Generalization of Rough Set Theory Using a Finite Number of a Finite d. g.'s

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Abstract: This paper is concerned with introducing and studying the new approximation operators based on a finite family of d. g.'s which are the core concept in this paper. In addition, we study generalization of some Pawlak's concepts and we offer generalize the definition of accuracy measure of approximations by using a finite family of d. g.'s.

Keywords: Digraph, Mixed degree set, n-lower approximation, n-upper approximation and n-accuracy measure.

2000 Mathematics Subject Classification: 04A05, 54A05, 05C20.

1. Introduction and preliminaries

Rough set theory was developed by Pawlak in 1982 [8], since then it has been widely applied in many fields, such as machine learning data mining and pattern recognition and the original rough set theory was developed framework of set theory algebra and logic. We relied on proposition in [21] and we built on some of the results in [1], [3], [4], [6], [7], [10], [11], [12], [13], [14], [15], [17], [18], [19], [20] and [22].

A directed graph (d. g.) [16] is pair \( D = (V(D), E(D)) \) where \( V(D) \) is a non-empty set (called vertex set) and \( E(D) \) of ordered pairs of elements of \( V(D) \) (called edge set). An edge of the from \((a, a)\) is called a loop. If \( a \in V(D) \), the out-degree of \( a \) is \( |\{u \in V(D) : (a, u) \in E(D)\}|\) and in-degree of \( a \) is \( |\{u \in V(D) : (u, a) \in E(D)\}|\). A subd. g. of a d. g. \( D \) is a d. g. each of whose vertices belong to \( V(D) \) and each of whose edges belong to \( E(D) \). An empty d. g. [2] if the vertices set and edge set is empty. The out-degree of \( a \) is denoted by \( OD(a) \) and defined by: \( OD(a) = |\{u \in V(D) : (a, u) \in E(D)\}|\) and in-degree of \( a \) is denoted by \( ID(a) \) and defined by: \( ID(a) = |\{u \in V(D) : (u, a) \in E(D)\}|\). Let \( D = (V(D), E(D)) \) be a d. g. The mixed degree system of a vertex \( a \in V(D) \) is denoted by \( MDS(a) \) and defined by: \( MDS(a) = \{ODS(a), IDS(a)\} \). Let \( D = (V(D), E(D)) \) be a d. g. The mixed degree system of a vertex \( a \in V(D) \) is denoted by \( MDS(a) \) such that \( MD(a) \in MDS(a) \). The lower and upper approximations of \( H \) using mixed degree systems are denoted by \( L_{MD}(V(H)) \) and \( U_{MD}(V(H)) \) for some \( MD(a) \subseteq V(H) \) and \( U_{MD}(V(H)) = \{a \in V(D) : \forall MD(a) \cap V(H) \neq \emptyset\} \).

2. New approximation operators based on a finite family of d. g.'s

In this section, some of their definitions and propositions about new approximation operators on a family of d. g.'s are studied and we gave examples in the case of properties that are not true in general.

Definition 2.1. Let \( D = \{D_i : i = 1, 2, 3, \ldots, n\} \) be a finite family of arbitrary non-empty finite d. g.'s. The \( n \)-lower and \( n \)-upper approximations of \( H \subseteq D \) according to \( D \) are denoted by \( L_{MD}(V(H)) \) and \( U_{MD}(V(H)) \), respectively and defined by:

\[
L_{MD}(V(H)) = \bigcup_{i=1}^{n} L_{MD}(V(H)), U_{MD}(V(H)) = \bigcap_{i=1}^{n} U_{MD}(V(H)).
\]

Definition 2.2. Let \( D = \{D_i : i = 1, 2, 3, \ldots, n\} \) be a finite family of d. g.'s which are the core concept in this paper. In addition, we study generalization of some Pawlak's concepts and we offer generalize the definition of accuracy measure of approximations by using a finite family of d. g.'s.

Example 2.3. Let \( D = \{D_i : i = 1, 2, 3\} \) be three d. g.'s defined as: \( V(D_1) = V(D_2) = V(D_3) = \{a_1, a_2, a_3, a_4, a_5, a_6\} \), \( E(D_1) = \{(a_1, a_2), (a_2, a_3), (a_3, a_4), (a_4, a_5), (a_5, a_6), (a_6, a_1), (a_1, a_3), (a_3, a_5), (a_5, a_2), (a_2, a_4), (a_4, a_3), (a_3, a_1)\} \), \( E(D_2) = \{(a_1, a_2), (a_2, a_3), (a_3, a_4), (a_4, a_5), (a_5, a_1), (a_1, a_3), (a_3, a_5), (a_5, a_2), (a_2, a_4), (a_4, a_3), (a_3, a_1)\} \) and \( E(D_3) = \{(a_1, a_2), (a_2, a_3), (a_3, a_4), (a_4, a_5), (a_5, a_1)\} \).
The mixed degree systems based on $D_1$ are given by:

$$MD_{m}(\omega_1) = \{\{\omega_1, \omega_2, \omega_3\}, \{\omega_1, \omega_3\}\}, MD_{m}(\omega_2) = \{\{\omega_2, \omega_3\}\}, MD_{m}(\omega_3) = \{\{\omega_2, \omega_3\}\} \text{ and } MD_{m}(\omega_4) = \{\{\omega_2, \omega_5\}\} \text{ and } MD_{m}(\omega_5) = \{\{\omega_2, \omega_5\}\}$$

The mixed degree systems based on $D_2$ are given by:

$$MD_{m}(\omega_1) = \{\{\omega_1, \omega_2, \omega_3\}, \{\omega_1, \omega_3\}\}, MD_{m}(\omega_2) = \{\{\omega_2, \omega_3\}\}, MD_{m}(\omega_3) = \{\{\omega_2, \omega_3\}\} \text{ and } MD_{m}(\omega_4) = \{\{\omega_2, \omega_3\}\} \text{ and } MD_{m}(\omega_5) = \{\{\omega_2, \omega_3\}\}.$$
Proposition 2.4. Let $D = \{D_i; i = 1, 2, 3, \ldots, n\}$ be a finite family of arbitrary non-empty finite d. g. ’s, then the following hold for every $H, K \subseteq D$:

(1) $L_d(V(D)) = V(D)$,
(2) $L_d(V(H)) \subseteq L_d(V(K))$,
(3) $L_d(V(H) \cup V(K)) \supseteq L_d(V(H)) \cap L_d(V(K))$,
(4) $L_d(V(H) \cup V(K)) \supseteq L_d(V(H)) \cup L_d(V(K))$,
(5) $L_d(V(H) - V(K)) \subseteq L_d(V(H)) - L_d(V(K))$.

Proof.

The boundary according to $D$, for all $H \subseteq D$, are given in the table:

<table>
<thead>
<tr>
<th>$V(H)$</th>
<th>$Bd_d(V(H))$</th>
<th>$Bd_d(V(H))$</th>
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</tbody>
</table>

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1045
(L_2) By (L_2) in Proposition (2.3.1) in [21], we have
\[ L_{\text{mod}}(V(D)) = V(D) \setminus \mathcal{N} = \{1, 2, \ldots, n\} \]
\[ \Rightarrow \bigcup_{i=1}^{n} L_{\text{mod}}(V(D)) = V(D) \setminus \mathcal{N} \]
\[ \Rightarrow L_{\text{mod}}(V(D)) = V(D). \]

(L_3) Let \[ H \subseteq K, \text{ then by (L_3) in Proposition (2.3.1) in [21], we get} \]
\[ L_{\text{mod}}(V(H)) \subseteq L_{\text{mod}}(V(K)) \forall i = 1, 2, \ldots, n \]
\[ \Rightarrow \bigcup_{i=1}^{n} L_{\text{mod}}(V(H)) \subseteq \bigcup_{i=1}^{n} L_{\text{mod}}(V(K)) \]
\[ \Rightarrow L_{\text{mod}}(V(H)) \subseteq L_{\text{mod}}(V(K)). \]

(L_4) Let \[ V(K) \subseteq V(H) \text{ and } V(H) \subseteq V(K), \text{ then by Proposition (2.3.1) in [21], we have that} \]
\[ L_{\text{mod}}(V(H)) \subseteq L_{\text{mod}}(V(K)) \wedge L_{\text{mod}}(V(H) \cap V(K)) \subseteq L_{\text{mod}}(V(K)) \]
\[ \Rightarrow \bigcup_{i=1}^{n} L_{\text{mod}}(V(H)) \subseteq \bigcup_{i=1}^{n} L_{\text{mod}}(V(K)) \wedge \bigcup_{i=1}^{n} L_{\text{mod}}(V(H)) \subseteq \bigcup_{i=1}^{n} L_{\text{mod}}(V(K)) \]
\[ \Rightarrow L_{\text{mod}}(V(H)) \subseteq L_{\text{mod}}(V(K)) \bigwedge L_{\text{mod}}(V(H) \cap V(K)) \subseteq L_{\text{mod}}(V(K)). \]

(L_5) By substituting \[ V(D) \setminus \{i\} \text{ for } V(D) \text{ in } (L_2), \text{ we have} \]
\[ L_{\text{mod}}(V(D)) = \bigcup_{i=1}^{n} L_{\text{mod}}(V(D)) \setminus \mathcal{N} = \{i\} \]
\[ \Rightarrow L_{\text{mod}}(V(D)) = \bigcup_{i=1}^{n} L_{\text{mod}}(V(D)) \setminus \{i\} \]
\[ \Rightarrow L_{\text{mod}}(V(D)) = \bigcup_{i=1}^{n} L_{\text{mod}}(V(D)) \setminus \{i\} \]
\[ \Rightarrow L_{\text{mod}}(V(D)) = \bigcup_{i=1}^{n} L_{\text{mod}}(V(D)) \setminus \{i\}. \]

Remark 2.5. Let \[ D = \{D_i; i = 1, 2, 3, \ldots, n\} \text{ be a finite} \]
\[ \text{family of arbitrary non-empty finite d. g. s. and } K \subseteq D, \text{ then the following are not true in general:} \]

(L_6) \[ L_{\text{mod}}(V(H)) \subseteq V(H), \]
\[ \text{then by Proposition (2.3.1)} \]
\[ \Rightarrow \bigcup_{i=1}^{n} L_{\text{mod}}(V(H)) \subseteq \bigcup_{i=1}^{n} L_{\text{mod}}(V(H)) \]
\[ \Rightarrow L_{\text{mod}}(V(H)) \subseteq L_{\text{mod}}(V(H)). \]

Example 2.6. \[ H = (V(H), E(H)) : V(H) = \{a_1, a_2\}, \text{ then} \]
\[ L_{\text{mod}}(V(H)) = \{a_1, a_2\}. \]

Therefore, \[ L_{\text{mod}}(V(H)) \neq V(H). \]

(L_7) \[ H = (V(H), E(H)) : V(H) = \{a_1, a_2, a_3\}, \text{ then} \]
\[ L_{\text{mod}}(V(H)) = \{a_1, a_2, a_3\}. \]

Therefore, \[ L_{\text{mod}}(V(H)) \neq V(H). \]

(L_8) \[ H = (V(H), E(H)) : V(H) = \{a_1, a_2, a_3, a_4\}, \text{ then} \]
\[ L_{\text{mod}}(V(H)) = \{a_1, a_2, a_3, a_4\}. \]

Therefore, \[ L_{\text{mod}}(V(H)) \neq V(H). \]

(L_9) \[ H = (V(H), E(H)) : V(H) = \{a_1, a_2, a_3, a_4, a_5\}, \text{ then} \]
\[ L_{\text{mod}}(V(H)) = \{a_1, a_2, a_3, a_4, a_5\}. \]

Therefore, \[ L_{\text{mod}}(V(H)) \neq V(H). \]
(U₁₀) Let \( H = (V(H), E(H)) \) where \( V(H) = \{ (σ₁, σ₂) \} \) and \( E(H) = \{ (σ₁, σ₂) \} \). Then \( L_2(V(H)) = \{ (σ₁, σ₂) \} \) and \( U_2(L_2(V(H))) = \{ (σ₁, σ₂) \} \). Therefore, \( V(H) \not\subseteq U_2(L_2(V(H))) \).

(U₁₂) In Example (2.7), Let \( H = (V(H), E(H)) \) where \( V(H) = \{ (σ₁, σ₂), (σ₁, σ₃), (σ₁, σ₄), (σ₁, σ₅) \} \) and \( E(H) = \{ (σ₁, σ₂), (σ₁, σ₃), (σ₁, σ₄), (σ₁, σ₅), (σ₁, σ₆) \} \). Then \( U_2(V(H)) = \{ (σ₁, σ₂), (σ₁, σ₃), (σ₁, σ₄), (σ₁, σ₅), (σ₁, σ₆) \} \). Therefore, \( U_2(V(H)) \not\subseteq U_2(L_2(V(H))) \).

(U₂) Let \( H = (V(H), E(H)) \) where \( V(H) = \{ (σ₁, σ₂), (σ₁, σ₃), (σ₁, σ₄), (σ₁, σ₅) \} \) and \( E(H) = \{ (σ₁, σ₂), (σ₁, σ₃), (σ₁, σ₄), (σ₁, σ₅), (σ₁, σ₆) \} \). Then \( V(H) \cup E(H) = \{ (σ₁, σ₂), (σ₁, σ₃), (σ₁, σ₄), (σ₁, σ₅), (σ₁, σ₆) \} \). Therefore, \( U_2(V(H)) \not\subseteq U_2(L_2(V(H))) \).

Example 2.7. Let \( D = \{ D_i : i = 1, 2, 3 \} \) be three d. g. ‘s defined as: \( V(D) = V(D_1) = V(D_2) = V(D_3) = \{ σ₁, σ₂, σ₃, σ₄, σ₅, σ₆ \} \), \( E(D_1) = \{ (σ₁, σ₂), (σ₁, σ₃), (σ₁, σ₄), (σ₁, σ₅), (σ₁, σ₆), (σ₂, σ₃), (σ₂, σ₄), (σ₂, σ₅), (σ₂, σ₆), (σ₃, σ₄), (σ₃, σ₅), (σ₃, σ₆), (σ₄, σ₅), (σ₄, σ₆), (σ₅, σ₆) \} \) and \( E(D_2) = \{ (σ₁, σ₂), (σ₁, σ₃), (σ₁, σ₄), (σ₁, σ₅), (σ₁, σ₆), (σ₂, σ₃), (σ₂, σ₄), (σ₂, σ₅), (σ₂, σ₆), (σ₃, σ₄), (σ₃, σ₅), (σ₃, σ₆), (σ₄, σ₅), (σ₄, σ₆), (σ₅, σ₆) \} \).

The mixed degree systems based on \( D_1 \) are given by:
\[
MD_{m_1}(σ₁) = \{ [σ₁, σ₂, σ₃], [σ₂, σ₃] \}, MD_{m_2}(σ₂) = \{ [σ₁, σ₂, σ₃], [σ₂, σ₃] \}, MD_{m_3}(σ₃, σ₄) = \{ [σ₁, σ₂, σ₃, σ₄] \}, MD_{m_4}(σ₄, σ₅) = \{ [σ₁, σ₂, σ₃, σ₄, σ₅] \}.
\]

The mixed degree systems based on \( D_2 \) are given by:
\[
MD_{m_1}(σ₁) = \{ [σ₁, σ₂, σ₃], [σ₂, σ₃] \}, MD_{m_2}(σ₂) = \{ [σ₁, σ₂, σ₃, σ₄] \}, MD_{m_3}(σ₃, σ₄) = \{ [σ₁, σ₂, σ₃, σ₄, σ₅] \}, MD_{m_4}(σ₄, σ₅) = \{ [σ₁, σ₂, σ₃, σ₄, σ₅] \}.
\]

The mixed degree systems based on \( D_3 \) are given by:
\[
MD_{m_1}(σ₁) = \{ [σ₁, σ₂, σ₃], [σ₂, σ₃] \}, MD_{m_2}(σ₂) = \{ [σ₁, σ₂, σ₃, σ₄] \}, MD_{m_3}(σ₃, σ₄) = \{ [σ₁, σ₂, σ₃, σ₄, σ₅] \}, MD_{m_4}(σ₄, σ₅) = \{ [σ₁, σ₂, σ₃, σ₄, σ₅] \}.
\]

The lower approximation, for all \( H \subseteq D_3 \), are given in the table:

<table>
<thead>
<tr>
<th>V(H)</th>
<th>L₀(V(H))</th>
<th>L₂(V(H))</th>
<th>L₄(V(H))</th>
<th>L₆(V(H))</th>
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</table>

The mixed degree systems based on \( D_3 \) are given by:
\[
MD_{m_1}(σ₁) = \{ [σ₁, σ₂, σ₃], [σ₂, σ₃] \}, MD_{m_2}(σ₂) = \{ [σ₁, σ₂, σ₃, σ₄] \}, MD_{m_3}(σ₃, σ₄) = \{ [σ₁, σ₂, σ₃, σ₄, σ₅] \}, MD_{m_4}(σ₄, σ₅) = \{ [σ₁, σ₂, σ₃, σ₄, σ₅] \}.
\]
The upper approximation, for all $H \subseteq D$, are given in the table:

<table>
<thead>
<tr>
<th>$V(H)$</th>
<th>$U_{u_1}(V(H))$</th>
<th>$U_{u_2}(V(H))$</th>
<th>$U_{u_3}(V(H))$</th>
<th>$U_{u_4}(V(H))$</th>
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</thead>
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The boundary according to $D$, for all $H \subseteq D$, are given in the table:

<table>
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<tr>
<th>$V(H)$</th>
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<th>$Bd_{u_2}(V(H))$</th>
<th>$Bd_{u_3}(V(H))$</th>
<th>$Bd_{u_4}(V(H))$</th>
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3. Generalization of some Pawlak’s concepts and definition using a finite number of d.g.'s

In this section, we introduced generalization of some Pawlak’s concepts, offer some definition using a finite number of d. g.’s and we gave examples to illustrate these definitions.

Definition 3.1. Let \( D = \{ D_i; i = 1, 2, 3, \ldots, n \} \) be a finite family of non-empty d. g.’s. A subd. g. \( H \subseteq D \) is called:

(a) \( R \)-definable (or \( R \)-exact) d. g. if \( Bd_d(V(H)) = \phi \).
(b) \( R \)-rough d. g. if \( Bd_d(V(H)) \neq \phi \).

Remark 3.2. On the contrary to the case of classical rough set theory, there exists some subd. g. \( H \subseteq D \) for which \( Bd_d(V(H)) = \phi \) but \( U_d(V(H)) = U_d(V(H)) \). For example, consider \( H = (V(H), E(H)); V(H) = \{ a_2, a_3, a_5 \}, E(H) = \{ \{ a_2, a_2 \}, \{ a_2, a_3 \}, \{ a_2, a_5 \}, \{ a_3, a_3 \}, \{ a_3, a_5 \}, \{ a_5, a_5 \} \} \). Let \( U_d(V(H)) = \{ a_2, a_3, a_5 \} \) and \( Bd_d(V(H)) = \phi \).

In classical rough set theory, it is obvious that the intersection, the union and the difference of two definable sets is also definable [5].

Remark 3.3. On the contrary to the case of classical rough set theory, the intersection (union and difference) of two \( R \)-definable d. g.’s is not necessarily \( R \)-definable as the following example illustrates.

Example 3.4. According to Example (2.3)

(a) \( H, K \) are two \( R \)-definable but \( H \cap K \) not \( R \)-definable.
(b) \( R \)-rough graph if \( \eta_d(V(H)) = \phi \).

Table 3.1: \( \eta_d(V(H)), \eta_d(V(H)), \eta_d(V(H)) \) and \( \eta_d(V(H)) \) for all \( H \subseteq D \).

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<tr>
<th>( V(H) )</th>
<th>( \eta_d(V(H)) )</th>
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<tr>
<td>( { a_2 } )</td>
<td>4/5</td>
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<td>( { a_3 } )</td>
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<td>( { a_5 } )</td>
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<td>( { a_2, a_3 } )</td>
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Using the accuracy of the approximations \( \eta_d(V(H)) \). Another definition of \( R \)-rough and \( R \)-exact graphs is introduced as follows:

Definition 3.6. Let \( D = \{ D_i; i = 1, 2, 3, \ldots, n \} \) be a finite family of non-empty arbitrary d. g.’s. A subd. g. \( H \subseteq D \) is called:

(a) \( R \)-definable (or \( R \)-exact) graph if \( \eta_d(V(H)) = 1 \).
(b) \( R \)-rough graph if \( 0 \leq \eta_d(V(H)) < 1 \).

Example 3.7. According to Example (2.3), we have the following table

Table 3.1: \( \eta_d(V(H)), \eta_d(V(H)), \eta_d(V(H)) \) and \( \eta_d(V(H)) \) for all \( H \subseteq D \).
In the above table, for instance, we see that the degree of exactness of the subd. g. \( H = (V(H), E(H)) \); \( V(H) = \{ \sigma_2, \sigma_3, \sigma_5 \} \), \( E(H) = \{ (\sigma_2, \sigma_3), (\sigma_2, \sigma_5), (\sigma_3, \sigma_5) \} \) by using \( D_1 \) equals to 40% and by using \( D_2 \) equals to 40% and by using \( D_3 \) equals to 80%. But when we use \( D = \{ D_i; i = 1, 2, 3 \} \), the degree of exactness of the subd. g. \( H \) equals to 100%.

**Proposition 3.8.** Let \( D = \{ D_i; i = 1, 2, 3, \ldots, n \} \) be a finite family of arbitrary non-empty d. g. and \( H \subseteq D \), then the following statements are true:
(a) \( L_{ol}(V(H)) \subseteq L_{ol}(V(H)) \)
(b) \( U_{al}(V(H)) \subseteq U_{al}(V(H)) \)
(c) \( Bd_{al}(V(H)) \subseteq Bd_{al}(V(H)) \).

**Proof.** The proof (a) and (b) by Definition (2.1). The proof of (c), on use can use (a) and (b).

**Corollary 3.9.** Let \( D = \{ D_i; i = 1, 2, 3, \ldots, n \} \) be a finite family of arbitrary non-empty d. g. \( \sigma \)'s and \( H \subseteq D \), then \( \eta(H) \geq \max \{ \rho(H); i = 1, 2, \ldots, n \} \).

**Proof.** By using Proposition (3.8) (c), we have
\[
\frac{|Bd_{al}(V(H))|}{|V(D)|} \leq \frac{1}{|V(D)|} \frac{|Bd_{al}(V(H))|}{\rho(H)}
\]
\[
\eta(H) \geq \max \{ \rho(H); i = 1, 2, \ldots, n \}
\]

**Example 3.11.** In Example (2.3), we have,
(a) R-bottom equal (\( H \equiv_K K \)) if \( L_{ol}(V(H)) = L_{ol}(V(K)) \)
(b) R-top equal (\( H \equiv_K K \)) if \( U_{al}(V(H)) = U_{al}(V(K)) \)
(c) R-equall (\( H \equiv_K K \)) if (\( H \equiv_{K'} K \)) and \( H \equiv_{K''} K \).

**Definition 3.10.** Let \( D = \{ D_i; i = 1, 2, 3, \ldots, n \} \) be a finite family of non-empty arbitrary d. g., then the subd. g. \( H, K \subseteq D \) are called:
(a) R-bottom equal (\( H \equiv_K K \)) if \( L_{ol}(V(H)) = L_{ol}(V(K)) \)
(b) R-top equal (\( H \equiv_K K \)) if \( U_{al}(V(H)) = U_{al}(V(K)) \)
(c) R-equal (\( H \equiv_K K \)) if (\( H \equiv_{K'} K \)) and \( H \equiv_{K''} K \).
then \( L_d(V(H)) = \{ \sigma_4, \sigma_5 \} \), \( L_d(V(K)) = \{ \sigma_2, \sigma_4, \sigma_5 \} \). Therefore, \( L_d(V(H)) \subseteq L_d(V(K)) \).

(b) \( H \) is called R-top included in \( K \) (denoted by \( H \subseteq K \)) if \( U_d(V(H)) \subseteq U_d(V(K)) \).

Let \( H = (V(H), E(H)): V(H) = \{ \sigma_2, \sigma_3 \}, E(H) = \{ (\sigma_2, \sigma_3), (\sigma_3, \sigma_3) \} \) and \( K = (V(K), E(K)): V(K) = \{ \sigma_4, \sigma_5 \}, E(K) = \{ (\sigma_4, \sigma_5), (\sigma_5, \sigma_5), (\sigma_4, \sigma_4), (\sigma_5, \sigma_3) \} \) then \( U_d(V(H)) = \phi, U_d(V(K)) = \{ \sigma_3 \} \). Therefore, \( U_d(V(H)) \subseteq U_d(V(K)) \).

(c) \( H \) is called R-roughly included in \( K \) (denoted by \( H \subseteq R K \)) if \((H \subseteq K) \) and \((H \subseteq R K) \).

Let \( H = (V(H), E(H)): V(H) = \{ \sigma_1, \sigma_2, \sigma_5 \}, E(H) = \{ (\sigma_1, \sigma_2), (\sigma_1, \sigma_5), (\sigma_2, \sigma_2), (\sigma_5, \sigma_5) \}, K = (V(K), E(K)): V(K) = \{ \sigma_3, \sigma_4, \sigma_5 \}, E(K) = \{ (\sigma_3, \sigma_4), (\sigma_4, \sigma_4), (\sigma_3, \sigma_3), (\sigma_4, \sigma_5), (\sigma_5, \sigma_5) \} \) then \( L_d(V(H)) = \{ \sigma_1, \sigma_2, \sigma_5 \}, L_d(V(K)) = \{ \sigma_3, \sigma_4, \sigma_5 \} \), \( U_d(V(H)) = \{ \sigma_1, \sigma_2, \sigma_5 \} \) \( U_d(V(K)) = \{ \sigma_3, \sigma_4, \sigma_5 \} \). Therefore, \( L_d(V(H)) \subseteq L_d(V(K)) \) and \( U_d(V(H)) \subseteq U_d(V(K)) \).

Definition 3.14. Let \( D = \{ D_i; i = 1, 2, 3, \ldots, n \} \) be a finite family of non-empty arbitrary d. g. and subd. g. ‘s \( H, K \in D \).

Then

(a) \( \sigma \in \mathcal{E}_R(V(H)) \) if \( \sigma \in \mathcal{E}_d(V(H)) \).

(b) \( \sigma \in \mathcal{E}_R(U_d(V(H))) \).

(c) \( \sigma \in \mathcal{E}_R(V(H)) \) if \( \sigma \in \mathcal{E}_d(U_d(V(H))) \) and \( \sigma \in \mathcal{E}_d(V(H)) \).

Example 3.15. In Example (2.3), we have,

(a) \( \sigma \in \mathcal{E}_R(V(H)) \) if \( \sigma \in \mathcal{E}_d(U_d(V(H))) \).

Let \( H = (V(H), E(H)): V(H) = \{ \sigma_4, \sigma_5 \}, E(H) = \{ (\sigma_4, \sigma_4) \}, \) then \( L_d(V(H)) = \{ \sigma_4, \sigma_5 \}, U_d(V(H)) = \{ \sigma_4, \sigma_5 \} \). Therefore, \( \sigma \in \mathcal{E}_R(V(H)) \).

(b) \( \sigma \in \mathcal{E}_R(U_d(V(H))) \).

Let \( H = (V(H), E(H)): V(H) = \{ \sigma_1, \sigma_2, \sigma_5 \}, E(H) = \{ (\sigma_1, \sigma_2), (\sigma_1, \sigma_5), (\sigma_2, \sigma_2), (\sigma_5, \sigma_5) \} \), then \( U_d(V(H)) = \{ \sigma_1, \sigma_2, \sigma_5 \} \). Therefore, \( \sigma \in \mathcal{E}_R(U_d(V(H))) \).

(c) \( \sigma \in \mathcal{E}_R(V(H)) \) if \( \sigma \in \mathcal{E}_d(U_d(V(H))) \) and \( \sigma \in \mathcal{E}_d(U_d(V(H))) \).

Let \( H = (V(H), E(H)): V(H) = \{ \sigma_2, \sigma_3, \sigma_5 \}, E(H) = \{ (\sigma_2, \sigma_3), (\sigma_3, \sigma_5), (\sigma_3, \sigma_3), (\sigma_5, \sigma_3), (\sigma_5, \sigma_5) \} \), then \( L_d(V(H)) = \{ \sigma_2, \sigma_3, \sigma_5 \} \). Therefore, \( \sigma \in \mathcal{E}_R(V(H)) \).

4. Conclusions

A generalization of approximation operators in rough set theory is introduced using a finite number of a finite d. g. ’s and based on mixed degree systems. Proposition (3.8) and Corollary (3.9) show the effectiveness of this new approach in increasing the accuracy of the approximation of d. g. ’s since \( \eta_i(V(H)) \geq \max \{ \eta_i(V(H)): i = 1, 2, \ldots, n \} \). It is clear from Proposition (3.8) that by using the lower and upper approximation defined in Definition (2.1); we decrease the boundary region of this d. g. by using the lower and upper approximation defined in Definition (2.2.1) in [21].

References


