 0 \\ 0 & if\ t < 0 \end{cases}$

And $\lim_{n\to\infty} \Delta_i^{\ n}(t) = 0 \ \forall\, i = 1, \ldots, 7$, where Δ^n denotes the n – th iterate of Δ. And let C be the set of all mappings $A_1, A_2, \ldots\ldots, A_3 : W^3 \to W$ and $B_1, B_2, \ldots\ldots, B: W \to W$ such that

i. $B(W)$ is complete of W containing $A_1\left(A_2\left(\ldots\ldots\left(A_n(W \times W \times W \ldots\ldots\right.\right.\right.$

ii. $A_1, A_2, \ldots\ldots, A_n$ and B are continuous and public commute mappings .

Theorem (2.6):

Let $(W, d, \preccurlyeq)$ be a partially ordered metric space. If $A_1, A_2, \ldots\ldots, A_n : W^3 \to W$ and $B: W \to W$ are mappings lies in C such that $A_1, A_2, \ldots\ldots, A_n$ having public mixed B_monotone property. Suppose that. For all $e, f, g, u, v, w \in W$ with $e \succcurlyeq u, f \preccurlyeq v$ and $g \succcurlyeq w$,

$d\left(A_1\left(\left(A_2\left(\ldots\ldots\left(A_{n(e,f,g)}\right)\ldots\ldots\right)\right)\right),\right.$

$\left.A_1\left(\left(A_2\left(\ldots\ldots\left(A_{n(u,v,w)}\right)\ldots\ldots\right)\right)\right)\right) \preccurlyeq$

$\max\{\Delta_1 d(B(e), B(u)), \Delta_2 d(B(f), B(v))$

$, \Delta_3 d(B(g), B(w)),$

$\Delta_4 d\left(A_1\left(\left(A_2\left(\ldots\ldots\left(A_{n(e,f,g)}\right)\ldots\ldots\right)\right)\right), B(e)\right),$

$\Delta_5 d\left(A_1\left(\left(A_2\left(\ldots\ldots\left(A_{n(g,f,e)}\right)\ldots\ldots\right)\right)\right), B(g)\right),$

$\Delta_6 d\left(A_1\left(\left(A_2\left(\ldots\ldots\left(A_{n(u,v,w)}\right)\ldots\ldots\right)\right)\right), B(u)\right),$

$\Delta_7 d\left(A_1\left(\left(A_2\left(\ldots\ldots\left(A_{n(w,v,u)}\right)\ldots\ldots\right)\right)\right), B(w)\right)\}$

Where $\Delta_1, \Delta_2, \ldots\ldots, \Delta_7 \in \Gamma$. If there exist $e_o, f_o, w_o \in W$ such that

$B(e_o) \preccurlyeq A_1\left(\left(A_2\left(\ldots\ldots\left(A_{n(e_o,f_o,g_o)}\right)\ldots\ldots\right)\right)\right)$

$B(f_o) \succcurlyeq A_1\left(\left(A_2\left(\ldots\ldots\left(A_{n(f_o,e_o,f_o)}\right)\ldots\ldots\right)\right)\right)$ and

(2.1)

$B(z_o) \preccurlyeq A_1\left(\left(A_2\left(\ldots\ldots\left(A_{n(g_o,f_o,e_o)}\right)\ldots\ldots\right)\right)\right)$

Then, $A_1, A_2, \ldots\ldots, A_n$ and B having public tripled coincidence point.

Proof:

Consider $e_o, f_o, g_o \in W$ satisfy (2.1).We can construct sequences as,

Define: $B(e_1) \preccurlyeq A\left(\left(A_2\left(\ldots\ldots\left(A_{n(e_\circ,f_\circ,g_\circ)}\right)\ldots\ldots\right)\right)\right)$

$B(f_1) \preccurlyeq A_1\left(\left(A_2\left(\ldots\ldots\left(A_{n(f_\circ,e_\circ,f_\circ)}\right)\ldots\ldots\right)\right)\right)$

$B(g_1) \preccurlyeq A_1\left(\left(A_2\left(\ldots\ldots\left(A_{n(g_\circ,f_\circ,e_\circ)}\right)\ldots\ldots\right)\right)\right)$

And hence, we get:

$B(e_0) \preccurlyeq B(e_1)$

$B(f_0) \succcurlyeq B(f_1)$

$B(g_0) \preccurlyeq B(g_1)$

As the same way define

$B(e_2) = A_1\left(\left(A_2\left(\ldots\ldots\left(A_{n(e_1,f,g_1)}\right)\ldots\ldots\right)\right)\right)$

$B(f_2) = A_1\left(\left(A_2\left(\ldots\ldots\left(A_{n(f_1,e_1,f_1)}\right)\ldots\ldots\right)\right)\right)$

$B(g_2) = A_1\left(\left(A_2\left(\ldots\ldots\left(A_{n(g_1,f_1,e_1)}\right)\ldots\ldots\right)\right)\right)$

But $A_1, A_2, \ldots\ldots, A_n$ having public B_mixed monotone property.Then, we get

$B(e_0) \preccurlyeq B(e_1) \preccurlyeq B(e_2)$

$B(f_0) \succcurlyeq B(f_1) \succcurlyeq B(f_2)$

$B(g_0) \preccurlyeq B(g_1) \preccurlyeq B(g_2)$

We continue operations where we get the sequences.

$< B(e_n) >, < B(f_n) >$ and $< B(g_n) >$ in $B(W)$ and satisfy the following

$B(e_n) = A_1\left(A_2\left(\ldots\ldots\left(A_{n(e_{n-1},f_{n-1},g_{n-1})}\right)\ldots\ldots\right)\right),$

$\preccurlyeq B(e_{n+1}) = A_1\left(\left(A_2\left(\ldots\ldots\left(A_{n(e_n,f_n,g_n)}\right)\ldots\ldots\right)\right)\right)$

$B(f_{n+1}) = A_1\left(\left(A_2\left(\dots\dots\left(A_{n(f_n,e_n,f_n)}\right)\dots\dots\right)\right)\right),$

$\preccurlyeq B(f_n) = A_1\left(\left(A_2\left(\dots\dots\left(A_{n(f_{n-1},e_{n-1},f_{n-1})}\right)\dots\dots\right)\right)\right)$

$B(g_n) = A_1\left(\left(A_2\left(\dots\dots\left(A_{n(g_{n-1},f_{n-1},e_{n-1})}\right)\dots\dots\right)\right)\right),$

$\preccurlyeq B(g_{n+1}) = A_1\left(\left(A_2\left(\dots\dots\left(A_{n(g_n,f_n,e_n)}\right)\dots\dots\right)\right)\right)$

We will take two cases during the proof

Case (1):

If, $(B(e_{n+1}), B(f_{n+1}), B(g_{n+1})) = (B(e_n), B(f_n), B(g_n))$ For some $n \in \mathbb{N}$,then$A_1\left(A_2\left(\dots\dots\left(A_{n(e_n,f_n,g_n)}\right)\dots\dots\right)\right) = B(e_n)$

$A_1\left(\left(A_2\left(\dots\dots\left(A_{n\,(f_n,e_n,f_n)}\right)\dots\dots\right)\right)\right) = B(f_n)$and

$A_1\left(\left(A_2\left(\dots\dots\left(A_{n(g_n,f_n,e_n)}\right)\dots\dots\right)\right)\right) = B(g_n)$

Hence, $(e_n, f_n, g_n)$is a public tripled coincidence point of $A_1, A_2, \dots\dots, A_n$ $and B.$

Case (2):

If $(B(e_{n+1}), B(f_{n+1}), B(g_{n+1})) \neq (B(e_n), B(f_n), B(g_n))$,then $\forall\, n \in N$,either $B(e_n) \neq B(e_{n+1})$ or $B(f_n) \neq B(f_{n+1})$ or $B(g_n) \neq B(e_{n+1})$

Now,

$d(B(e_{n+1}), B(e_n))$

$$= d\begin{pmatrix} A_1\left(A_2\left(\dots\dots\left(A_{n(e_n,f_n,g_n)}\right)\dots\dots\right)\right), \\ A_1\left(A_2\left(\dots\dots\left(A_{n(e_{n-1},f_{n-1},g_{n-1})}\right)\dots\dots\right)\right)\end{pmatrix}$$

$$\leq \max\{\Delta_1\, d(\; B(e_n),\; B(e_{n-1})), \Delta_2\, d(B(f_n), B(f_{n-1})), \Delta_3\, d(\; B(g_n),\; B(g_{n-1})), \Delta_4\, d\left(A_1\left(A_2\left(\dots\dots\left(A_{n(e_n,f_n,g_n)}\right)\dots\dots\right)\right), B(e_n)\right),$$
$$\Delta_7\, d\left(A_1\left(A_2\left(\dots\dots\left(A_{n(g_n,f_n,e_n)}\right)\dots\dots\right)\right), B(g_n)\right),$$
$$\Delta_6\, d\left(A_1\left(A_2\left(\dots\dots\left(A_{n(e_{n-1},f_{n-1},g_{n-1})}\right)\dots\dots\right)\right), B(e_{n-1})\right),$$
$$\Delta_7\, d\left(A_1\left(A_2\left(\dots\dots\left(A_{n(g_{n-1},f_{n-1},e_{n-1})}\right)\dots\dots\right)\right), B(g_{n-1})\right)\}$$

$$= \max\{\Delta_1\, d\left(B_1\left(B_2\left(\dots\dots\left(B_{n(e_n)}\right)\dots\dots\right)\right), B(e_{n-1})\right),$$
$$\Delta_2\, d(B(f_n), B(f_{n-1})), \Delta_3\, d(B(g_n), B(g_{n-1})), \Delta_4\, d(B(e_{n+1}), B(e))$$
$$\Delta_5\, d(\; B(g_{n+1}), B(g_n)), \Delta_6\, d(\; B(e_n), B(e_{n-1})), \Delta_7\, d(B(g_n), B(g_{n-1}))\}$$

Let$h_{1(t)} = \max\{\Delta_{1(t)}, \Delta_{6(t)}\}$, $h_{2(t)} = \max\{\Delta_{3(t)}, \Delta_{7(t)}\}$,and$\Delta_{(t)} \in \Gamma$

$$= \max\{h_1\, d(B(e_n), B(e_{n-1})), \Delta_2\, B(g(f_n), B(f_{n-1})), h_2 d(B(g), B(g_{n-1})), \Delta_4\, d(B(e_{n+1}),\; B(e_n)), \Delta_5\, d(B(g_{n+1}), B(g_n))\}$$

$$\leq \max\{h_1\, d(B(e_n), B(e_{n-1})), \Delta_2\, d(\; B(f_n), B(f_{n-1})), h_2 d(B(g_n), B\;\;(g_{n-1})), \Delta_4\, d(B(e_{n+1}), B(e_n)), \Delta\, d(B(f_{n+1}), B(f_n)), \Delta_5\, d(B(g_{n+1}), B(g_n))\}$$

Now,

$$d(\; B(f_n), B(f_{n+1}))$$
$$= d\begin{pmatrix} A_1\left(A_2\left(\dots\dots\left(A_{n(f_{n-1},e_{n-1},f_{n-1})}\right)\dots\dots\right)\right), \\ A_1\left(A_2\left(\dots\dots\left(A_{n(f_n,e_n,f_n)}\right)\dots\dots\right)\right)\end{pmatrix}$$

$$\leq \max\{h_3 d(\; B(f_{n-1}), B(f_n)), \Delta_2\, d(\; B(e_{n-1}), B(e_n)),$$
$$h_4 d\left(A_1\left(A_2\left(\dots\dots\left(A_{n(f_{n-1},e_{n-1},f_{n-1})}\right)\dots\dots\right)\right),\;\; B(f_{n-1})\right),$$
$$h_5 d\left(A_1\left(A_2\left(\dots\dots\left(A_{n(f_n,e_n,f_n)}\right)\dots\dots\right)\right), B(f_n)\right)\}$$

where$h_{3(t)} = \max\{\Delta_{1(t)}, \Delta_{3(t)}\}$, $h_{4(t)} = \max\{\Delta_{4(t)}, \Delta_{5(t)}\}$, $h_{5(t)} = \max\{\Delta_{6(t)}, \Delta_{7(t)}\}$, $h_{6(t)} = \max\{h_{3(t)}, h_{4(t)}\}$ and $\Delta_{(t)} \in \Gamma = \max\{h_6 d(B(f_{n-1}), B(f_n)),$ $\Delta_2\, d(\; B(e_{n-1}), B(e_n)), h_5 d(\; B(f_{n+1}), B(f_n))\}$

$$\leq \max\{\Delta_2\, d(B(z_{n-1}),\;\; B(z_n)), h_6 d(B(f_{n-1}), B(f_n)), \Delta\, d(B(g_{n-1}), B\;\;(g_n)), \Delta\, d(B(e_{n+1}), B(e_n)), h_5 d(\; B(f_{n+1}), B(f_n)), \Delta\, d(B(g_{n+1}), B(g_n))\}$$

Also we have:

$d(B(g_{n+1}), B(g_n))$

$$= d\begin{pmatrix} A_1\left(A_2\left(\dots\dots\left(A_{n(g_n,f_n,e_n)}\right)\dots\dots\right)\right), \\ A_1\left(A_2\left(\dots\dots\left(A_{n(g_{n-1},f_{n-1},e_{n-1})}\right)\dots\dots\right)\right)\end{pmatrix}$$

$$\leq \max\{\Delta_1\, d(\; B(g_n), B(g_{n-1})), \Delta_2\, d(\; B(f_n), B(f_{n-1})), \Delta_3\, d(\; B(e_n), B(e_{n-1})), \Delta_4\, d\left(A\left(A_2\left(\dots\dots\left(A_{n(g_n,f_n,e_n)}\right)\dots\dots\right)\right), B(g_n)\right),$$
$$\Delta_5\, d\left(\; A_1\left(A_2\left(\dots\dots\left(A_{n(e_n,f_n,g_n)}\right)\dots\dots\right)\right), B(e_n)\right),$$
$$\Delta_6\, d\begin{pmatrix} A_1\left(A_2\left(\dots\dots\left(A_{n(g_{n-1},f_{n-1},e_{n-1})}\right)\dots\dots\right)\right), \\ B(g_{n-1})\end{pmatrix},$$
$$\Delta_7\, d\begin{pmatrix} A_1\left(A_2\left(\dots\dots\left(A_{n(e_{n-1},f_{n-1},g_{n-1})}\right)\dots\dots\right)\right), \\ B(e_{n-1})\end{pmatrix}\}.$$

$$= \max\{\Delta_1\, d(\; B(g_n), B(g_{n-1})), \Delta_2\, d(B(f_n), B(f_{n-1})), \Delta_3\, d(B(e_n), B(e_{n-1})), \Delta_4\, d(B(g_{n+1}), B(g_n)), \Delta_5\, d(B(e_{n+1}), B(e_n)), \Delta_6\, d(B(g_n), B(g_{n-1})), \Delta_7\, d(B(e_n), B(e_{n-1}))\}.$$

$$\leq \max\{h_7 d(B(g_n), B(g_{n-1})), \Delta_2\, d(\; B(g_n\;), B(g_{n-1})), h_8 d(B(e_n), g(e_{n-1})), \Delta_4\, d(B(g_{n+1}), B(g_n)), \Delta_5\, d(B(e_{n+1}), B(e_n))\}$$

where,$h_{7(t)} = \max\{\Delta_{1(t)}, \Delta_{6(t)}\}$

$h_{8(t)} = \max\{\Delta_{3(t)}, \Delta_{7(t)}\}$

$$\leq \max\{h_7 d(B\;\;(g_n), B(g_{n-1})), \Delta_2\, d(B(f_n), B(f_{n-1})), h_8 d(B(e_n),\;\; B(\;\; e_{n-1})), \Delta_4\, d(B(g_{n+1}), g(z_n)), \emptyset d(B(f_{n+1}), B(f_n)), \Delta_5\, d(B(\;\; e_{n+1}), B_{(e_n)})\}$$

Let $\varphi_{(t)} = \max\{\Delta_{(t)}, \Delta_{1(t)}, \dots\dots, \Delta_{7(t)}, h_{1(t)}, \dots\dots, h_{8(t)}\}$, then we have

$\max\{d(B(e_{n+1}), B(e_n)), d(B(f_{n+1}), B(f_n)), d(B(g_{n+1}), B(g_n))\}$

$$\leq \max\{\varphi d(B(\;\; e_n), B(e_{n-1})), \varphi d(B\;(f_n), B(f_{n-1})), \varphi d(B(g_n), B(g_{n-1})), \varphi d(B(e_{n+1}), B(e_n)), \varphi d(B(f_{n+1}), B(f_n)), \varphi d(B(g_{n+1}), B(g_n))\} \quad (2.2)$$

$$< \max\{d(\;\; B(e_n), B(e_{n-1})), B\;\;(g(f_n), (f_{n-1})), d(B(g_n), g(g_{n-1})), d(B(e_{n+1}), B(e_n)), d(B(f_{n+1}), B(f_n)), d(B(g_{n+1}), B(g_n))\}$$

This leads

$$\max\{d(\;\; B(e_{n+1}), B(e_n)), d(B(f_{n+1}), B(f_n)), d(B(g_{n+1}), B(g_n))\} < \max\{d(B(e_n), B(e_{n-1})), d(\;\; B(f_n), B(f_{n-1})), d(B(g_n), B(g_{n-1}))\}$$

And hence, the equation (2.2) become

$\max\{d(B(e_{n+1}), B(e_n)), d(B(f_{n+1}), B(f_n)), d(B(g_{n+1}), B(g_n))\}$

$$\leq \max\{\varphi[\;\; B(g(e_n), B(e_{n-1})), d(B(f_n), B(f_{n-1})), d(\;\; B(g_n), B(g_{n-1}))]\}$$
$$\leq \max\{\varphi^2[d(B(e_{n-1}), B(e_{n-2})),$$

$$d\big(B(f_{n-1}),B(f_{n-2})\big),d\big(B(g_{n-1}),B(g_{n-2})\big)]\}$$
$$\vdots$$
$$\leq \max\{\varphi^n[d\big(B(e_1),B(e_o)\big),d\big(B(f_1),B(f_o)\big),d\big(B(g_1),B(g_o)\big)]\}$$

But $\lim_{n\to\infty}\max\varphi^n[d\big(B(e_1),B(e_0)\big),d\big(B(f_1),B(f_0)\big),d\big(B(g_1),B(g_o)\big)]=0$

Then, $\forall\,\epsilon>0$; $\varphi_{(\epsilon)}<\epsilon$, $\exists\, n_\circ\in N$ such that:

$$\varphi^n\{d\big(B(e_1),B(e_\circ)\big),d\big(B(f_1),B(f_o)\big),d\big(B(g_1),B(g_o)\big)\}<\epsilon-\varphi_{(\epsilon)}\quad\forall\, n\geq n_\circ$$

$$\max\{d\big(B(e_{n+1}),B(e_n)\big),d\big(B(f_{n+1}),B(f_n)\big),d\big(B(g_{n+1}),B(g_n)\big)\}<\epsilon-\varphi_{(\epsilon)}\qquad(2.3)$$

Now, To prove that, $\forall\, m\geq n\geq n_\circ$

$$\max\left\{\begin{matrix}d\big(B(e_n),B(e_m)\big),d\big(B(f_n),B(f_m)\big)\\ ,d\big(B(g_n),B(g_m)\big)\end{matrix}\right\}<\epsilon\qquad(2.4)$$

We will discuss the Cauchy sequence,

i. For $m=n+1$ and by using (2.3) we get (2.4).

ii. Suppose it is if $m=k$, i.e.

$$\max\{d\big(B(e_n),B(e_k)\big),d\big(B(f_n),B(f_k)\big),d\big(B(g_n),B(g_k)\big)\}<\epsilon$$

iii. Now, to prove it is true when $m=k+1$

$$\begin{aligned}d\big(B(e_n),B(e_{k+1})\big)&\leq d\big(B(e_n),B(e_{n+1})\big)+d\big(B(e_{n+1}),B(e_{k+1})\big)\\&<\epsilon-\varphi_{(\epsilon)}+d\begin{pmatrix}A_1\Big(A_2\big(\ldots\ldots\big(A_{n(e_n,f_n,g_n)}\big)\ldots\ldots\big)\Big)\\ ,A_1\Big(A\big(\ldots\ldots\big(A_{n(e_k,f_k,g_k)}\big)\ldots\ldots\big)\Big)\end{pmatrix}\end{aligned}$$

$$\leq\epsilon-\varphi_{(\epsilon)}+\max\{\triangle_1\, d\big(B(e_n),B(e_k)\big),\triangle_2\, d\big(B(f_n),B(f_k)\big),\triangle_3\, d\big(\triangle(g_n),\triangle(g_k)\big),$$
$$\triangle_4\, d\Big(A_1\Big(A_2\big(\ldots\ldots\big(A_{n(e_n,f_n,g_n)}\big)\ldots\ldots\big)\Big),B(e_n)\Big),$$
$$\triangle_5\, d\Big(A_1\Big(A_2\big(\ldots\ldots\big(A_{n(g_n,f_n,e_n)}\big)\ldots\ldots\big)\Big),B(g_n)\Big),$$
$$\triangle_6\, d\Big(A\Big(A_2\big(\ldots\ldots\big(A_{n(e_k,f_k,g_k)}\big)\ldots\ldots\big)\Big),B(e_n)\Big),$$
$$\triangle_7\, d\Big(A_1\Big(A_2\big(\ldots\ldots\big(A_{n(g_k,f_k,e_k)}\big)\ldots\ldots\big)\Big),B(g_n)\Big)\}$$

$$\leq\epsilon-\varphi_{(\epsilon)}+\max\{\varphi d\big(B(e_n),B(e_k)\big),\varphi d\big(B(f_n),B(f_k)\big),\varphi d\big(B(g_n),B(g_k)\big),\varphi d\big(B(e_{n+1}),B(e_n)\big),\varphi d\big(B(g_{n+1}),B(g_n)\big),\varphi d\big(B(e_{k+1}),B(e_k)\big),\varphi d\big(B(g_{k+1}),B(g_k)\big)\}$$

$$\leq\epsilon-\varphi_{(\epsilon)}+\varphi\max\{d\big(B(e_n),B(e_k)\big),d\big(B(f_n),B(f_k)\big),d\big(B(g_n),B(g_k)\big),d\big(B(e_{n+1}),B(e_n)\big),\big(B(g_{n+1}),B(g_n)\big),d\big(B(e_{k+1}),B(e_k)\big),d\big(B(g_{k+1}),B(g_k)\big)\}$$

$$\leq\epsilon-\varphi_{(\epsilon)}+\varphi\max\{d\big(B(e_n),B(e_k)\big),d\big(B(f_n),B(f_k)\big),d\big(B(g_n),B(g_k)\big),d\big(B(e_{n+1}),B(e_n)\big),d\big(B(f_{n+1}),B(f_n)\big),d\big(B(g_{n+1}),B(g_n)\big),d\big(B(e_{k+1}),B(e_k)\big),d\big(B(f_{k+1}),B(f_k)\big),d\big(B(g_{k+1}),B(g_k)\big)\}$$

$<\epsilon-\varphi_{(\epsilon)}+\varphi\max\{\epsilon,\epsilon-\varphi_{(\epsilon)}\}$ by (i) and (ii)

$<\epsilon-\varphi_{(\epsilon)}+\varphi_{(\epsilon)}=\epsilon$

This leads, $d\big(B(e_n),B(e_{k+1})\big)<\epsilon$

As the same way, we get

$$d\big(B(f_n),B(f_{k+1})\big)<\epsilon$$

and $d\big(B(g_n),B(g_{k+1})\big)<\epsilon$

Hence,

$$\max\{d\big(B(e_n),B(e_{k+1})\big),d\big(B(f_n),B(f_{k+1})\big),d\big(B(g_n),B(g_{k+1})\big)\}<\epsilon.$$

For all $m\geq n$, (iii) holds, therefor $<B(e_n)>,<B(f_n)>$ and $<B(g_n)>$ are Cauchy sequences in $g(W)$ which is complete. Therefore, there exists $L_1,L_2,L_3\in g(W)$ such that: $B(e_n)\to L_1$, $B(f_n)\to L_2$ and $B(g_n)\to L_3$

When, $L_1=B(e)$, $L_2=B(f)$ and $L_3=B(g)$ for some $e,f,g\in W$

Now to prove that (L_1,L_2,L_3) is public tripled coincidence point

Since $A_1,A_2,\ldots\ldots,A_n$ and B lies in C, we have

$$B\big(B(e_{n+1})\big)=B\Big(A_1\Big(A_2\big(\ldots\ldots\big(A_{n(e_n,f_n,g_n)}\big)\ldots\ldots\big)\Big)\Big)$$

Which is converge to $A_1\Big(A_2\big(\ldots\ldots\big(A_n(L_1,L_2,L_3)\big)\ldots\ldots\big)\Big)$

Now since $B(e_n)\to L_1\Rightarrow B\big(B(e_n)\big)\to B(L_1)$

But the limit point is unique:

$$B(L_1)=A_1\Big(A_2\big(\ldots\ldots\big(A\ (L_1,L_2,L_3)\big)\ldots\ldots\big)\Big)$$

As the same way, we get:

$$B(L_2)=A_1\Big(A_2\big(\ldots\ldots\big(A_n(L_2,L_1,L_2)\big)\ldots\ldots\big)\Big)$$

$$B(L_3)=A_1\Big(A_2\big(\ldots\ldots\big(A_n\,(L_3,L_2,L_1)\big)\ldots\ldots\big)\Big).$$

Therefore, the point (L_1,L_2,L_3) is public tripled coincidence point of $A_1,A_2,\ldots\ldots,A_n$ and B.

Corollary(2.7)

Let $(W,d,\preccurlyeq)$ be a partially ordered metric space. If $A_1,A_2,\ldots\ldots,A_n:W^3\to W$ and $B:W\to W$ are mappings lies in C such that $A_1,A_2,\ldots\ldots,A_n$ having public mixed g_monotone property. Suppose that, for all $x,y,z,u,v,w\in X$ with $x\preccurlyeq u, y\preccurlyeq v$ and $z\succcurlyeq w$,

$$d\Big(A_1\Big(A_2\big(\ldots\ldots\big(A_{n(x,y,z)}\big)\ldots\ldots\big)\Big),A_1\Big(A_2\big(\ldots\ldots\big(A_{n(u,v,w)}\big)\ldots\ldots\big)\Big)\Big)\preccurlyeq$$
$$\max\{k_1d\big(B(x),B(u)\big),k_2d\big(B(y),B(v)\big),k_3d\big(B(z),B(w)\big),$$
$$k_4d\Big(A_1\Big(A_2\big(\ldots\ldots\big(A\ _{n(x,y,z)}\big)\ldots\ldots\big)\Big),B(x)\Big),$$
$$k_5d\Big(A_1\Big(A_2\big(\ldots\ \ldots\big(A_{n(z,y,x)}\big)\ldots\ldots\big)\Big),B(z)\Big)$$
$$k_6d\Big(A_1\Big(A_2\big(\ldots\ldots\big(A_{n(u,v,w)}\big)\ldots\ldots\big)\Big),B(u)\Big),$$
$$k_7d\Big(A_1\Big(A_2\big(\ldots\ldots\big(A_{n(w,v,u)}\big)\ldots\ldots\big)\Big),B(w)\Big)\}$$

where $k_1,k_2,\ldots\ldots,k_7\in[0,1)$ and $\sum_{i=1}^{7}k_i<1$

If there exist $x_\circ,y_\circ,z_\circ\in X$, such that

$$B(x_\circ)\leq A_1\Big(A_2\big(\ldots\ldots\big(A_{n(x_\circ,y_\circ,z_\circ)}\big)\ldots\ldots\big)\Big)$$

$$B(y_\circ)\geq A_1\Big(A_2\big(\ldots\ldots\big(A_{n(y_\circ,x_\circ,y_\circ)}\big)\ldots\ldots\big)\Big)\text{ and}$$

$$B(z_\circ)\leq A_1\Big(A_2\big(\ldots\ldots\big(A_{n(z_\circ,y_\circ,x_\circ)}\big)\ldots\ldots\big)\Big)$$

Then $A_1,A_2,\ldots\ldots,A_n$ and B having a public tripled coincidence point.

Corollary(2.8)

Let $(W,d,\preccurlyeq)$ be a partially ordered metric space. If $A_1,A_2,\ldots\ldots,A_n:W^3\to W$ and $g:W\to W$ are mappings lies in C such that $A_1,A_2,\ldots\ldots,A_n$ having public mixed B_

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monotone.Suppose that, for all $x,y,z,u,v,w \in W$with $x \succcurlyeq u, y \preccurlyeq v \; and \; z \succcurlyeq w$,

$$d(A_1\left(A_2(\dots\dots(A_{n(x,y,z)})\dots\dots)\right), A_1\left(A_2(\dots\dots(A_{n(u,v,w)})\dots\dots)\right) \preccurlyeq$$

$$\max\Delta\{d(B(x),B(u)), d(B(y),B(v), d(B(z),B(w),$$
$$d\left(A_1\left(A_2(\dots\dots(A_{n(x,y,z)})\dots\dots)\right), B(x)\right),$$
$$d\left(A_1\left(A_2(\dots\dots(A_{n(z,y,x)})\dots\dots)\right), B(z)\right)$$
$$d\left(A_1\left(A_2(\dots\dots(A_{n(u,v,w)})\dots\dots)\right), B(u)\right),$$
$$d\left(A_1\left(A_2(\dots\dots(A_{n(w,v,u)})\dots\dots)\right), B(w)\right)\},$$

Where, $\Delta \epsilon \Gamma$.If there exist $x_\circ, y_\circ, z_\circ \in W$ such that

$$B(x_\circ) \preccurlyeq A_1\left(A_2(\dots\dots(A_{n(x_\circ,y_\circ,z_\circ)})\dots\dots)\right)$$
$$B(y_\circ) \succcurlyeq A_1\left(A_2(\dots\dots(A_{n(y_\circ,x_\circ,y_\circ)})\dots\dots)\right) \text{and}$$
$$B(z_\circ) \preccurlyeq A_1\left(A_2(\dots\dots(A_{n(z_\circ,y_\circ,x_\circ)})\dots\dots)\right)$$

Then$A_1, A_2, \dots\dots, A_n \; and \; B$ having a public tripled coincidence point

Corollary(2.9):

Let $(W,d,\preccurlyeq)$ be a partially ordered metric space. If $A_1, A_2, \dots\dots, A_n: W^3 \to W$ and $g: W \to W$ are mappings lies in C such that $A_1, A_2, \dots\dots, A_n$having public B_mixed monotone . Suppose that, for all $x,y,z,u,v,w \in W$with $x \succcurlyeq u, y \preccurlyeq v \; and \; z \succcurlyeq w$,

$$d(A_1\left(A_2(\dots\dots(A_{n(x,y,z)})\dots\dots)\right), A_1\left(A_2(\dots\dots(A_{n(u,v,w)})\dots\dots)\right) \preccurlyeq$$

$$k \max\{d(B(x),B(u)), d(B(y),B(v)), d(B(z),B(w)),$$
$$d\left(A_1\left(A_2(\dots\dots(A_{n(x,y,z)})\dots\dots)\right), B(x)\right),$$
$$d\left(A_1\left(A_2(\dots\dots(A_{n(z,y,x)})\dots\dots)\right), B(z)\right)$$
$$d\left(A_1\left(A_2(\dots\dots(A_{n(u,v,w)})\dots\dots)\right), B(u)\right),$$
$$d\left(A_1\left(A_2(\dots\dots(A_{n(w,v,u)})\dots\dots)\right), B(w)\right)\}$$

where $k \, \epsilon \, [0,1)$. If there exist $x_\circ, y_\circ, z_\circ \in W$ such that

$$B(x_\circ) \preccurlyeq A_1\left(A_2(\dots\dots(A_{n(x_\circ,y_\circ,z_\circ)})\dots\dots)\right)$$
$$B(y_\circ) \succcurlyeq A_1\left(A_2(\dots\dots(A_{n(y_\circ,x_\circ,y_\circ)})\dots\dots)\right) \text{and}$$
$$B(z_\circ) \preccurlyeq A_1\left(A_2(\dots\dots(A_{n(z_\circ,y_\circ,x_\circ)})\dots\dots)\right)$$

Then,$A_1, A_2, \dots\dots, A_n \; andB$ having a public tripled coincidence point.

Corollary(2.10)

Let $(W,d,\preccurlyeq)$ be a partially ordered metric space.If $A_1, A_2, \dots\dots, A_n: X^3 \to X$ and $B_1, B_2, \dots\dots, B_n: X \to X$ are mappings lies in C such that $A_1, A_2, \dots\dots, A_n$ having public B_mixed monotone. Suppose that for all $x,y,z,u,v,w \in W$with $x \succcurlyeq u, y \preccurlyeq v \; and \; z \succcurlyeq w$,

$$d(A_1\left(A_2(\dots\dots(A_{n(x,y,z)})\dots\dots)\right), A_1\left(A_2(\dots\dots(A_{n(u,v,w)})\dots\dots)\right) \preccurlyeq$$

$$k_1\, d(B(x),B(u)) + k_2 d(B(y),B(v)) + k_3 d(B(z),B(w)) +$$
$$k_4 d\left(A_1\left(A_2(\dots\dots(A_{n(x,y,z)})\dots\dots)\right), B(x)\right) +$$
$$k_5 d\left(A_1\left(A_2(\dots\dots(A_{n(z,y,x)})\dots\dots)\right), B(z)\right)$$
$$k_6 d\left(A_1\left(A_2(\dots\dots(A_{n(u,v,w)})\dots\dots)\right), B(u)\right) +$$
$$k_7 d\left(A_1\left(A_2(\dots\dots(A_{n(w,v,u)})\dots\dots)\right), B(w)\right)$$

where $k_1, k_2, \dots\dots, k_7 \, \epsilon \, [0,1) \; and \; \sum_{i=1}^{7} k_i < 1$.If there exist $x_\circ, y_\circ, z_\circ \in W$ such that

$$B(x_\circ) \leq A_1\left(A_2(\dots\dots(A_{n(x_\circ,y_\circ,z_\circ)})\dots\dots)\right)$$
$$B(y_\circ) \geq A_1\left(A_2(\dots\dots(A_{n(y_\circ,x_\circ,y_\circ)})\dots\dots)\right) \text{and}$$
$$B(z_\circ) \leq A_1\left(A_2(\dots\dots(A_{n(z_\circ,y_\circ,x_\circ)})\dots\dots)\right)$$

Then, $A_1, A_2, \dots\dots, A_n \; andB$ having a public tripled coincidence point.

Also, you can get others results:
If $B(x) = x$ and $A_1, A_2, \dots\dots, A_n$having public mixed monotone in above theorems, then we get the public tripled fixed point theorems.

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