Quite Comparable Deflection Analysis in Composite Beam Using Analytically and by 2-D ABAQUS Software

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Abstract: ***In this work present that quite comparable deflection result analysis in composite beam (while beam is statically and dynamically loaded) using analytically and by 2-D ABAQUS software. Composite Structures are widely used due to its strength and stiffness to weight ratio. In this work Classical Beam Theory and Shear Deformation Theory is considered for studying the deflection of a composite beam. The composite is a sandwich type reinforced concrete beam. The deflection of the composite beam is calculated analytically. The result is also compared using the 2D ABAQUS Finite Element software. The results are quite comparable. In this project the result for natural frequency and their shapes are also given.***

Keywords: Composite Beam, Classical Beam Theory, Shear Deformation Theory, 2-D ABAQUS software

1. Introduction

A composite is commonly defined as a combination of two or more distinct materials, each of which retains its own distinctive properties, to create a new material with properties that cannot be achieved by any of the components acting alone. Using this definition, it can be determined that a wide range of engineering materials fall into this category. For example, concrete is a composite because it is a mixture of Portland cement and aggregate. Fiberglass sheet is a composite since it is made of glass fibers imbedded in a polymer.

Composite materials are said to have two phases. The reinforcing phase is the fibers, sheets, or particles that are embedded in the matrix phase. The reinforcing material and the matrix material can be metal, ceramic, or polymer. Typically, reinforcing materials are strong with low densities while the matrix is usually a ductile, or tough, material.

The advantages of composite is that they usually exhibits the best qualities of their constituents and often some qualities that neither constituents possesses. The properties that can be improved by forming a composite material include, strength, stiffness, corrosion, resistance, wear, resistance, attractiveness, weight, fatigue life, temperature dependent behaviour, thermal-insulation, thermal-conductivity, accaustical insulation etc. Composite material have a long history of its usage. Their beginning are known but all recorded history contains references to some form of composite materials, for eg, straw was used by Israelites to strengthen mud bricks. Plywood was used by ancient Egyptians when they that wood would be rearranged to achieve superior strength and resistance to thermal expansion as well as to swelling owing to the presence of moisture. Medieval swords and armour were constructed with different layers of materials. More recently fibre reinforced rigid composite that has high strength to weight and stiffness to weight ratio have become important in weight sensitive such as aircraft and space vehicles.

2. Classical Beam Theory

Frequently, as engineers try to optimize the use of materials, they design composite beams made from two or more materials. The design rationale is quite straight forward. For bending loading, stiff, strong, heavy or expensive material must be far away from the neutral axis at places where its effect will be greatest. The weaker, lighter or les expensive material will be placed in the central part of the beam. At one extreme is a steel-reinforced concrete beam, where weight is not a major concern, but strength and cost are. At the other extreme is a sandwich beam used eg in an aircraft with fibre-reinforced laminate cover sheets and a foam core. In that case, stiffness and weight are essential but cost not.

First we consider elementary beam equations: The cross-section area A can have various geometries but must be symmetric to the x_3-axis. The fibre reinforcement of the beam is parallel to the x_1-axis and the volume fraction is a function of the cross-section coordinates x_2, x_3, ie. $v_f = v_f(x_2, \ x_3)$.The symmetry condition yields $v_f(x_2, x_3) = v_f(-x_2, x_3)$ and $E_1(x_2, x_3) = E_1(-x_2, x_3)$.

With the known equation for the strain ε_1 and the stress σ_1 at x_1= constant.

$$\varepsilon_1(x_3) = \epsilon_1 + x_3\kappa_1,$$
$$\sigma_1(x_2, x_3) = \epsilon_1 E_1(x_2, x_3) + x_3\kappa_1 E_1(x_2, x_3)$$

Follow the stress resultant N(x_1), M(x_1) of a beam.

$$N = \epsilon_1 \int_{(A)} E_1(x_2, x_3)dA + \kappa_1 \int_{(A)} x_3 E_1(x_2, x_3)\, dA,$$

M = $\epsilon_1 \int_{(A)} x_3 E_1(x_2,x_3)dA$ + $\kappa_1 \int_{(A)} x_3^2 E_1(x_2,x_3)$dA

The effective longitudinal modulus of elasticity is
$E_1 = E_f v_f + E_m v_m$ = $E_m + \phi(x_2,x_3)(E_f - E_m)$
and with $E_f = const,\ E_m = const,\ \phi(x_2,x_3)$ = $\phi(-x_2,x_3)$ it follows that
N = a$\epsilon_1 + b\kappa_1$, a = $E_m A + (E_f - E_m)\int_{(A)} \phi$ $(x_2,x_3)dA$
M = b$\epsilon_1 + d\kappa_1$, b = $(E_f - E_m)\int_{(A)} \phi\,(x_2,x_3)x_3^2 dA$
I = $\int_{(A)} x_3^2\, dA$, d = $E_m I + (E_f - E_m)\int_{(A)} \phi\,(x_2,x_3)x_3^2 dA$

The inverse of the stress resultant are
$\epsilon_1 = \frac{dN - dM}{ad - b^2}$, $\kappa_1 = \frac{aM - bN}{ad - b^2}$
and the stress equation has the form
$$\sigma_1(x_2,x_3) = \frac{dN - bM + (aM - bN)x_3}{ad - b^2} E_1(x_2,x_3)$$

Taking into consideration the different moduli E_f and E_m, the fibre and matrix stresses are
$$\sigma_f(x_3) = \frac{dN - bM + (aM - bN)x_3}{ad - b^2} E_f,$$
$$\sigma_m(x_3) = \frac{dN - bM + (aM - bN)x_3}{ad - b^2} E_m$$
In the case of a double symmetric geometry and fibre volume fraction function ϕ, b = 0 and the equation can be simplified
$$\epsilon_1 = N/a,\ \kappa_1 = M/d,$$
$$\sigma_f(x_3) = \left(N/a + x_3 M/d\right) E_f,$$
$$\sigma_m(x_3) = \left(N/a + x_3 M/d\right) E_m$$
For a uniform fibre distribution, $\phi\,(x_2,x_3)$ = const
a = $E_m A + +(E_f - E_m)\phi A = E_1 A$,
b = 0,
d = $E_m I + (E_f - E_m)\phi I = E_1 I$
and the stress relations for fibre and matrix
$$\sigma_f(x_3) = \left(\frac{N}{A} + x_3\frac{M}{I}\right)\left(\frac{E_f}{E_1}\right),$$
$$\sigma_m(x_3) = \left(\frac{N}{A} + x_3\frac{M}{I}\right)\left(\frac{E_m}{E_1}\right),$$

If $E_f = E_m$ and $E_1 = E$, becomes the classical stress formula for isotropic beam with axial and lateral loadings
$$\sigma(x_3) = \frac{N}{A} + x_3\frac{M}{I}$$

3. Shear Deformation Theory

The structural behavior of many usual beams may be satisfactorily approximated by the classical Euler-Bernoulli theory.But short and moderately thick beams or laminated composite beams which l/h ratios are not rather large cannot be well treated in the frame of the classical theory. To overcome this shortcoming Timoshenko extended the classical theory by including the effect of transverse shear deformation. However, since Timoshenko's beam theory assumed constant shear strain through the thickness h a shear correction factor is required to correct the shear strain energy.

In this we study the influence of transverse shear deformation upon the bending of laminated beams. The similarity of elastic behavior of laminate and sandwich beams with transverse shear effects included allows us generally to transpose the results from laminate to sandwich beams. When applied to beams, the first order shear deformation theory is known as Timoshenko's beam theory.

The strains of the Timoshenko's beam are
$$\varepsilon_1 = \frac{\partial u_1}{\partial x_1}, \quad = \frac{du}{dx_1} + x_3\frac{d\varphi_1}{dx_1},$$
$$\varepsilon_2 \equiv 0, \quad \varepsilon_3 \equiv 0,$$
$$\varepsilon_5 = \frac{\partial u_1}{\partial x_3} + \frac{\partial w}{\partial x_1} = \varphi_1 + \frac{dw}{dx_1},$$
$$\varepsilon_4 \equiv 0, \quad \varepsilon_6 \equiv 0,$$
i.e. we only have one longitudinal and one shear strain
$$\varepsilon_1(x_1,x_3) = \varepsilon_1(x_1) + x_3\kappa_1(x_1),$$
$$\varepsilon_5(x_1,x_3) = \varphi_1(x_1) + w'(x_1),$$
$$\varepsilon_1(x_1) = \frac{du}{dx_1}, \qquad \kappa_1(x_1) = \frac{d\varphi_1(x_1)}{dx_1}$$
When the transverse shear strain are neglected it follows with $\varepsilon_5 \approx 0$ that the relations is $\varphi_1(x_1) = -w'(x_1)$, and that is the bernoulli's kinematics. In the general case of an unsymmetric laminated Timoshenko's beam loaded orthogonally to the laminated plane and $N \neq 0,\ M \neq 0$, the constitutive equations (stress resultants – strain relations) are given by

N = $\bar{A}_{11}\epsilon_1 + \bar{B}_{11}\kappa_1$, M = $\bar{B}_{11}\epsilon_1 + \bar{D}_{11}\kappa_1$,
Q = $\kappa^s \bar{A}_{55}\gamma^s$

With $\bar{A}_{11} = bA_{11}$, $\bar{B}_{11} = bB_{11}$,
$\bar{D}_{11} = bD_{11}$, $\gamma^s = \varepsilon_5$

$Q_{11}^{(k)} = C_{11}^{(k)} = E_1^{(k)}$, $Q_{55}^{(k)} \equiv C_{55}^{(K)} = G_{13}^{(k)}$

$$\bar{A}_{11} = b\sum_{k=1}^{n} C_{11}^{(k)}\left(x_3^{(k)} - x_3^{(k-1)}\right) = b\sum_{k=1}^{n} C_{11}^{(k)} h^{(k)},$$

$$\bar{A}_{55} = \mathrm{b}\sum_{k=1}^{n} C_{55}^{(k)}\left(x_3^{(k)} - x_3^{(k-1)}\right) = \mathrm{b}\sum_{k=1}^{n} C_{55}^{(k)} h^{(k)},$$

$$\bar{B}_{11} = \frac{b}{2}\sum_{k=1}^{n} C_{11}^{(k)} \left({x_3^{(k)}}^2 - {x_3^{(k-1)}}^2\right),$$

$$\bar{D}_{11} = \frac{b}{3}\sum_{k=1}^{n} C_{11}^{(k)} \left({x_3^{(k)}}^3 - {x_3^{(k-1)}}^3\right)$$

κ^s is the shear deformation factor

The variational formulation for a lateral loaded symmetric laminate beam is

$$\pi(\omega,\varphi) = \frac{1}{2}\int_0^L \left[\bar{D}_{11}\left(\frac{d\varphi}{dx_1}\right)^2 + \kappa^s \bar{A}_{55}\left(\varphi + \frac{d\omega}{dx_1}\right)^2\right] dx_1 - \int_0^L q_0 \sin\left(\frac{\pi x_1}{L}\right)\omega dx_1$$

The approximate functions are

$$\bar{\omega}(x_1) = a_1 \sin\left(\frac{\pi x_1}{L}\right),$$

$$\bar{\varphi}(x_1) = b_1 cos\left(\frac{\pi x_1}{L}\right)$$

So,

$$\bar{\pi}(\bar{\omega},\bar{\varphi}) = \frac{1}{2}\int_0^L \left[\bar{D}_{11}\left(b_1\frac{\pi}{L}\right)^2 sin^2\frac{\pi x_1}{L} + \kappa^s \bar{A}_{55}(\right. dx_1 - \int_0^L q_0 \sin\left(\frac{\pi x_1}{L}\right)\left(a_1 \sin\left(\frac{\pi x_1}{L}\right)\right) dx_1 = \tilde{\pi}(a_1, b_1)$$

ABAQUS

ABAQUS is a suite of powerful engineering simulation programs, based on the finite element method, that can solve problems ranging from relatively simple linear analyses to the most challenging nonlinear simulations. ABAQUS contains an extensive library of elements that can model virtually any geometry. It has an equally extensive list of material models that can simulate the behavior of most typical engineering materials including metals, rubber, polymers, composites, reinforced concrete, crushable and resilient foams, and geotechnical materials such as soils and rock. Designed as a general-purpose simulation tool, ABAQUS can be used to study more than just structural (stress/displacement) problems. It can simulate problems in such diverse areas as heat transfer, mass diffusion, thermal management of electrical components (coupled thermal-electrical analyses), acoustics, soil mechanics (coupled pore fluid-stress analyses), and piezoelectric analysis.

ABAQUS is simple to use and offers the user a wide range of capabilities. Even the most complicated analyses can be modeled easily. For example, problems with multiple components are modeled by associating the geometry defining each component with the appropriate material models. In most simulations, including highly nonlinear ones, the user need only provide the engineering data such as the geometry of the structure, its material behavior, its boundary conditions, and the loads applied to it. In a nonlinear analysis ABAQUS automatically chooses appropriate load increments and convergence tolerances. Not only does it choose the values for these parameters, it also continually adjusts them during the analysis to ensure that an accurate solution is obtained efficiently. The user rarely has to define parameters for controlling the numerical solution of the problem.

The Abaqus Products

ABAQUS consists of two main analysis products—ABAQUS/Standard and ABAQUS/Explicit. There are also four special-purpose add-on analysis products for ABAQUS/Standard—ABAQUS/Aqua, ABAQUS/Design, ABAQUS/AMS, and ABAQUS/Foundation. In addition, the ABAQUS Interface for MOLDFLOW and the ABAQUS Interface for MSC.ADAMS are interfaces to MOLDFLOW and ADAMS/Flex, respectively. ABAQUS also provides translators that convert entities from third-party preprocessors to input for ABAQUS analyses and that convert output from ABAQUS analyses to entities for third-party postprocessors. ABAQUS/CAE is the complete ABAQUS environment that includes capabilities for creating ABAQUS models, interactively submitting and monitoring ABAQUS jobs, and evaluating results. ABAQUS/Viewer is a subset of ABAQUS/CAE that includes just the postprocessing functionality. The relationship between these products is shown in Figure 1–1.

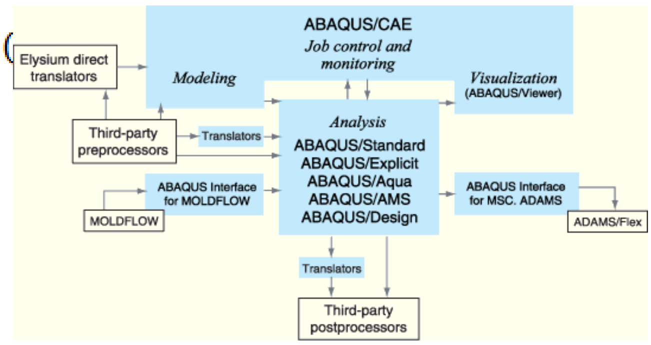


Figure 1–1: ABAQUS products

4. Formulation Used

The neutral axis x_1 of the beam is in an unknown distance αh from the top, $A_c \equiv A_m$ is the effective area of the concrete above the x_1-axis. The strains will vary linearly from the x_1-axis and the stresses will equal strain times the respective moduli. The stress resultant N (x_1)must be zero.

Now,

$$\sigma_f A_f - \frac{1}{2}\sigma_m(\alpha h) b\, \alpha h = 0,$$

$$\sigma_m(\alpha h) = \sigma_m{}^{max}$$

With the known equations for the strain ε_1 and the stress σ_1,

$$\varepsilon_1(x_3) = \epsilon_1 + x_3\kappa_1$$

$$\sigma_1(x_2, x_3) = \epsilon_1 E_1(x_2, x_3) + x_3\kappa_1 E_1(x_2, x_3)$$

$$\sigma_f = (h - \alpha h)\,\kappa_1 E_f,$$

$$\sigma_m(\alpha h) = \alpha h \kappa_1 E_m$$

i.e.

$$(h - \alpha h)E_f A_f - \frac{1}{2}(\alpha h)^2 b E_m = 0\,,$$

$$\alpha = \frac{E_f A_f}{E_m b h}\left(-1 + \sqrt{1 + 2\frac{bh}{A_f}\frac{E_m}{E_f}}\right)$$

Or

$$\alpha = \frac{1}{m}\left(-1 + \sqrt{1 + 2m}\right),$$

Where m $= \frac{E_m bh}{E_f A_f}$,

Now the bending moment is with

$$\sigma_f A_f = \frac{1}{2}\sigma_m b h^2 \alpha^2$$

$$M = (\sigma_f A_f)\left(h - \frac{\alpha h}{3}\right) = \sigma_f A_f h\left(1 - \frac{\alpha}{3}\right)$$

$$= \kappa_1 E_f A_f (h - \alpha h)\left(h - \frac{\alpha h}{3}\right),$$

The maximal stress in the concrete is

$$\sigma_m(\alpha h) = -\kappa_1 \alpha h E_m = M\frac{E_m \alpha h}{E_f A_f (h - \alpha h)\left(h - \frac{\alpha h}{3}\right)}$$

And the reinforcement stress is

$$\sigma_f = \kappa_1 (h - \alpha h)\, E_f = \frac{M}{\left(h - \frac{\alpha h}{3}\right) A_f}$$

The bending stiffness

$$\bar{D}_{11} = \frac{b}{3}\sum_{k=1}^{n} C_{11}^{(k)}\left({x_3^{(k)}}^3 - {x_3^{(k-1)}}^3\right)$$

The variational formulation for a lateral loaded symmetric laminate beam is

$$\pi(w,\psi) = \frac{1}{2}\int_0^L \left[\bar{D}_{11}\left(\frac{d\psi}{dx_1}\right)^2 + \kappa^s \bar{A}_{55}\left(\psi + \frac{dw}{dx_1}\right)^2\right]dx_1 - \int_0^L q_0\, w\, dx_1$$

$$\tilde{W}_{max} = \tilde{W}\left(x_1 = \frac{L}{2}\right) = \frac{4 q_0 L^4}{\bar{D}_{11}\pi^5}\left(1 + \frac{\bar{D}_{11}}{\kappa^s \bar{A}_{55}}\frac{\pi^2}{L^2}\right)$$

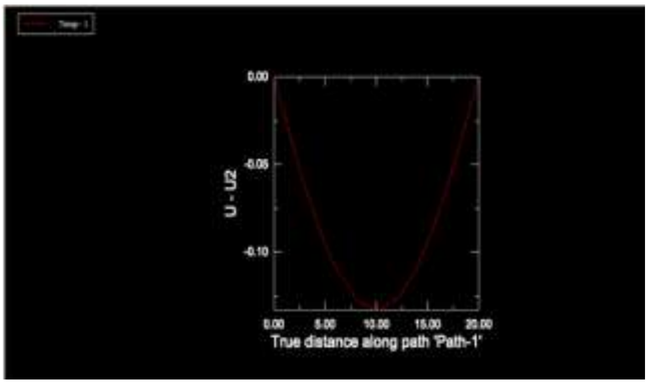
$$w = \frac{(100\tilde{w}) y_0}{h s^4 p_0}$$

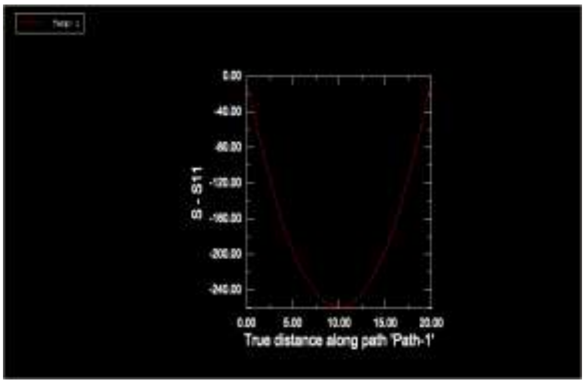
$$W_F = W_N L S \left(\frac{\rho}{Y_O}\right)^{\frac{1}{2}}$$

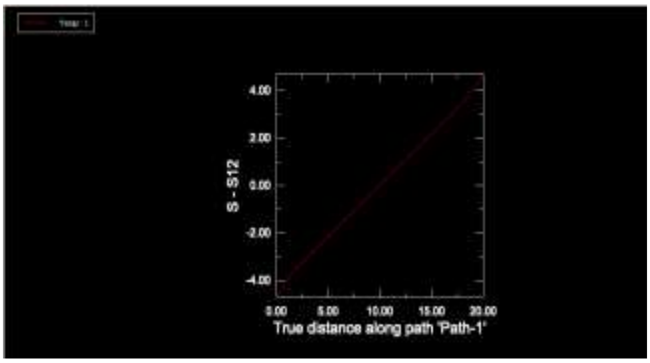
5. Result and Discussion

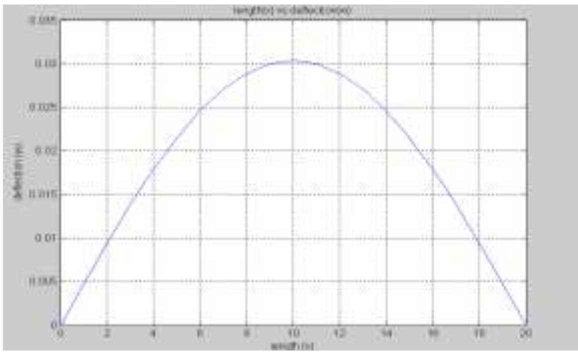
In this we have studied the static & dynamic analysis of composite beam. Composite beam used is reinforced concrete beam with steel reinforcement. The results finally shows the amount of deflection in a simply supported cross-ply laminated beam. The load applied was uniformly distributed. First the deflection is carried out analytically. The value of deflection comes out to 0.1863mm. The material properties of steel and concrete were defined. Using these properties the deflection of the composite beam was taken. Then the result was compared using abaqus. ABAQUS software was used to find out the deflection of the reinforced composite beams. The deflection from the abaqus software of the reinforced concrete beam comes to be 0.180mm approx. This shows that the deflection taken out analytically is correct. In this normal and shear stresses are calculated. The natural frequencies is presented in table.

Result for Static Analysis





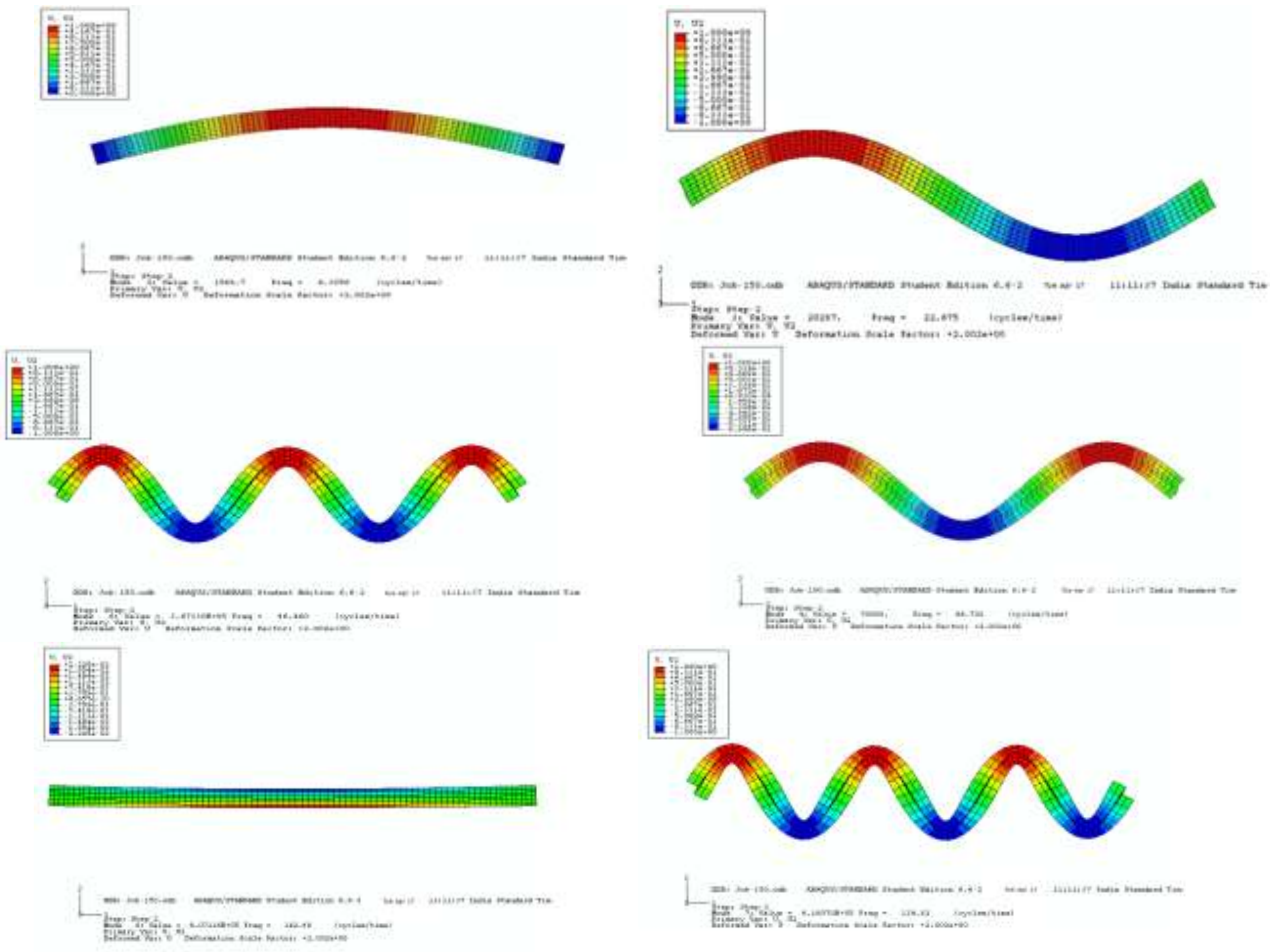


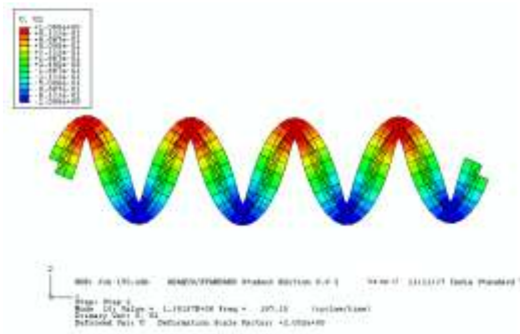


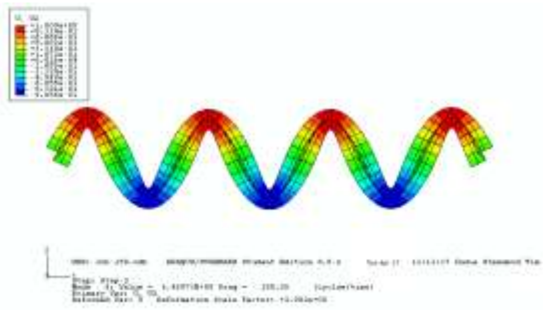
Graphical Results of Dynamic Analysis

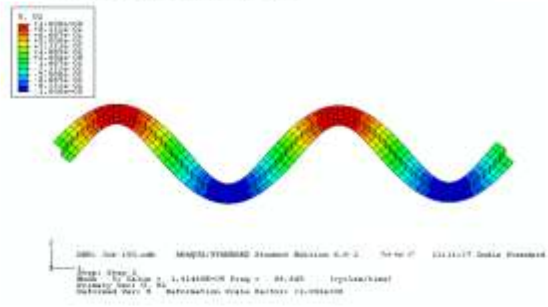
TABLE: Value of frequency with respect to mode number

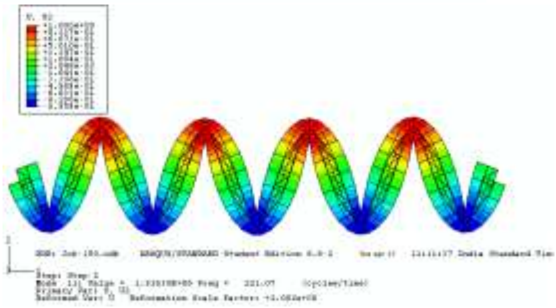
Serial number	Mode number	Frequency
1	1	0.4976
2	2	1.754
3	3	3.357
4	4	5.486
5	5	7.4398
6	6	9.760
7	7	11.048
8	8	11.984
9	9	14.462
10	10	17.086











6. Conclusion

Control of Deflection

The limiting deflections under two heads as given below:

(a) The maximum final deflection should not normally exceed span/250 due to all loads including the effect of temperatures, creeps and shrinkage measured from the as-cast level of the supports of floors, roofs and all other horizontal members.

(b) The maximum final deflection should not normally exceed the lesser of span/350 or 20mm including the effect of temperatures, creeps and shrinkage occuring after erections of partitions and the application of finishes.

It is essential that both the requirements are to be fulfilled for every structure.

7. Future Scope

1) Above technique can be used for other type of structure like concrete structures etc.
2) The present experimental analysis can be done for different types of beams made of different material.
3) The present experimental model can be used for developing the control strategies like FSDT strategies.
4) This technique can be used for other types of structures like trusses, plates, shells etc.

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