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Estimation of Location (μ) and Scale (λ) for Two-Parameter Rayleigh Distribution by Median Rank Regression Method

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Abstract: ***In this paper, we propose the estimation of Location (μ) and Scale (λ) parameters using the median ranks method (Benard's approximation) and also using two step least square estimation method. We also computed Average Estimate (AE), Variance (VAR), Standard Deviation (STD), Mean Absolute Deviation (MAD), Mean Square Error (MSE), Simulated Error (SE) and Relative Absolute Bias (RAB) for both the parameters under complete sample based on 1000 simulations to assess the performance of the estimators. In this paper, finally we recommended that Median Rank Regression Method shows best performance than Least Square Estimation Method.***

Keywords: Two parameter Rayleigh distribution, median ranks method (Benard's approximation), Least Square method, Monte-carlo Simulation.

1. Introduction

Lord Rayleigh (1880) introduced the Rayleigh distribution in connection with a problem in the field of acoustics. Since then, extensive work has taken place related to this distribution in different areas of science and technology. It has some nice relations with some of the well known distributions like Weibull, Chi-square or extreme value distributions. An important characteristic of the Rayleigh distribution is that its hazard function is an increasing function of time. It means that when the failure times are distributed according to the Rayleigh law, an intense aging of the equipment item takes place. Estimations, predictions and inferential issues for one parameter Rayleigh distribution have been extensively studied by several authors, see for example of Aarset, M. V. (1987). Swain, J., Venkataraman, S. & Wilson, J. R. (1988) proposed Least squares estimatimators and Weighted Least squares estimators of a Beta distribution. For an excellent review for the two distributions the readers are referred to Johnson, Kotz and Balakrishnan (1995). Abd-Elfattah studied the Efficiency of maximum likelihood estimators under different censored sampling schemes for Rayleigh distribution. We see also Dey and Das (2007), Dey (2009) for some references. Torabi and Bagheri (2010) consider different parameter estimation methods in extended generalized half logistic Distribution based on Complete and Censored Data. Recently, Khan, Provost and Singh (2010) consider two-parameter Rayleigh distribution and discussed some inferential issues. Rama Mohan, ch. and Anjaneyulu, G. V. S. R. (2011) studied how the least square method be good for estimating the parameters to Two Parameter Weibull Distribution from an optimally constructed grouped sample.

In this paper we consider two-parameter Rayleigh distribution: one location and one scale parameter, and it has the following Probability Density Function (PDF), Cumulative Distribution Function(CDF), Hazard Function(HF) and by Survival Function (SF) are given

f(x; λ, μ) = 2λ(X-μ); X> μ, λ> 0 (1.1)

F (x; λ, μ) = ; X> μ, λ> 0 (1.2)
H(x) = ; X> μ, λ> 0 (1.3)
S(x) = ; X> μ, λ>0 (1.4)

Estimation of Location (μ) and Scale (λ) parameters of two parameter Rayleigh distribution using Median Ranks Method.

Let X1<X2<X3<…<Xn be an ordered sample of size ‘N’ from two parameter Rayleigh distribution with the parameters Location (μ) and Scale (λ). Then the CDF is given in the equation (1.2) can be rewritten as

1-F(X) = (2.1)

Take logarithm on both side to (2.1), we get

[-ln(1-F(X))] = (2.2)

Take square root on both side to above obtained equation

X = μ + (2.3)

From the least squares parameter estimation method (also known as regression analysis). Let us consider

A = μ, (2.4)

Where F(x) is obtained by using the Median Ranks Method (also called as Benard's Approximation), which is a good approximation to the median rank estimator. The Benard's median rank was used because it showed the best performance and is the most widely used rank to estimate (F(x)). The procedure for ranking complete data is as follows:

1. List the time to failure data from small to large.
2. Use Benard's formula to assign median ranks to each failure.
3. Estimate the μ and λ by Equations.

Benard's Approximation is given by

F(X) = i=1, 2, 3... n. (2.5)

Where i $\rightarrow$ The ranked position of data point and
n $\rightarrow$ The total number of units in the sample
(2.6)
(2.7)

The F(X) is estimated from the median ranks. Once and obtained, the values of and can easily be obtained. Estimation of Location (μ) and Scale (λ) parameters of two parameter Rayleigh distribution using Least Square Method.

Let X1<X2<X3<…<Xnbe an ordered sample of size 'N' from two parameter Rayleigh distribution with the parameters Location (μ) and Scale (λ)
In the equation (2.3) from the least squares parameter estimation method (also known as regression analysis).

Let us consider
=, = and (3.1)

From the least squares parameter estimation method (also known as regression analysis). F(x) obtained using the formula

F(x) =, i = 1, 2, 3,..., n. (3.2)
= (3.3)
= (3.4)

The F(x) is estimated from least squares parameter. Once and obtained, the values of and can easily be obtained.
Comparison of Median Ranks Method estimators and Least square Method estimators.

The Median Rank Method estimates of Scale (σ) and Shape (θ) obtained from complete sample are compared with the Least Square estimates of Scale (σ) and Shape (θ) obtained from complete sample. Though the estimates from both the methods in closed form, it is difficult to obtain the Average Variance, Standard Deviation, Mean Square Error and Relative Absolute Bias of Median Ranks Method estimates as well as Least Square Method estimates, we simulate these values based on 1000 samples of size N = 10, 15, 20, 25, 30, 35, 40, 45, 50 generated from Rayleigh distribution with Scale (σ) and Shape (θ).Here to assess the performance of Median Ranks Method estimates, we compare these with the Least Square Method estimates.

2. Simulation Study

In order to obtain the median ranks method estimators of Scale (σ) and Shape (θ) and study the properties of their estimates through the Average Estimate (AE), Variance (VAR), Standard Deviation (STD), Mean Square Error (MSE) and Relative Absolute Bias (RAB) under In order to obtain the Median Ranks Method estimators of Scale (σ) and Shape (θ) and study the properties complete sample are given respectively by forgiven values of n, μ and λ. If is Median Ranks Method estimate of, m=1, 2 where is a general notation that can be replaced by based on sample l, (l=1, 2, …, r) then The Average Estimate (AE), Variance (VAR), Standard Deviation (STD), Mean Square Error (MSE) and Relative Absolute Bias (RAB) are given respectively by

Average Estimate (Variance) =
Standard Deviation) =
Mean Absolute Deviation =
Mean Square Error) =
Relative Absolute Bias) =

3. Observations from Simulation Results

The Average Estimate (AE), Variance (VAR), Standard deviation (STD), Mean
Square Error (MSE) and Relative Absolute Bias (RAB) are independent of true values of the parameters of of Location (μ) and Scale (λ).

Average Estimate (AE) of Location parameter () and

Scale parameter ($\hat{\lambda}$) by Median Ranks Method and Location parameter () and Scale parameter by Least Square Method were increasing when sample size (N) is increasing.

Variance (VAR) of Location parameter () and Scale parameter () by Median Ranks Method and Location parameter () and

Scale parameter ($\tilde{\lambda}$) by Least Square Method were decreasing when sample size (N) is increasing.

Standard Deviation (STD) of Location parameter () and Scale parameter () by Median Ranks Method and

Location parameter () and Scale parameter ($\tilde{\lambda}$) by Least Square Method were decreasing when sample size (N) is increasing.

Mean Absolute Deviation (MAD) of Location parameter () and Scale parameter () by Median Ranks Method and Location parameter () and Scale parameter ($\tilde{\lambda}$) by Least Square Method were decreasing when sample size (N) is increasing.

Mean Square Error (MSE) of Location parameter () and Scale parameter () by Median Ranks Method and Location parameter () and Scale parameter ($\tilde{\lambda}$) by Least Square Method were decreasing when sample size (N) is increasing.

Simulated Error(SE) in Average Estimate (AE), Variance (VAR), Standard Deviation (STD), Mean Square Error (MSE) and Relative Absolute Bias (RAB) of Location parameter () and Scale parameter () by Median Ranks Method and Location parameter () and Scale parameter ($\tilde{\lambda}$) by Least Square Method were decreasing when sample size (N) is increasing.

Relative Absolute Bias (RAB) of Location parameter () and Scale parameter () by Median Ranks Method were decreasing when sample size (N) is increasing and Location parameter()and Scale parameter ($\tilde{\lambda}$) by Least Square Method were increasing when sample size (N) is increasing.

Location parameter () by Median Ranks Method is having less variance(VAR) than to the Location parameter () by Least Square Method

Location parameter () and Scale parameter () by Median Ranks Method having less Standard deviation than to the Location parameter () and Scale parameter () by Least Square Method.

Simulated Error in The Average Estimate (AE) of Location parameter () and Scale parameter () by Median Ranks Method is having less than to the Location parameter() and Scale parameter ($\tilde{\lambda}$) by Least Square Method.

Simulated Error in Mean Absolute Deviation (MAD) of Scale parameter () by Median Ranks Method is having less than to the Scale parameter ($\tilde{\lambda}$) by Least Square Method.

Simulated Error in Mean Square Error (MSE) of Location parameter () by Median Ranks Method is having less than to the Location parameter () by Least Square Method.

Location parameter () by Median Ranks Method is having less Mean Absolute Deviation (MAD) than to the Location parameter () by Least Square Method.

Location parameter () and Scale parameter () by Median Ranks

Method having less Mean Square Error (MSE) than to the Location parameter () Scale parameter ($\tilde{\lambda}$) by Least Square Method.

The Average Estimate (AE), Variance (VAR), Standard deviation (STD), Mean Square Error (MSE) and Relative Absolute Bias (RAB) Simulated Error (SE) of Median rank method estimators and least square method estimators of location and scale and parameters under complete sample of 1000 simulations. Population parameters location=1.5 and scale =3 in Table-1 and Table-2.

Table 1: Median Ranks Estimators

Size	Parameters	AE (SE)	VAR	STD	MAD (SE)	MSE (SE)	RAB
10		1.552	0.0533	0.2308	0.2386	0.0507	0.1235
		-0.073			-0.0634	-0.0197	
		.					
		1.995	1.8192	1.3488	1.5574	2.6474	0.5191
		-0.4265			-0.1571	-0.6047	
15		1.6296	0.0482	0.22	0.1801	0.061	0.1443
		-0.0567			-0.0473	-0.0148	
		2.0183	1.2147	1.1021	1.35	2.0975	0.45
		-0.2846			-0.1402	-0.4595	
20		1.6276	0.0482	0.2195	0.1801	0.0613	0.1443
		-0.0491			-0.041	-0.0128	
		2.0183	1.2147	1.1021	1.35	2.0975	0.45
		-0.2464			-0.1214	-0.3979	
25		1.7093	0.1119	0.3346	0.2641	0.1513	0.2344

		-0.0669			-0.0436	-0.0186	
		2.3423	0.7144	0.8452	0.9027	1.1266	0.2239
		-0.1429			-0.0958	-0.2452	
30		1.7191	0.0415	0.2038	0.1447	0.0878	0.1816
		-0.0372			-0.027	-0.0103	
		2.1719	0.7591	0.8713	1.0491	1.4146	0.3497
		-0.1591			-0.1044	-0.1044	
35		1.7328	0.0353	0.188	0.0238	0.087	0.1848
		-0.0318			-0.1405	-0.0512	
		2.3342	0.83	0.911	0.9516	1.2454	0.3172
		-0.154			-0.1002	-0.2593	
40		1.7424	0.0308	0.1754	0.1138	0.075	0.187
		-0.0278			-0.0212	-0.0476	
		2.3423	0.7144	0.8452	0.9027	1.1266	0.3009
		-0.1336			-0.0896	-0.2294	
45		1.7522	0.0277	0.1664	0.1065	0.0906	0.1903
		-0.0248			-0.019	-0.045	
		2.4375	0.7811	0.8838	0.885	1.078	0.295
		.1317)			-0.082	-0.2049	
50		1.7645	0.0244	0.1563	0.9961	2.7859	0.0565
		-0.0221			-0.01692	-0.0504	
		2.4749	0.6931	0.8326	0.8275	0.955	0.2191
		-0.1177			0.0743	-0.179	

Table 2: Least Square Estimators

Size	Parameters	AE (SE)	VAR	STD	MAD (SE)	MSE (SE)	RAB
10		1.357	0.0672	0.2593	0.4409	0.081	0.1418
		-0.082			-0.082	-0.0361	
	$\tilde{\lambda}$	1.6302	0.4764	0.6903	1.1966	0.4458	0.3469
		-0.216			-0.216	-0.2066	
15	$\tilde{\lambda}$	1.5514	0.1242	0.3624	0.3413	0.1185	0.1924
		-0.091			-0.0655	-0.0276	
		2.2908	1.2696	1.1268	1.0608	1.8104	0.7006
		-0.2909			-0.1565	-0.5633	
20	$\tilde{\lambda}$	1.649	0.1222	0.3496	0.292	0.1383	0.2179
		-0.0782			-0.0528	-0.0224	
		2.6264	1.3435	1.1591	0.9981	2.5451	0.881
		-0.2592			-0.1295	-0.5565	
25	$\tilde{\lambda}$	1.7093	0.1119	0.3346	0.2641	0.1513	0.2344
		-0.0669			-0.0436	-0.0186	
		2.7756	1.1793	1.0859	0.9084	2.7594	0.9544
		-0.2172			-0.1135	-0.4661	
30	$\tilde{\lambda}$	1.7497	0.1011	0.3179	0.2457	0.16	0.2455
		-0.058			-0.03698	-0.0159	
		2.8921	1.0542	1.0267	0.8655	2.957	1.0148

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		-0.1875			-0.0973	-0.4007		
35		1.77	0.0888	0.298	0.224	0.1591	0.2477	
		-0.0504			-0.0329	-0.0136		
	$\tilde{\lambda}$	2.8378	0.9227	0.9606	0.7867	2.6862	0.9662	
		-0.1624			-0.0904	-0.362		
40	$\tilde{\lambda}$	1.7769	0.0783	0.2799	0.2035	0.153	0.2439	
		-0.0443			-0.0301	-0.0125		
		2.7714	0.8616	0.9282	0.7635	2.4565	0.9126	
		-0.1468			-0.0816	-0.3343		
45	$\tilde{\lambda}$	1.7915	0.0713	0.267	0.1932	0.1547	0.247	
		-0.0398			-0.0271	-0.0111		
		2.8443	0.8434	0.9184	0.7556	2.6318	0.954	
		-0.1369			-0.076	-0.327		
50	$\tilde{\lambda}$	1.7979	0.0644	0.2538	0.1797	0.1519	0.246	
		-0.0359			-0.0251	-0.01		
		2.8231	0.7728	0.8791	0.7126	2.5079	0.9341	
		-0.1243			-0.0714	-0.3008		

4. Conclusions

The Estimate of Location parameter () and Scale parameter () by Median Rank method are more accurate than to the of Location parameter () and Scale parameter ($\tilde{\lambda}$) by least square Method.

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