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Daftardar-Jafari Method for Solving the Damped Generalized Regularized Long-Wave Equation

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Abstract: ***In this work, the Daftardar-Jafari method has been submitted to deal with the approximated solutions for the nonlinear damped generalized regularized long-wave (DGRLW) equation with some given variable coefficient. by using this iterative technique we have got rid of any additional assumptions in order to get the solution of the nonlinear DGRLW equation where this solution has been evaluated in a series form with several computable components. Three numerical examples have been tested to show the easy-applicable scheme for the method where the third example has been solved by using the modified Daftardar-Jafari method in order to deal with an inhomogeneous case for the DGRLW equation. Also, our approximate solutions have been numerically reported and figured with the exact solution.***

Keywords: Damped generalized regularized long wave equation; Daftardar-Jafari method; Modified Daftardar-Jafari method; approximate solution; numerical simulation

1. Introduction

Partial differential equations used in representing different problems in engineering, physics, chemistry, and biological sciences. Most of these problems are often very difficult to be dealt and solved accurately, in last times, numerous mathematical techniques are used to examine and find the solutions of partial differential equations that appear in distinct physical phenomena for example; one can see [1–8].

The presented equation of the damped generalized regularized long-wave (DGRLW) with some variable coefficient describes the amplitude of the long-wave that is given in the following form [9]

$$f_t - (\varphi(x,t) f_{xt})_x - \alpha f_{xx} + f_x + f^p f_x = 0, \quad (1)$$

where $f(x,t)$ represents the long-wave amplitude for some position x and a time t, $\alpha > 0$ and $p \geq 1$ is an integer number.

In the physical sense, the dissipative term αf_{xx} in equation (1) plays a significant role in studying the nonlinear dispersive waves since this term describes a large number of important physical phenomena, just like shallow water waves, nonlinear optics that include mode-locked lasers, fiber optic communications, and dispersion-managed wave phenomena [8].

It is important to know there are several previous studies have shown the importance to study and solve this equation. Of these studies: The Adomian decomposition method (ADM) [9], the variational iteration method (VIM) [10], homotopy perturbation method (HPM) [11], the modified homotopy analysis method (MHAM) [12], a fully Galerkin method [13], Ritz Legendre Multiwavelet method [14] and others.

The Daftardar-Jafary method (DJM) has been proposed in 2006 by Varsha Daftardar-Gejji and Hossein Jafari [15], and it has been used for solving many important functional models of ODEs, PDEs and Integral and algebraic equations. In this work we aim to produce easy-applicable calculations for finding suitable approximate solutions for the DGRLW equation with some variable coefficient. The DJM does not require using any external assumption to obtain the solution iterative components as using the Adomian polynomials in the ADM. So, for this cause, the DJM has been used for solving various problems in different areas as in [16-20].

In this work, the following sections will be organized as: section two involves the basic idea of the Daftardar-Jafary method (DJM). Section three represents the implementing of the DJM to solve different cases of the DGRLW problem. The numerical results will be discussed in section four. Finally, the conclusion is presented in section five.

2. The basic idea of the DJM

In what follows, the basic steps of the DJM [15] will be presented for solving both types of the DGRLW equation: homogeneous and inhomogeneous.

2.1. The standard DJM

Let us begin by considering the homogeneous DGRLW equation (1) in the following operator form [9]

$$L_t(f) - L_x(\varphi(x,t) L_{xt}(f)) - \alpha L_{xx}(f) + L_x(f) + N(f) = 0, \quad (2)$$

where $L_t = \frac{\partial}{\partial t}$, $L_x = \frac{\partial}{\partial x}$, $L_{xx} = \frac{\partial^2}{\partial x^2}$, $L_{xt} = \frac{\partial^2}{\partial x \partial t}$ and $N(f) = L_x(\frac{f^{p+1}}{p+1})$, where L is the linear operator and N denotes the nonlinear operator for f which is the amplitude function. The inverse operator L_t^{-1} is represented as

$$L_t^{-1}(.) = \int_0^t (.)\, d\tau.$$

Therefore, equation (2) will be equalled to

$$f = f(x,0) + L_x(\varphi(x,t) L_{xt}(f)) + \alpha L_{xx}(f) - L_x(f) - N(f), \quad (3)$$

In successive iterative steps, the solution function f can be found by the summation of all the resulted components f_i, that means:

$$f = \sum_{i=0}^{\infty} f_i. \tag{4}$$

So, we can define the following series

$$G_0 = L_x(\varphi(x,t)L_{xt}(f_0)) + \alpha L_{xx}(f_0) - L_x(f_0) - N(f_0), \tag{5}$$

$$G_1 = [L_x(\varphi(x,t)L_{xt}(f_0+f_1)) + \alpha L_{xx}(f_0+f_1) - L_x(f_0+f_1) - N(f_0+f_1)] \\ -[L_x(\varphi(x,t)L_{xt}(f_0)) + \alpha L_{xx}(f_0) - L_x(f_0) - N(f_0)] \tag{6}$$

$\vdots$

$$G_m = [L_x(\varphi(x,t)L_{xt}(\sum_{i=0}^{m} f_i)) + \alpha L_{xx}(\sum_{i=0}^{m} f_i) - L_x(\sum_{i=0}^{m} f_i) - N(\sum_{i=0}^{m} f_i)] \\ -[L_x(\varphi(x,t)L_{xt}(\sum_{i=0}^{m-1} f_i)) + \alpha L_{xx}(\sum_{i=0}^{m-1} f_i) - L_x(\sum_{i=0}^{m-1} f_i) - N(\sum_{i=0}^{m-1} f_i)]. \tag{7}$$

Hence, in an iterative procedure, the components f_i can be found by

$$f_0 = f(x,0), \tag{8}$$
$$f_1 = G_0 \tag{9}$$
$$f_2 = G_1, \tag{10}$$
$$\vdots$$
$$f_m = G_{m-1}, \quad m = 1,2,\dots. \tag{11}$$

The approximate solution for equation (2) is defined in the k-term series solution $\varphi_k = \sum_{i=0}^{k-1} f_i$. When k approaches $+\infty$, then we will get the exact solution f as in (4).

2.2. The modified DJM

In this subsection, we will review the steps of the modified Daftardar-Jafari method (MDJM). First, let us begin by the inhomogeneous DGRLW equation in the operator form

$$L_t(f) - L_x(\varphi(x,t)L_{xt}(f)) - \alpha L_{xx}(f) + L_x(f) + N(f) = g, \tag{12}$$

For both sides of Eq. (12), when applying the inverse operator L_t^{-1} we have

$$f = s + L_x(\varphi(x,t)L_{xt}(f)) + \alpha L_{xx}(f) - L_x(f) - N(f), \tag{13}$$

where $s(x,t) = f(x,0) + \int_0^t g(x,\tau)\,d\tau$. As in other researches [21,22], in order to develop the DJM to get a better performance in somewhat we have used this modified technique to do the same for solving the inhomogeneous DGRLW problem. As the same idea for modifying the ADM [23-25], Wazwaz have proposed another way in order to modify the iterative procedure of the ADM. The same idea has been made to develop the DJM in order to solve the inhomogeneous DGRLW equation. Wazwaz idea was splitting the function s into two parts as $s = s_0 + s_1$. So, the iterative components can be found by

$$f_0 = s_0, \tag{14}$$
$$f_1 = s_1 + G_0 \tag{15}$$
$$f_2 = G_1, \tag{16}$$
$$\vdots$$
$$f_m = G_{m-1}, \quad m = 1,2,\dots. \tag{17}$$

Subsequently, from these resulted components we can get the approximate solution $\varphi_k = \sum_{i=0}^{k-1} f_i$ and when continuing approximating this series till $+\infty$, we get the exact solution f.

3. Solving the DGRLW equation by the DJM

In the following examples, the basic steps of the DJM are used to solve homogenous and inhomogeneous types of the DGRLW equation [9].

Example 1- Suppose the homogenous DGRLW equation (2), by setting $\alpha = 1, p = 1$ and

$$\varphi(x,t) = 1 - 3\frac{\cosh\left(\frac{1}{4}x - \frac{1}{3}t\right)\sinh\left(\frac{1}{4}x - \frac{1}{3}t\right)}{2\cosh^2\left(\frac{1}{4}x - \frac{1}{3}t\right) - 3}. \tag{18}$$

with the initial condition:

$$f(x,0) = \operatorname{sech}^2\left(\frac{1}{4}x\right). \tag{19}$$

and the exact solution:

$$f(x,t) = \operatorname{sech}^2\left(\frac{1}{4}x - \frac{1}{3}t\right). \tag{20}$$

Solution:

According to the standard DJM, the first term is $f_0(x,t) = \operatorname{sech}^2\left(\frac{1}{4}x\right)$, hence the next iterative component will be

$$f_1(x,t) = \frac{1}{16}t\operatorname{sech}^5\left(\frac{x}{4}\right)\left(-3\cosh\left(\frac{x}{4}\right) + \cosh\left(\frac{3x}{4}\right) + 2\left(5\sinh\left(\frac{x}{4}\right) + \sinh\left(\frac{3x}{4}\right)\right)\right).$$

To discuss convergence of the approximate solution, we have considered the following 2nd approximate solution

$$\varphi_2 = f_0 + f_1. \tag{21}$$

Example 2- In the following, let us consider the homogenous DGRLW equation (2) with setting $\alpha = 1, p = 2$ and

$$\varphi(x,t) = -\frac{1}{6}e^{-2x+4t}. \tag{22}$$

with the following initial condition:

$$f(x,0) = e^{-x}, \tag{23}$$

and the exact solution:

$$f(x,t) = e^{-x+2t}. \tag{24}$$

Solution:

By applying the standard DJM, we choose the initial term as $f_0(x,t) = e^{-x}$, and the next iterative components will be

$$f_1(x,t) = e^{-3x}t + 2e^{-x}t.$$

$$f_2(x,t) = \frac{1}{24}e^{-9x}(-15e^{4(t+x)} - 6e^{4t+6x} + 48e^{8x}t^2 + 18t^4 \\ + 28e^{2x}t^3(2+3t) + 6e^{6x}(1+36t^2 \\ + 16t^3 + 8t^4) + 5e^{4x}(3 + 12t^2 + 32t^3 + 24t^4)).$$

The other iterative components have been evaluated in the same manner, but for brevity, they are not listed.

In the next section, the convergence for the following 5th approximate solution will be discussed numerically

$$\varphi_5 = f_0 + f_1 + f_2 + f_3 + f_4, \tag{25}$$

Example 3- Let us take the inhomogeneous DGRLW equation (12) where

$$g(x,t) = (-t\cos(x) + xt\sin(x) + \cos(x) + \sin(x)\cos(x)e^{-t})e^{-t}, \tag{26}$$

and $\varphi(x,t) = xt, \alpha = 1, p = 1$ with

$$f(x,0) = \sin(x), \tag{27}$$

and the exact solution is:

$$f(x,t) = \sin(x)e^{-t}. \tag{28}$$

Solution:

According to the MDJM [22] we have

$$s = f(x,0) + L_t^{-1}(g(x,t)), \text{ so}$$

$$s = \sin(x) + \frac{1}{2}e^{-t}(2t\cos(x) + 2(-1 + e^t - t)x\sin(x) + \sin(2x)\sinh(t)) = s_0 + s_1$$

Hence, the initial term is $f_0(x,t) = s_0 = \sin(x)$, and the next iterative component will be evaluated as

$$f_1(x,t) = s_1 + \int_0^t ((\partial_x(\varphi\partial_{x,\tau}f_0) + \partial_{xx}f_0 - \partial_x f_0 - f_0\partial_x f_0))\,d\tau,$$

$$= \frac{1}{2}e^{-t}(2t\cos(x) + 2(-1 + e^t - t)x\sin(x) + \sin(2x)\sinh(t)) + \int_0^t ((\partial_x(\varphi\partial_{x,\tau}f_0) + \partial_{xx}f_0 - \partial_x f_0 - f_0\partial_x f_0))\,d\tau$$

$$f_1(x,t) = -e^{-t}((e^t(t-x) + (1+t)x)\sin(x) + t\cos(x)(-1 + e^t + e^t\sin(x)) - \cos(x)\sin(x)\sinh(t)).$$

$$f_2(x,t) = -t\cos(x) + \frac{1}{2}t^2\cos(x) + \frac{1}{2}t^2(\cos(x) + \cos(2x) - \sin(x)) - t\sin(x) + \frac{1}{2}t^2\sin(x)$$
$$+ \frac{1}{4}e^{-2t}\left(-4e^t(2 + e^t(-2+t) + t)x\cos(x) + \left(-1 + e^{2t}(1-2t)\right)\cos(2x) - 4e^t(3 + e^t(-3+t) + 2t)\sin(x)\right)$$
$$- \frac{1}{4}e^{-2t}\left(2e^t(e^t(t^2 - 4x) + 2(2 + 2t + t^2)x)\cos(x) + \left(1 + 2t + e^{2t}(-1 + 2t^2)\right)\cos(2x) - 2e^t\left(e^t(6 + t^2) - 2(3 + 3t + 2t^2)\right)\sin(x)\right)$$
$$- t\left(-\cos(x) - \sin(x) - \cos(x)\sin(x)\right) + \cdots.$$

The 3rd approximate solution is equalled to the series

$$\varphi_3 = f_0 + f_1 + f_2. \qquad (29)$$

The convergence of this approximate result will be discussed numerically in the next section.

4. Numerical Results

In this section, the approximate solutions for the homogeneous and inhomogeneous DGRLW equation as they are mentioned in the examples of the previous section are measured by evaluating the absolute error values. The symbolic manipulator *Mathematica* has been used for dealing with the algorithms of the DGRLW examples numerically and plotting there approximate solutions. Table 1 shows the values of the absolute error between the 2nd order approximate solution (21) and the exact solution of Ex. 1. Figs. 1 and 2 are reviewing the 3D plotted graphs for the DJM approximate solution and the exact solution for Ex. 1, respectively. Fig. 3 represents a comparison between the approximate and the exact solutions of Ex. 1 when $t = 0.1$.

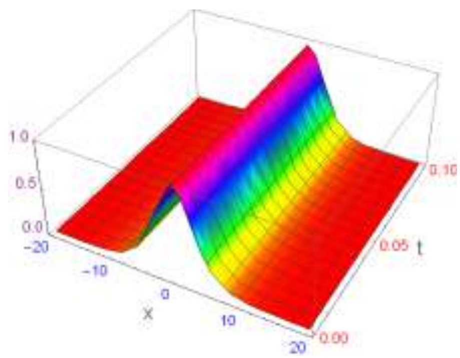

Figure 1: 3D plotted graph for the 2nd approximate solution (21) of Ex. 1

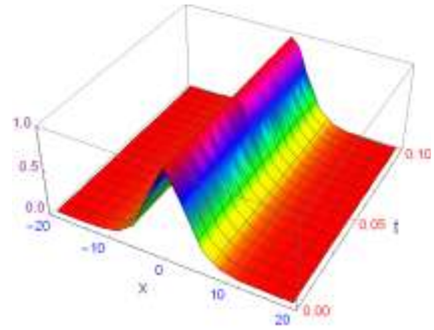

Figure 2: 3D plotted graph for the exact solution (20) of Ex. 1

Table 1: The numerical values of the absolute error for the 2nd approximate solution (21) for Ex. 1

t_i/x_i	-5	-2.5	0	2.5	5
0.02	9.23782×10^{-4}	1.51047×10^{-3}	2.45556×10^{-3}	1.24775×10^{-3}	6.71997×10^{-4}
0.04	1.81877×10^{-3}	3.01715×10^{-3}	4.82224×10^{-3}	2.50124×10^{-3}	1.31502×10^{-3}
0.06	2.68507×10^{-3}	4.52101×10^{-3}	7.10011×10^{-3}	3.76148×10^{-3}	1.929×10^{-3}
0.08	3.52277×10^{-3}	6.023×10^{-3}	9.28923×10^{-3}	5.02947×10^{-3}	2.51384×10^{-3}
0.1	4.33199×10^{-3}	7.52405×10^{-3}	1.13897×10^{-2}	6.30625×10^{-3}	3.06949×10^{-3}

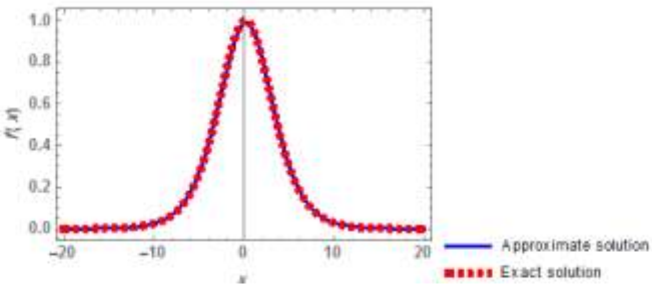

Figure 3: Comparison between the approximate and exact solutions for Ex. 1

Table 2 reviews the absolute error values between the 5th order approximate solution (25) and the exact solution for Ex. 2.

Figs. 4 and 5 are showing the 3D plotted graphs for the 5th order approximate solution (25) and the exact solution for Ex. 2.

Fig. 6 compares the approximate and the exact solutions of Ex. 2 when $t = 0.1$.

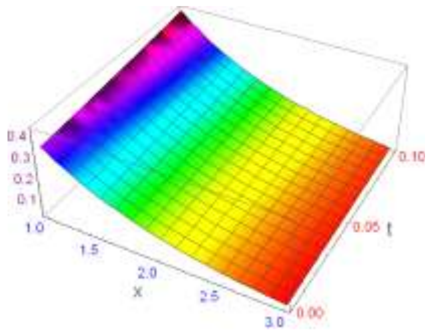

Figure 4: 3D plotted graph for the 5th approximate solution (25) of Ex. 2

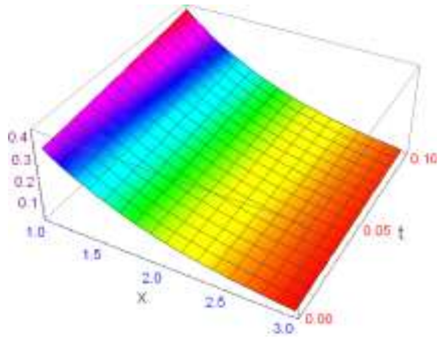

Figure 5: 3D plotted graph for the exact solution (24) of Ex. 2

Table 2: The numerical values of the absolute error for the 5^{th} approximate solution (25) for Ex. 2

t_j/x_i	1.2	1.4	1.6	1.8	2
0.02	1.10866×10^{-5}	1.35671×10^{-6}	7.45087×10^{-7}	1.6175×10^{-7}	7.52109×10^{-8}
0.04	4.52434×10^{-5}	1.24518×10^{-5}	1.45585×10^{-6}	4.54762×10^{-7}	3.79061×10^{-7}
0.06	1.19489×10^{-4}	1.51771×10^{-5}	3.79472×10^{-6}	3.44969×10^{-6}	1.3864×10^{-6}
0.08	1.66774×10^{-4}	5.51526×10^{-6}	1.89852×10^{-5}	9.23821×10^{-6}	2.54733×10^{-6}
0.1	1.46723×10^{-4}	6.16848×10^{-5}	4.61825×10^{-5}	1.707×10^{-5}	2.71325×10^{-6}

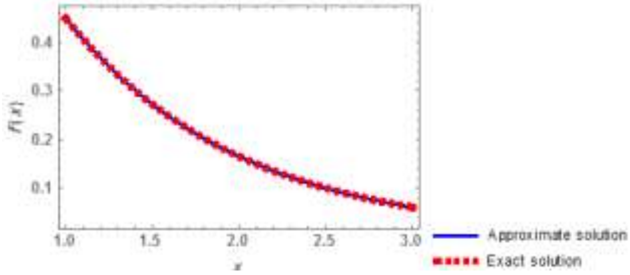

Figure 3: Comparison between the approximate and exact solutions for Ex. 2

Table 3 is showing the absolute error values between the 3^{rd} order approximate solution (29) and the exact solution for Ex. 3.

Figs. 7 and 8 represent the 3D plotted graphs for the 3^{rd} order approximate solution (29) and the exact solution for Ex. 3.

Fig. 9 is the comparison between the approximate and the exact solutions of Ex. 3 when $t = 0.1$.

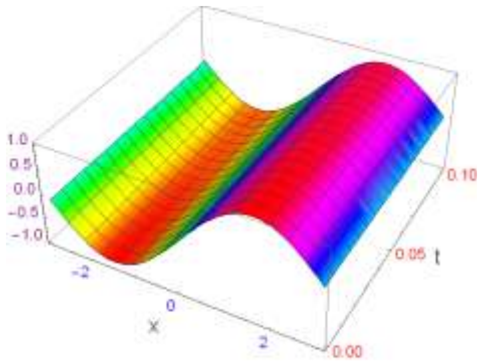

Figure 7: 3D plotted graph for the 3^{rd} approximate solution (29) of Ex. 3

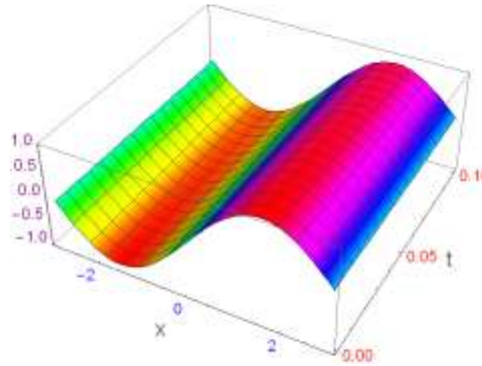

Figure 8: 3D plotted graph for the exact solution (28) of Ex. 3

Table 3: The numerical values of the absolute error for the 3^{rd} approximate solution (29) for Ex. 3

t_j/x_i	0.2	0.4	0.6	0.8	1
0.02	3.59674×10^{-4}	2.8323×10^{-4}	1.67619×10^{-4}	2.16589×10^{-5}	1.43917×10^{-4}
0.04	1.37435×10^{-3}	1.04359×10^{-3}	5.65129×10^{-4}	2.0575×10^{-5}	6.67271×10^{-4}
0.06	2.95081×10^{-3}	2.15195×10^{-3}	1.04031×10^{-3}	2.81706×10^{-4}	1.7027×10^{-3}
0.08	5.00018×10^{-3}	3.48549×10^{-3}	1.44639×10^{-3}	9.09325×10^{-4}	3.37679×10^{-3}
0.1	7.4378×10^{-3}	4.92746×10^{-3}	1.65168×10^{-3}	2.044×10^{-3}	5.81041×10^{-3}

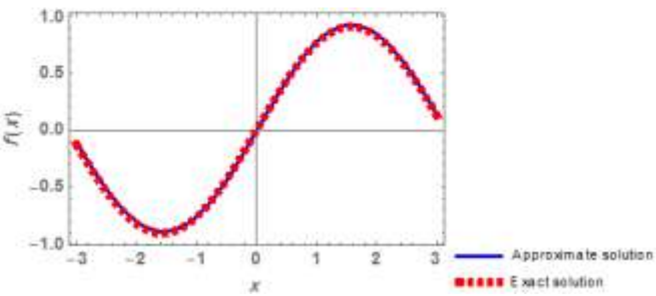

Figure 9: Comparison between the approximate and exact solutions for Ex. 3

5. Conclusion

In this paper, the standard and modified DJM are used to solve the homogeneous and inhomogeneous DGRLW equations with a given variable coefficient. These techniques appear to be very promising to get the solutions of nonlinear partial differential equations without any linearization, discretization or perturbation techniques.

There is no use for any auxiliary parameters or additional polynomials to get help in solving this kind of problems as in the other known classical iterative methods. Finally, the numerical results and graphs for this problem show the validity and convergence of the standard and modified DJM for each example.

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