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Promoting Students' Achievement and Mathematical Flexibility Through Pre-Within-Post Problem Posing Tasks

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Abstract: ***This study investigated the effect of utilizing pre-within-post problem posing activities in enhancing students' achievement and mathematical flexibility in Differential Calculus. This study was conducted in two semesters composed of 75 regular third year BSEd Mathematics students during the first semester of school year 2017-2018 and 57 irregular engineering students enrolled in special class in Differential Calculus during the second semester of school year 2016-2017 at the University of Science and Technology of Southern Philippines-CDO campus. Using quasi-experimental pretest-posttest control group design in gathering quantitative data, a 16-item achievement test and a 4-item Multiple Solutions Tasks (MST) test intended to measure students' level of mathematical flexibility were given to the participants before and after the experiment. Data collected were analyzed using mean, standard deviation and two-way analysis of covariance (ANCOVA). Results of the analysis revealed that the use of pre-within-post problem posing activities helped students' improve their achievement and flexibility scores in Differential Calculus. The students' level of achievement and mathematical flexibility increased as influenced by their exposure to activities using problem posing activities. Hence, the researcher recommended that mathematics teachers may use pre-within-post problem posing activities in teaching mathematics to help improve students' mathematical flexibility. Furthermore, similar studies may be conducted to show associations between students' mental ability and levels of mathematical flexibility and study how technology rich classroom environment can effectively foster mathematical flexibility.***

Keywords: mathematical flexibility, pre-within-post problem posing, multiple solutions task test

1. Introduction

Inquiry-oriented mathematics instruction which includes problem posing and problem solving tasks in mathematics classes are considered essential and effective ways of developing achievement and creativity among students (Kim, 2009). Yuan & Sriraman (2010) and Kontorovich (2011) linked problem posing skills with creativity and citing flexibility, fluency and originality as creativity categories. However, Silver (1997) as cited by Bonotto and Danto (2014) and Siswono (2010) argued that creativity lies in the interplay between problem posing and problem solving and classified problem posing according to whether it takes place before (pre-solution), during (within-solution) or after (post-solution). Polya's four-step problem solving are heuristic strategies which are rules of thumb in successful problem solving (Schoenfed, 2014). However, Polya considered problem solving in a mathematics class impoverished without enriching it with problem posing hence the present study integrated both activities.

Problem posing is recognized as an important component of mathematics teaching and learning (NCTM, 2007). Problem posing involves generation of new problems and questions aimed at exploring a given situation and reformulation of a problem during the process of solving.

Lavy and Shriki (2007) conducted a study to determine the development of mathematical knowledge and problem solving skills of prospective teachers' as a result of their engagement in problem posing activity. Analysis of the data revealed that the prospective teachers developed their ability to examine definition and attributes of mathematical objects, connections among mathematical concepts and validity of an argument. However, they tend to focus on common posed problems because they are afraid of their inability to prove the answers to their problems posed. This finding suggests that overemphasizing the importance of providing a formal proof prevents the development of mathematical knowledge and problem solving skills.

Singer, Pelczer and Voica (2014) explored different types of behavior during the problem posing process by looking at the ways students value the problem data in solving and extending their own posed problems. Based on the outcomes of these analyses they explained the differences in students' success and failure in the problem posing approaches in relation to the level of understanding the solution of a problem and the novelty of the posed problems. They noticed that the more the student advances in the abstract dimension of the problem and its context, the more mathematically relevant are the newly obtained versions. The abstraction level of the solution process determines the novelty of the newly posed problems and it seems to be a good predictor of the child's creative potential.

Georgiev and Nedyalkova (2014) studied group creativity and development of mathematical problem posing and solving capabilities. The work treats the impact of problem posing and solving activities on development of group creativity in Secondary School Math Labs formed by students and teachers. The problem was studied analyzing problems posed and solved by different Labs for relatively longer period of few months. This action was compared with classical group Math competition, where the same students from the Labs solved problems for shorted period of few

hours. The results show that the impact of problem posing and solving activities on development of group creativity can be manifested more effectively when students have less time restrictions.

Tan (2015) conducted a study on the influence of problem posing and sense making on students' conceptual understanding, procedural knowledge, retention and anxiety towards mathematics. She concluded that problem posing and sense making was effective in enhancing students' conceptual knowledge and retention and not in procedural knowledge. Furthermore, she concluded that problem posing and sense making significantly reduced students' mathematics anxiety and they have a positive regard with problem posing and sense making. The present study centered on the investigation of the effect of integrating problem posing activities in the Differential Calculus work text on students' flexibility in mathematics.

2. Design, Setting and Participants of the Study

The study utilized a mixed method of research which includes the quantitative quasi-experimental control group and qualitative design. The quantitative part of this study examined the effect of integrating pre-within-post problem posing activities in the work text in Differential Calculus. The study was conducted at University of Science and Technology of Southern Philippines (USTP). Two (2) sections in Differential Calculus offered in the second semester of school year 2016-2017 and first semester of school year 2017-2018 was considered participants of this study which was handled by the researcher. The first set of participants are mixed engineering students enrolled in a special class in Differential Calculus and the second set are BSEd Math students enrolled in a regular offering of the same subject. One (1) section was randomly assigned using fishbowl method as the experimental group subjected to pre-within-post problem posing activities integrated in work text in Differential Calculus while the other section was considered as the control group which was taught using post problem posing activities but no work text used.

3. Results and Discussion

Table 1: Mean, Standard Deviation and Descriptive Level of Students' Achievement in Differential Calculus

Group of Learners	Experimental Group (n=75)				Control Group (n = 57)				Row Mean			
	$\bar{x}$	sd	$\bar{y}$	sd	$\bar{x}$	sd	$\bar{y}$	sd	$\bar{x}$	sd	$\bar{y}$	sd
BSEd Math Students	8.97	1.09	21.64	5.20	8.64	1.20	18.23	3.93	8.81	1.15	19.94	4.57
Mixed Engineering Students	9.33	3.78	19.69	4.15	9.48	2.33	18.85	3.61	9.41	3.06	19.27	3.88
Column Mean	9.15	2.44	21.00	4.67	9.06	1.76	18.54	3.77				
Descriptive Level	Beginning		Approaching Proficiency		Beginning		Developing					

Legend:	Mean Intervals	Description	Perfect Score is 40
	38 – 40	Excellent	
	31 – 37	Proficient	
	21 – 30	Approaching Proficiency	
	11 – 20	Developing	
	1 – 10	Beginning	

Table 1 above shows the analysis of the mean scores and standard deviation of the students' achievement in Differential Calculus. It can be observed from the table that pretest scores of both experimental and control groups from the two stages of implementation obtain scores which are all in beginning level. The experimental and control group of the regular BSEd-Mathematics students obtained a mean score difference of 0.33 which indicates that the experimental group have a slightly higher prior knowledge as compared to the control group. The mixed Engineering experimental group of students obtained a pretest mean score lower than the control group which suggest that the control group showed a little higher prior knowledge as compared to the experimental group.

During the posttest, both the experimental and control groups showed an increase of their achievement scores. The first experimental group which comprises the regular third year BSEd Mathematics obtain a score which has a descriptive level of approaching proficiency. The other experimental group which comprises the mixed Engineering students shows an improvement of their achievement scores however still in the developing level. Both control groups showed improvement of their achievement scores but same as the second experimental group, they are on the developing level. The first experimental group showed the highest increase as compared to all other groups and all experimental groups showed a higher increase of achievement scores compared to the control groups.

Overall, both the experimental and control groups showed an increase of their achievement scores. However, the experimental group yielded a higher increase of the posttest scores of 11.85 as compared to the control group. To determine if the treatment have significant effect further analysis using analysis of covariance is done.

Table 2: Mean, Standard Deviation and Descriptive Level of Students' Mathematical Flexibility in Differential Calculus

Group of Learners	Experimental Group (n=75)				Control Group (n = 57)				Row Mean			
	$\bar{x}$	sd	$\bar{y}$	sd	$\bar{x}$	sd	$\bar{y}$	sd	$\bar{x}$	sd	$\bar{y}$	sd
BSEd Math Students	5.81	1.70	9.06	1.37	5.88	1.78	7.91	1.08	5.84	1.74	8.49	1.23
Mixed Engineering Students	3.49	2.40	8.83	2.32	3.80	2.68	8.36	2.61	3.65	2.54	8.60	2.47
Column Mean	4.65	2.05	8.95	1.85	4.84	2.23	8.14	1.84				
Descriptive Level	Developing		Approaching Proficiency		Developing		Developing					

Legend:	Mean Intervals	Description	Perfect Score is 16
	15.50 – 16.00	Excellent	
	12.5 – 15.49	Proficient	
	8.50 – 12.49	Approaching Proficiency	
	4.50 – 8.49	Developing	
	1.00 – 4.49	Beginning	

3.1 Sections headings

Section headings come in several varieties:

1. first level headings: 1. Heading 1

Table 2 above shows the level of students' mathematical flexibility during the pretest and posttest. It can be observed from the table that during the pretest, the regular mathematics education group of participants obtain a mathematical flexibility scores which are in the developing level while the special class engineering group are still in the beginning level as indicated by their pretest scores. Taken altogether, the experimental and control groups of this study showed a developing level of mathematical flexibility before the start of the experiment. This result suggests that students in both groups have partial understanding of the concepts in Calculus.

In the posttest, both experimental groups showed an increase of their flexibility scores which reaches an approaching proficiency level of mathematical flexibility. The first experimental group showed a better increase as compared to the second experimental group but this goes to show that exposure to the Calculus work text with pre-within-post problem posing activities helped the students improved their mathematical flexibility especially that in the exercise students were tasked to solve problems in many different ways.

Taken jointly, the same result showed that the experimental groups reached the approaching proficiency level and the control groups only reach the developing level in terms of their mathematical flexibility scores in the posttest. This indicates that both groups improved but the experimental group showed higher increase as compared to the control groups which means that the pre-within-post problem posing activities helped the students improved their mathematical flexibility scores. The control group also showed an increase because they are also exposed to problem solving and problem posing activities.

4. Conclusions and Recommendations

The researchers concluded that the use of pre-within-post problem posing activities helped students' improve their level of mathematical flexibility and the BSEd-Mathematics students performed better than the mixed engineering students in terms of their mathematical flexibility. The type of participants and the treatment have mixed effect on students' flexibility test scores in Differential Calculus and students had a favorable response on the use of pre-within-post problem posing activities. Based on the findings and conclusions of the study, the researchers recommended that mathematics teachers may use pre-within-post problem posing activities in teaching content subjects in mathematics to improve students' flexibility scores, teachers may regularly design activities in their mathematics classes which requires students' to posit novel solutions to problems in order to develop their creative potential and USTP mathematics teachers may develop their own work text with pre-within-post problem posing activities. Similar studies may be conducted to show the association between students' mental ability and level of mathematical flexibility and study how technology integration can effectively foster mathematical flexibility and how students joining mathematics competition exhibit and develop their flexibility of their solutions.

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