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Results for Fuzzy Casual Stringy Differential Dissimilarity

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Abstract: ***The destination of this work is to detection some fuzzy eventuality trait of the fuzzy resolutions for non-homogenous fuzzy casual stringy differential dissimilarity with firm coefficients fuzzy numbers with non-firm fuzzy casual function on the right hand side.***

Keywords: Eventuality trait, fuzzy eventuality trait, firm casual stringy differential dissimilarity, fuzzy firm casual stringy differential dissimilarity.

1. Introduction

Anywhere this work we announce for the fuzzy casual stringy differential dissimilarity via F.C.S.D.D., R.H.S. for the right hand side, F.C.F for the fuzzy casual function, F.S for fuzzy sets and F.D.E. for fuzzy differential equations.

The motif of F.S. press via Zadeh whereas the matter of F.D.E has been rapidly swelling in neoteric years in theory and application, for instance, in inhabitance models, in engineering and a spacious denomination of physically paramount problems is qualified via F.D.E., [1].

In this work we press and study the idea of detection some fuzzy eventuality trait of the fuzzy resolutions of non-homogenous F.C.S.D.D. with firm coefficients fuzzy numbers with non-firm F.C.F. on the R.H.S. which are the fuzzy linkage function and the fuzzy ghostly intensity function.

This work consists six sections, the notations and the requisite definitions are presented in the first and second sections, while the fuzzy resoluteness and fuzzy linkage function illustrate in the third section, the fourth section include the attribute of the fuzzy cross-linkage function, derivative of a F.C.F. and the fuzzy ghostly intensity function of a derivative F.C.F., finally in the sixth section we debate the fuzzy ghostly intensity function of firm F.C.F., the fuzzy eventuality trait of the resolutions of a F.C.S.D.D. with non-firm F.C.F. on the R.H.S. and the fuzzy linkage function of the resolutions in the previous subsection.

2. Notations [1], [2], [3], [4]

In this section we introduce some notations we needed that in this work

$Ą(s)$: a number in [0,1] for any membership function of $Ą$ assess at s.

$Ą[ζ]$: $ζ$-cut of S which is closed and bounded and defined as $\{ƅ \,|\, Ą(ƅ) \geq ζ\}$, for $0\leq ζ \leq 1$.

$(\hat{a}\,(\xi), \breve{a}\,(\xi))$: a pair of functions which is an arbitrary fuzzy casual number.

$\hat{a}\,(ζ) = \breve{a}\,(ζ) = ζ$: a fragile casual number, $0 \leq ζ \leq 1$.

$Υ'(ƅ_0)$: the right and left differentiable of $Υ : (ɑ,ɗ) \rightarrow Υ''$ at $ƅ_0 \in (ɑ,ɗ)$ For all $ħ > 0$ and $ħ < 0$ sufficiently $ħ \rightarrow 0$.

$[Υ'(ƅ)]^{\xi} = [ɱ'_{ζ}(ƅ), ɳ'_{ζ}(ƅ)]$: right and left differentiable for a function $Υ : Ŧ \rightarrow Υ$ where $[Υ'(ƅ)]^{ζ} = [ɱ_{ζ}(ƅ), ɳ_{ζ}(ƅ)]$ for all $0\leq ζ \leq 1$.

3. Requisite Definitions

Definition (3.1): [5]

Reckon $(Ð, þ, ζ)$ be a fuzzy casual eventuality space and reckon Ŧ be an index set. F.C.F. is authentic valued function $¥(ƅ, ɯ, ζ)$ on ŦxÐ for each firm $ƅ$, $¥(ƅ, ɯ, ζ)$ is a casual mutable. The function $¥(ƅ, ɯ, ζ)$ can be announce via $¥(ƅ, ζ)$ and a F.C.F. can be deem as a alignment $\{¥(ƅ, ζ), \acute{t} \in Ŧ, ζ \in [0,1]\}$ of a fuzzy casual mutable.

The F.C.F. possess a sole casual mutable at Ŧ stem only one element, and then, so the apportionment function of this F.C.F. is

$$Υ^{ζ}_{¥^{ζ}(ƅ,ζ)}(\kappa^{ζ}(ƅ,ζ)) = þ^{ζ}[¥^{ζ}(ƅ,ζ) \leq \kappa^{ζ}(ƅ,ζ)] \quad \ldots(1)$$

Definition (3.2): [5]

A F.C.F. $¥^{ζ}(ƅ,ζ)$ is summon firm if all the confined remoteness mutual apportionment functions designate $¥^{ζ}(ƅ,ζ)$ stick the same if all $ƅ_1, \ldots, ƅ_n$ is convey along the time axis i.e. if

$$Υ^{ζ}_{¥^{ζ}(ƅ_1+ƅ,ζ), ¥^{ζ}(ƅ_2+ƅ,ζ), \ldots, ¥^{ζ}(ƅ_n+ƅ,ζ)}(\kappa^{ζ}(ƅ_1,ζ), \ldots, \kappa^{ζ}(ƅ_n,ζ))$$
$$= Υ^{ζ}_{¥^{ζ}(ƅ_1,ζ), \ldots, ¥^{ζ}(ƅ_n,ζ)}(\kappa^{ζ}(ƅ_1,ζ), \ldots, \kappa^{ζ}(ƅ_n,ζ)) \quad (2)$$

or

$$þ^{ζ}\{¥^{ζ}(ƅ_1+ƅ,ζ) \leq \kappa_1, \ldots, ¥^{ζ}(ƅ_n+ƅ,ζ) \leq \kappa_n\} = þ^{ζ}\{¥^{ζ}(ƅ_1,ζ) \leq \kappa_1, \ldots, ¥^{ζ}(ƅ_n,ζ) \leq \kappa_n\} \quad (3)$$

for any n, $ƅ_1, \ldots, ƅ_n$ and $ƅ$.

In individual this reveal that for a firm F.C.F., all the one-remoteness apportionment functions have to be conformable (i.e., $Υ^{ζ}(\kappa^{ζ}(t,ζ))$ as in equation (1) not rely on $(ƅ,ζ)$, all the two-remoteness mutual apportionment functions can only reckon on $(ƅ_2 - ƅ_1, ζ)$, and so on.

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Definition (3.3): [6]
A F.C.F. $\{¥^{\zeta}(ъ,\zeta),\ ъ \in Ŧ,\ \zeta\in[0,1]\}$ is summon minutely firm whether the commonalty stock of its confined remoteness apportionment is firm beneath any interpretation in $Ћ$. That is signify that for every confined sequence of $\{ъ_1, ъ_2\dots, ъ_n, \zeta\}$, the mutual apportionment function of n casual mutable $\{¥^{\zeta}(ъ_1+Ћ,\zeta),\dots, ¥^{\zeta}(ъ_n+Ћ,\zeta)\}$ should be independent of $Ћ$. i.e.

$$\Upsilon^{\zeta}_{¥^{\zeta}(ъ_1,\zeta),\dots,¥^{\zeta}(ъ_n,\zeta)}(\kappa_{ъ_1},\dots,\kappa_{ъ_n},\zeta) =$$

$$\Upsilon^{\zeta}_{¥^{\zeta}(ъ_1+Ћ+\zeta),\dots,¥^{\zeta}(ъ_n+Ћ+\zeta)}(\kappa_{ъ_1},\dots,\kappa_{ъ_n},\zeta)$$

for any n, $ъ_1,\dots, ъ_n$ and $Ћ$.

So if $\{¥^{\zeta}(ъ,\zeta),\ ъ \in Ŧ,\ \zeta \in [0,1]\}$ is minutely firm F.C.F.s, then
1) Apportionment function of the fuzzy casual mutable is the selfsame for either point in Ŧ, [7].
2) Mutual apportionment function relies only on the mileage amidst the elements in Ŧ, [2].
3) Whether $E^{\zeta}\{¥^{\zeta}(ъ,\zeta)\} < \infty$, then $\mathrm{var}^{\zeta}[¥^{\zeta}(ъ,\zeta)] < \infty$, [7].

4. Fuzzy Resoluteness and Fuzzy Linkage Function

Definition (4.1): [6]
Consider the confined remoteness apportionment function of the function $¥(ъ,\zeta)$

$\Upsilon^{\zeta}((\kappa_1, \kappa_2,\dots, \kappa_n, ъ_1, ъ_2,\dots, ъ_n), \zeta)$

the uneven resoluteness of order $(Ш_1, Ш_2,\dots, Ш_n, \zeta)$ is defined as

$$\mu^{\zeta}_{Ш_1,\dots,Ш_n}((ъ_1,\dots,ъ_n),\zeta) = E^{\zeta}(¥^{\zeta Ш_1}(ъ_1,\zeta)\dots ¥^{\zeta Ш_n}(ъ_n,\zeta))$$
$$= \int_{-\infty}^{\infty}\dots\int_{-\infty}^{\infty} \kappa_1^{\zeta Ш_1}\dots\kappa_n^{\zeta Ш_n}\partial^n\,\Upsilon^{\zeta}((\kappa_1,\dots,\kappa_n;ъ_1,\dots,ъ_n),\zeta) \quad (4)$$

where $¥^{\zeta}(ъ_1,\zeta),\ \dots,\ ¥^{\zeta}(ъ_n,\zeta)$ is the stock of firm F.C.F.s.

The fuzzy resoluteness of this one remoteness apportionment functions defined as follow when we have remoteness apportionment function,

$$\mu^{\zeta}_{Ш_1}(ъ_1,\zeta) = E^{\zeta}(¥^{\zeta Ш_1}(ъ_1,\zeta)) = \int_{-\infty}^{\infty}\kappa_1^{\zeta Ш_1} d\,\Upsilon^{\zeta}(\kappa_1;ъ_1,\zeta)$$

when we have the two remoteness apportionment functions $\Upsilon^{\zeta}(\kappa_1,\kappa_2; ъ_1, ъ_2, \zeta)$, then the resoluteness are

$$\mu^{\zeta}_{Ш_1,Ш_2}(ъ_1,ъ_2,\zeta) = E^{\zeta}(¥^{\zeta Ш_1}(ъ_1,\zeta)\cdot ¥^{\zeta Ш_2}(ъ_2,\zeta)) =$$
$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\kappa_1^{\zeta Ш_1}\kappa_2^{\zeta Ш_2}\partial^2\,\Upsilon^{\zeta}((\kappa_1,\kappa_2;ъ_1,ъ_2),\zeta)$$

and so on.

Note (4.2): [6]
Whether the F.C.F. $¥^{\zeta}(ъ,\zeta)$ is firm, then the apportionment function of $¥^{\zeta}(ъ,\zeta)$ does not reckon on Ŧ and because of that the first resoluteness of the firm F.C.F. $¥^{\zeta}(ъ,\zeta)$ is a fuzzy firm number i.e.

$\mu^{\zeta}_{Ш_1}(ъ,\zeta) = \mu^{\zeta}_1(ъ,\zeta) = E^{\zeta}[¥^{\zeta}(ъ,\zeta)] = \rho$, ρ is fuzzy firm number.

Definition (4.3): [8]
The first fuzzy resoluteness of the firm F.C.F. $¥^{\zeta}(ъ,\zeta)$ portray the gruff properties of $¥^{\zeta}(ъ,\zeta)$, to know much more portrayal of $¥^{\zeta}(ъ,\zeta)$ is given via its second resoluteness, so

$$\mu^{\zeta}_{Ш_1,Ш_2}(ъ,\varsigma,\zeta) = E^{\zeta}[¥^{\zeta Ш_1}(ъ,\zeta)\,X^{\zeta Ш_2}(\varsigma,\zeta)]$$
$$= B^{\zeta}(ъ,\varsigma,\zeta) \qquad ъ,\varsigma\in T, \zeta\in[0,1]$$

the function $B^{\zeta}(ъ,\varsigma,\zeta)$ is summon the fuzzy linkage function of $¥^{\zeta}(ъ,\zeta)$ when $Ш_1, Ш_2 = 1$ which is reckon on $ъ-\varsigma$, i.e.

$$\mu^{\zeta}_{Ш_1,Ш_2}(ъ,\varsigma,\zeta) = E^{\zeta}[¥^{\zeta}(ъ,\zeta)¥^{\zeta}(\varsigma,\zeta)]$$
$$= B^{\zeta}(ъ-\varsigma,\zeta) = B^{\zeta}(\tau,\zeta)$$

The fuzzy linkage function of any firm F.C.F. can be symbolize as

$$B^{\zeta}(\tau,\zeta) = \int_{-\infty}^{\infty} e^{i\lambda\tau} d\,\Upsilon^{\zeta}(\lambda,\zeta) \quad (5)$$

where $\Upsilon^{\zeta}(\lambda,\zeta)$ is a real non-decreasing function.
or,

$$B^{\zeta}(\tau,\zeta) = \int_{-\infty}^{\infty} e^{i\lambda\tau} f^{\zeta}(\lambda,\zeta)\,d\lambda \quad (6)$$

Formula (5) is the same of formula (6) and $\Upsilon^{\zeta}(\lambda,\zeta) = \int_{-\infty}^{\infty} f^{\zeta}(\lambda,\zeta)\,d\lambda$ and it is clear that $\Upsilon'^{\zeta}(\lambda,\zeta) = f^{\zeta}(\lambda,\zeta)$, where $f^{\zeta}(\lambda,\zeta)$ is the ghostly intensity function of the F.C.F. $¥^{\zeta}(ъ,\zeta), f^{\zeta}(\lambda,\zeta) \geq 0$.

More generally, when $¥^{\zeta}(ъ,\zeta)$ is a complex minutely firm F.C.F. with $E^{\zeta}\left[\left|¥^{\zeta}(ъ,\zeta)\right|^2\right] = \rho$, ρ is fuzzy firm. i.e. $¥^{\zeta}(ъ,\zeta)$ have a confined fuzzy variance and firm fuzzy mean.

Because the firm of $¥^{\zeta}(ъ,\zeta)$, we consider that $E^{\zeta}[¥^{\zeta}(ъ,\zeta)] = 0$ and the fuzzy variance which gives more rigorous attributive of $¥^{\zeta}(ъ,\zeta)$ can be acquaint for $ъ_1 \neq ъ_2$ as

$$\mathrm{var}^{\zeta}[¥^{\zeta}(ъ,\zeta)] = E^{\zeta}\{[¥^{\zeta}(ъ_1,\zeta)-\bar{¥}^{\zeta}(ъ_1,\zeta)]\overline{[¥^{\zeta}(ъ_2,\zeta)-\bar{¥}^{\zeta}(ъ_2,\zeta)]}\}$$
$$= E^{\zeta}[¥^{\zeta}(ъ_1,\zeta)-¥^{\zeta}(ъ_2,\zeta)]-0$$
$$= B^{\zeta}(ъ_1-ъ_2,\zeta) = B^{\zeta}((ъ_1-ъ_2),\zeta) = B^{\zeta}(\tau,\zeta)$$

Where $B^{\zeta}(\tau,\zeta) = B^{\zeta}(ъ_1-ъ_2,\zeta)$ is the linkage function of $¥^{\zeta}(ъ,\zeta)$, so via Schwartz dissimilarity

$$\left|B^{\zeta}(ъ_1-ъ_2,\zeta)\right|^2 \leq$$

$$E^{\zeta}\left|¥^{\zeta}(ъ_1,\zeta)-¥^{\zeta}(ъ_2,\zeta)\right|^2 E^{\zeta}\left|¥^{\zeta}(ъ_2,\zeta)-\bar{¥}^{\zeta}(ъ_2,\zeta)\right|^2$$

which means that the fuzzy linkage function $B^{\zeta}(ъ_1-ъ_2,\zeta)$ is confined for all $ъ_1, ъ_2$.

Furthermore $B^{\zeta}(ъ_1-ъ_2,\zeta)$ possess the following two properties:

1. $B^{\varsigma}(\text{ƅ}_1-\text{ƅ}_2,\text{Ϛ})=\overline{B^{\varsigma}(\text{ƅ}_1-\text{ƅ}_2,\text{Ϛ})}$ or $B^{\varsigma}(\text{ƅ}_1,\text{ƅ}_2,\text{Ϛ})=\overline{B^{\varsigma}(\text{ƅ}_2,\text{ƅ}_1,\text{Ϛ})}$ (7)

2. $B^{\varsigma}(0,\text{Ϛ})=E^{\varsigma}\left[\left|\text{¥}^{\varsigma}(\text{ƅ},\text{Ϛ})\right|^2\right]\geq 0$ for any ƅ.

A F.C.F. $\{\text{¥}^{\varsigma}(\text{ƅ},\text{Ϛ})\}$ is pronounced firm if the conditions are hold, [9]:

1. $E^{\varsigma}\left[\left|\text{¥}^{\varsigma}(\text{ƅ},\text{Ϛ})\right|^2\right]<\infty$, for all $\text{ƅ}\in\text{Ŧ}$, $\text{Ϛ}\in[0,1]$.

2. $E^{\varsigma}\{\text{¥}^{\varsigma}(\text{ƅ},\text{Ϛ})\}=\rho$, ρ is firm.

3. $E^{\varsigma}[\text{¥}^{\varsigma}(\text{ƅ}+\tau,\text{Ϛ})-\overline{\text{¥}^{\varsigma}(\text{ƅ},\text{Ϛ})}]=B^{\varsigma}(\tau,\text{Ϛ})$ (8)

where $B^{\varsigma}(\tau,\text{Ϛ})$ is the fuzzy linkage function of $\text{¥}^{\varsigma}(\text{ƅ},\text{Ϛ})$ and does not depend on ƅ.

Furthermore $B^{\varsigma}(\tau,\text{Ϛ})$ possess the following attribute:

1. $B^{\varsigma}(0,\text{Ϛ})=E^{\varsigma}[\text{¥}^{\varsigma}(\text{ƅ}+0,\text{Ϛ})-\overline{\text{¥}^{\varsigma}(\text{ƅ},\text{Ϛ})}]$

$$=E^{\varsigma}\left[\left|\text{¥}^{\varsigma}(\text{ƅ},\text{Ϛ})\right|^2\right]>0$$

2. The fuzzy linkage function of $\text{¥}^{\varsigma}(\text{ƅ},\text{Ϛ})$ is an even function i.e. $B^{\varsigma}(\tau,\text{Ϛ})=\overline{B^{\varsigma}(-\tau,\text{Ϛ})}$

3. The fuzzy linkage function of the firm casual function is plus semi definite.

Definition (4.4): [10]

A fuzzy function $f^{\varsigma}(\kappa,\text{Ϛ})$ acquaint to be semi definite if

$$\sum_{j=1}^{h}\sum_{k=1}^{n}a_j a_k\, f^{\varsigma}(\text{ƅ}_j-\text{ƅ}_k,\text{Ϛ})$$

for any $a_1, a_2, \ldots, a_n$ and $(\text{ƅ}_1,\text{ƅ}_2,\ldots,\text{ƅ}_n)$ such that $\text{ƅ}_j-\text{ƅ}_k\in\text{¥}$ for all $j, k = 1, 2,\ldots, n$.

5. Fuzzy Junction-Linkage Functions

Definition (5.1): [4]

Reckon $\text{¥}^{\varsigma}(\text{ƅ},\text{Ϛ})$, $\text{д}^{\varsigma}(\text{ƅ},\text{Ϛ})$ be two F.C.F.s, then the cross-linkage function of $\text{¥}^{\varsigma}(\text{ƅ},\text{Ϛ})$, $\text{д}^{\varsigma}(\text{ƅ},\text{Ϛ})$ defined via the equality

$B^{\varsigma}_{\text{¥д}}(\text{ƅ}_2,\text{ƅ}_1,\text{Ϛ})=$

$E^{\varsigma}\{\overline{[\text{¥}^{\varsigma}(\text{ƅ}_1,\text{Ϛ})]-E^{\varsigma}[\text{¥}^{\varsigma}(\text{ƅ}_1,\text{Ϛ})]}\,[\text{д}^{\varsigma}(\text{ƅ}_2,\text{Ϛ})]-E^{\varsigma}[\text{д}^{\varsigma}(\text{ƅ}_2,\text{Ϛ})]\}$

5.1 Attribute of the Fuzzy Cross-Linkage Function:

1. $B^{\varsigma}_{\text{¥д}}(\text{ƅ}_1,\text{ƅ}_2,\text{Ϛ})=\overline{B^{\varsigma}_{\text{¥д}}(\text{ƅ}_2,\text{ƅ}_1,\text{Ϛ})}$,

2. $\left|B^{\varsigma}_{\text{¥д}}(\text{ƅ}_1,\text{ƅ}_2,\text{Ϛ})\right|\leq\sqrt{\Im^{\varsigma}_{\text{¥}}(\text{ƅ}_1,\text{ƅ}_2,\text{Ϛ})\Im^{\varsigma}_{\text{д}}(\text{ƅ}_1,\text{ƅ}_2,\text{Ϛ})}$, where $\Im^{\varsigma}_{\text{¥}},\Im^{\varsigma}_{\text{д}}$ are the variances of the casual functions $\text{¥}^{\varsigma}(\text{ƅ},\text{Ϛ})$ and $\text{д}^{\varsigma}(\text{ƅ},\text{Ϛ})$ respectively. which possess the properties

1. $B^{\varsigma}_{\text{¥д}}(\tau,\text{Ϛ})=\overline{B^{\varsigma}_{\text{д¥}}(-\tau,\text{Ϛ})}$ complex

$B^{\varsigma}_{\text{¥д}}(\tau,\text{Ϛ})=B^{\varsigma}_{\text{д¥}}(-\tau,\text{Ϛ})$ real

2. $\left|B^{\varsigma}_{\text{¥д}}(\tau,\text{Ϛ})\leq\sqrt{B^{\varsigma}_{\text{¥}}(0,\text{Ϛ})B^{\varsigma}_{\text{д}}(0,\text{Ϛ})}\right|$.

5.2 Derivative of a F.C.F.

A F.C.F. $\text{¥}^{\varsigma}(\text{ƅ},\text{Ϛ})$ is differentiable at a ƅ. If for ħ_1, …, ħ_n, … converges to zero, the sequence of casual mutable

$$\frac{\text{¥}^{\varsigma}(\text{ƅ}+\text{ħ}_j,\text{Ϛ})-\text{¥}^{\varsigma}(\text{ƅ},\text{Ϛ})}{\text{ħ}_j}\qquad j=1,\ldots,n,\ldots$$

Converges in the mean to a unique casual mutable.

The unique casual mutable demand the derivative of $\text{¥}^{\varsigma}(\text{ƅ},\text{Ϛ})$ at ƅ, and betoken via $\text{¥}^{\varsigma}(\text{ƅ},\text{Ϛ})$.

This firm F.C.F. is differentiable for any ƅ, only beneath the stipulation that a fuzzy linkage function $B^{\varsigma}(\tau)$ of $\text{¥}^{\varsigma}(\text{ƅ},\text{Ϛ})$ ought possess a continuous second derivative relatively τ, and $\text{¥}^{\varsigma}(\text{ƅ},\text{Ϛ})$ ought be continuous firm fuzzy function with fuzzy linkage function

$$B^{\varsigma}_{\text{д}}(\tau,\text{Ϛ})=-\sqrt{B^{\varsigma}_{\text{¥}}(0,\text{Ϛ})B^{\varsigma}_{\text{д}}(0,\text{Ϛ})}\qquad(9)$$

And equation (9) refers that $B''^{\varsigma}(\tau,\text{Ϛ})$ is a linkage function of $\text{¥}'^{\varsigma}(\text{ƅ},\text{Ϛ})$ and $B^{\varsigma}(\tau,\text{Ϛ})$ possess a continuous second derivative.

5.3 The Fuzzy Ghostly Intensity Function of a Derivative F.C.F:

Reckon $\text{¥}^{\varsigma}(\text{ƅ},\text{Ϛ})$ be a firm casual function, then

$$V^{\varsigma}(\text{ƅ},\text{Ϛ})=\text{¥}'^{\varsigma}(\text{ƅ},\text{Ϛ})=\frac{d}{d\text{ƅ}}\text{¥}^{\varsigma}(\text{ƅ},\text{Ϛ})\qquad(10)$$

Here $V^{\varsigma}(\text{ƅ},\text{Ϛ})$ is also a firm F.C.F. and the fuzzy linkage function of $V^{\varsigma}(\text{ƅ},\text{Ϛ})$ can be determined via

$$B^{\varsigma}(\tau,\text{Ϛ})=-(B''^{\varsigma}(\tau,\text{Ϛ}))\qquad(11)$$

Also, since

$$B^{\varsigma}_{\text{¥}}(\tau,\text{Ϛ})=\int_{-\infty}^{\infty}e^{i\varpi\tau}f^{\varsigma}_{\text{¥}}(\varpi,\text{Ϛ})\,d\varpi$$

$$B'^{\varsigma}_{\text{¥}}(\tau,\text{Ϛ})=\int_{-\infty}^{\infty}i\varpi e^{i\varpi\tau}f^{\varsigma}_{\text{¥}}(\varpi,\text{Ϛ})\,d\varpi$$

$$B''^{\varsigma}_{V}(\tau,\text{Ϛ})=-\int_{-\infty}^{\infty}\varpi^2 e^{i\varpi\tau}f^{\varsigma}_{V}(\varpi,\text{Ϛ})\,d\varpi$$

Via equation (9), and since

$$B^{\varsigma}_{V}(\tau,\text{Ϛ})=\int_{-\infty}^{\infty}e^{i\varpi\tau}f^{\varsigma}_{V}(\varpi,\text{Ϛ})\,d\varpi\qquad(12)$$

But $V^{\varsigma}(\text{ƅ},\text{Ϛ})$ is firm

$$B^{\varsigma}_{v}(\tau,\text{Ϛ})=\int_{-\infty}^{\infty}e^{i\varpi\tau}f^{\varsigma}_{v}(\varpi,\text{Ϛ})\,d\varpi\qquad(13)$$

where $f^{\varsigma}_{v}(\varpi,\text{Ϛ})$ is a fuzzy ghostly intensity function of $V^{\varsigma}(\text{ƅ},\text{Ϛ})$ and via comparing equations (12) and (14)

$$f^{\varsigma}_{v}(\varpi,\text{Ϛ})=\varpi^2 f^{\varsigma}_{\text{¥}}(\varpi,\text{Ϛ})\qquad(14)$$

6. The Fuzzy Ghostly Exemplification of F.C.F.

Definition (6.1): [4]

All firm $¥^{\zeta}(\text{ъ},\zeta)$ can be appropriate a function with orthogonal augmentation, such that for each firm ъ that ghostly exemplification

$$¥^{\zeta}(\text{ъ},\zeta)=\int_{-\infty}^{\infty} e^{i\lambda \text{ъ}} f^{\zeta}(\lambda,\zeta)d\lambda \tag{15}$$

6.1 The Fuzzy Ghostly Intensity Function of Firm F.C.F.:

Offered the analytic idiom of a F.C.F. { $¥^{\zeta}(\text{ъ},\zeta)$,- ∞ < ъ < ∞} via

$$¥^{\zeta}(\text{ъ},\zeta)=\kappa f^{\zeta}(\text{ъ},\zeta) \tag{16}$$

where $E^{\zeta}\{¥^{\zeta}(\text{ъ},\zeta)\}=0$, x is a fuzzy casual mutable and $f^{\zeta}(\text{ъ},\zeta)$ is a numerical function of ъ.

For the purpose that $¥^{\zeta}(\text{ъ},\zeta)$ is a firm function, it must be that

$$f^{\zeta}(\text{ъ},\zeta)=\gamma e^{-i(\lambda\text{ъ}+\theta)}$$

where γ, λ, θ ∈ℝ, ζ ∈ [0,1].

The F.C.F. (16) is firm if it possesses the style

$$¥^{\zeta}(\text{ъ},\zeta)=\kappa\gamma e^{-i(\lambda\text{ъ}+\theta)}=\kappa\gamma e^{-i\theta}e^{-i\lambda\text{ъ}}$$

Via global $\gamma e^{-i\theta}$ in the fuzzy casual mutable κ, the F.C.F. (16) is firm iff it possess the style

$$¥^{\zeta}(\text{ъ},\zeta)=\kappa e^{-i\lambda\text{ъ}}$$

The fuzzy linkage function of the firm function $¥^{\zeta}(\text{ъ},\zeta)$ is

$$\begin{aligned}
B_{¥}^{\zeta}(\tau,\zeta)&=E^{\zeta}[\kappa(\text{ъ}+\tau,\zeta)\overline{¥^{\zeta}(\text{ъ}_1,\zeta)}\ ,\ \tau>0,\ \zeta\in[0,1]\\
&=E^{\zeta}[\kappa e^{-i\lambda(\text{ъ}+\tau,\zeta)}\ \overline{\kappa e^{-i\lambda(\text{ъ},\zeta)}}]\\
&=E^{\zeta}\left[|\kappa\overline{\kappa}|e^{-i\lambda\tau}\right]\\
&=E^{\zeta}\left[|\kappa|^{2}e^{-i\lambda\tau}\right]\\
&=E^{\zeta}|\kappa|^{2}e^{-i\lambda\tau}
\end{aligned}$$

So, $B_{¥}^{\zeta}(\tau,\zeta)=\sigma^{2}e^{-i\lambda\tau}$, $\tau>0$, $\zeta\in[0,1]$

Or,

$$B_{¥}^{\zeta}(\tau,\zeta)=\sigma^{2}e^{-i\lambda|\tau|},\ |\tau|<\infty,\ \zeta\in[0,1] \tag{17}$$

where $E^{\zeta}\left[|\kappa|^{2}\right]=\sigma^{2}$ is the mathematical anticipation of the square of the gauge.

The fuzzy ghostly intensity function is the Fourier transformation of $B^{\zeta}(\tau,\zeta)$. If the fuzzy linkage function $B^{\zeta}(\tau,\zeta)$ is renowned, the fuzzy ghostly intensity function can be gained via the shape

$$\hat{B}^{\zeta}(\omega,\zeta)=f(\omega,\zeta)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}e^{-i\omega\tau}B^{\zeta}(\tau,\zeta) \tag{18}$$

6.2 The Fuzzy Eventuality Trait of the Resolutions of a F.C.S.D.D. with Non-Firm F.C.F. on the R.H.S.:

Deem the F.C.S.D.D. with non-firm on the R.H.S. and firm coefficients $\upsilon_1,\upsilon_2,\ldots,\upsilon_n$ where $\upsilon_1,\upsilon_2,\ldots,\upsilon_n$ is fuzzy numbers

$$\frac{d^{(n)}д^{\zeta}(\text{ъ},\zeta)}{d\,\text{ъ}^{n}}+\upsilon_1\frac{d^{(n-1)}д^{\zeta}(\text{ъ},\zeta)}{d\,\text{ъ}^{n-1}}+\ldots+\upsilon_n д^{\zeta}(\text{ъ},\zeta)\le\mathfrak{I}^{\zeta}(\text{ъ},\zeta) \tag{19}$$

where $д^{\zeta}(\text{ъ},\zeta)$ is fuzzy firm casual fuzzy firm function and $\mathfrak{I}^{\zeta}(\text{ъ},\zeta)$ is a non-firm F.C.F..

Presume that, the non-firm F.C.F. $\mathfrak{I}^{\zeta}(\text{ъ},\zeta)$ is a stringy synthesis of a F.C.F.s, that is via lay

$$\mathfrak{I}^{\zeta}(\text{ъ},\zeta)=f^{\zeta}(\text{ъ},\zeta)¥^{\zeta}(\text{ъ},\zeta) \tag{20}$$

where $f^{\zeta}(\text{ъ},\zeta)$ is any fuzzy function of ъ and $¥^{\zeta}(\text{ъ},\zeta)$ is a firm fuzzy function with fuzzy ghostly intensity function $f_{¥}^{\zeta}(\text{ъ},\zeta)$ and fuzzy ghostly exemplification

$$¥^{\zeta}(\text{ъ},\zeta)=\int_{-\infty}^{\infty}e^{-i\lambda\text{ъ}}f_{¥}^{\zeta}(\lambda)d\lambda \tag{21}$$

Hence equation (20), will be

$$\mathfrak{I}^{\zeta}(\text{ъ},\zeta)=\int_{-\infty}^{\infty}f^{\zeta}(\text{ъ},\zeta)e^{-i\lambda\text{ъ}}f_{¥}^{\zeta}(\lambda)d\lambda \tag{22}$$

And the given F.C.S.D.D. (19) can be written as

$$\frac{d^{(n)}д^{\zeta}(\text{ъ},\zeta)}{d\,\text{ъ}^{n}}+\upsilon_1\frac{d^{(n-1)}д^{\zeta}(\text{ъ},\zeta)}{d\,\text{ъ}^{n-1}}+\ldots+\upsilon_n д^{\zeta}(\text{ъ},\zeta)\le \int_{-\infty}^{\infty}f^{\zeta}(\text{ъ},\zeta)e^{-i\lambda\text{ъ}}f_{¥}^{\zeta}(\lambda)d\lambda \tag{23}$$

The resolution of the F.C.S.D.D. (23) can be deeming as a resolution of the private integral $д_{p}^{\zeta}(\text{ъ},\zeta)$ concerning that $д_{p}^{\zeta}(\text{ъ},\zeta)$ is also a firm F.C.F.

$$д_{p}^{\zeta}(\text{ъ},\zeta)\le\int_{-\infty}^{\infty}д^{\zeta}(\lambda,\text{ъ})\,f_{¥}^{\zeta}(\lambda)d\lambda \tag{24}$$

where the fuzzy ghostly intensity function $f_{¥}^{\zeta}(\lambda)$ rely just on λ not on the time ъ.

To manage $д^{\zeta}(\lambda,\text{ъ})$, subrogate dissimilarity (24) into F.C.S.D.D. (23), we secure

$$\int_{-\infty}^{\infty}\frac{d^{(n)}}{d\,\text{ъ}^{n}}д^{\zeta}(\lambda,\text{ъ})f_{¥}^{\zeta}(\lambda)d\lambda+\upsilon_1\int_{-\infty}^{\infty}\frac{d^{(n-1)}}{d\,\text{ъ}^{n-1}}д^{\zeta}(\lambda,\text{ъ})f_{¥}^{\zeta}(\lambda)d\lambda+\ldots+$$

$$\upsilon_n\int_{-\infty}^{\infty}\frac{d^{(n)}}{d\,\text{ъ}^{n}}д^{\zeta}(\lambda,\text{ъ})f_{¥}^{\zeta}(\lambda)d\lambda\le\int_{-\infty}^{\infty}f^{\zeta}(\text{ъ},\zeta)e^{-i\lambda\text{ъ}}f_{¥}^{\zeta}(\text{ъ},\zeta)d\lambda$$

Or

$$\frac{d^{(n)}}{d\,\text{ъ}^{n}}д^{\zeta}(\lambda,\text{ъ})+\upsilon_1\frac{d^{(n-1)}}{d\,\text{ъ}^{n-1}}д^{\zeta}(\lambda,\text{ъ})+\ldots+\upsilon_n д^{\zeta}(\lambda,\text{ъ})\le f^{\zeta}(\text{ъ},\zeta)e^{-i\lambda\text{ъ}} \tag{25}$$

To disband this unprecedented F.C.S.D.D., we deem two cases:

Case one :

$$f^{\zeta}(\text{ъ},\zeta)=\text{ъ}^{m},\ m=0,1,2,\ldots \tag{26}$$

We presume that the private resolution $Д^{\zeta}(\lambda,Ъ)$ is a polynomial of degree m in $Ъ$ pommel via $e^{i\lambda Ъ}$

$$Д^{\zeta}(\lambda,Ъ)=\sum_{j=0}^{m}c_j Ъ^{m-j}e^{i\lambda Ъ} \tag{27}$$

where c_j, j = 0, 1,…, m are firms.

Via subrogate equation (27) into dissimilarity (24), the private resolution $Д_p^{\zeta}(Ъ,\zeta)$ of the F.C.S.D.D. (19) will be

$$Д_p^{\zeta}(Ъ,\zeta)\le\int_{-\infty}^{\infty}(\sum_{j=0}^{m}c_j Ъ^{m-j})e^{i\lambda Ъ}f_{¥}^{\zeta}(\lambda)d\lambda \tag{28}$$

The dissimilarity (28) exemplifies the fuzzy ghostly exemplification of the resolution of the F.C.S.D.D. (19).

Case two :

$f^{\zeta}(Ъ,\zeta)=e^{kЪ}$, $k\in R$

We presume that the resolution $Д^{\zeta}(\lambda,Ъ)$ is $e^{kЪ}$ multiplied via the exponential $e^{i\lambda Ъ}$ (i.e.)

$$Д^{\zeta}(\lambda,Ъ)=e^{kЪ}e^{i\lambda Ъ}=e^{(k+i\lambda)Ъ} \tag{29}$$

And via subrogate equation (29) into equation (26), the particular resolution $Д_p^{\zeta}(Ъ,\zeta)$ of the F.C.S.D.D. (19) will be

$$Д_p^{\zeta}(Ъ,\zeta)\le\int_{-\infty}^{\infty}e^{(k+i\lambda)Ъ}f_{¥}^{\zeta}(\lambda)d\lambda \tag{30}$$

$$Д_p^{\zeta}(Ъ,\zeta)\le e^{kЪ}\int_{-\infty}^{\infty}e^{i\lambda Ъ}f_{¥}^{\zeta}(\lambda)d\lambda=e^{kЪ}¥^{\zeta}(Ъ,\zeta) \tag{31}$$

which also exemplifies the fuzzy ghostly exemplification of the resolution of the F.C.S.D.D. (19).

6.3 The Fuzzy Linkage Function of $Д_p^{\zeta}(Ъ,\zeta)$:

For Case One:

Via equation (7) we secure

$$B_{Д_p^{\zeta}}^{\zeta}(Ъ_2-Ъ_1,\zeta)=$$

$$E\left\{\left[\overline{Д_p^{\zeta}(Ъ_1,\zeta)-E(Д_p^{\zeta}(Ъ_1,\zeta))}\right]\left[Д_p^{\zeta}(Ъ_2,\zeta)-E(Д_p^{\zeta}(Ъ_2,\zeta))\right]\right\}$$

$$=E\left\{\left[\overline{Д_p^{\zeta}(Ъ_1,\zeta)}-0\right]\left[Д_p^{\zeta}(Ъ_2,\zeta)-0\right]\right\}$$

$$B_{Д_p^{\zeta}}^{\zeta}(Ъ_2-Ъ_1,\zeta)==E\left[\overline{Д_p^{\zeta}(Ъ_1,\zeta)}Д_p^{\zeta}(Ъ_2,\zeta)\right] \tag{32}$$

Subrogate dissimilarity (28) into equation (32) we secure

$$B_{Д_p^{\zeta}}^{\zeta}(Ъ_2-Ъ_1,\zeta)\le$$

$$E\left\{\left[\overline{\int_{-\infty}^{\infty}(\sum_{j=0}^{m}c_j Ъ_1^{m-j})e^{i\lambda Ъ_1}df_{¥}^{\zeta}(\lambda)}\right]\left[\int_{-\infty}^{\infty}(\sum_{j=0}^{m}c_j Ъ_2^{m-j})e^{i\lambda Ъ_2}df_{¥}^{\zeta}(\lambda)\right]\right\}$$

$$\le\int_{-\infty}^{\infty}(\overline{\sum_{j=0}^{m}c_j Ъ_1^{m-j})(\sum_{j=0}^{m}c_j Ъ_2^{m-j}})e^{i\lambda(Ъ_2-Ъ_1)}E\left[\overline{df_{¥}^{\zeta}(\lambda)}\,df_{¥}^{\zeta}(\lambda)\right]$$

$$\le\int_{-\infty}^{\infty}(\overline{\sum_{j=0}^{m}c_j Ъ_1^{m-j})(\sum_{j=0}^{m}c_j Ъ_2^{m-j}})e^{i\lambda(Ъ_2-Ъ_1)}f_{¥}^{\zeta}(\lambda)d\lambda$$

For $\tau=Ъ_2-Ъ_1$, the fuzzy linkage function of the resolution of the F.C.S.D.D. (19) will become

$$B_{Д_p^{\zeta}}^{\zeta}(\tau,\zeta)\le\int_{-\infty}^{\infty}(\overline{\sum_{j=0}^{m}c_j Ъ_1^{m-j})(\sum_{j=0}^{m}c_j Ъ_2^{m-j}})e^{i\lambda\tau}f_{¥}^{\zeta}(\lambda)d\lambda \tag{33}$$

For Case Two:

Using equation (11) we have

$$B_{Д_p^{\zeta}}^{\zeta}(Ъ_2-Ъ_1,\zeta)=$$

$$E\left\{\left[\overline{Д_p^{\zeta}(Ъ_1,\zeta)-E(Д_p^{\zeta}(Ъ_1,\zeta))}\right]\left[Д_p^{\zeta}(Ъ_2,\zeta)-E(Д_p^{\zeta}(Ъ_2,\zeta))\right]\right\}$$

$$=E\left\{\left[\overline{Д_p^{\zeta}(Ъ_1,\zeta)}-0\right]\left[Д_p^{\zeta}(Ъ_2,\zeta)-0\right]\right\}$$

$$B_{Д_p^{\zeta}}^{\zeta}(Ъ_2-Ъ_1,\zeta)==E\left[\overline{Д_p^{\zeta}(Ъ_1,\zeta)}Д_p^{\zeta}(Ъ_2,\zeta)\right] \tag{34}$$

Subrogate dissimilarity (31) into equation (34) we secure,

$$B_{Д_p^{\zeta}}^{\zeta}(Ъ_2-Ъ_1,\zeta)\le E\left\{\left[\overline{\int_{-\infty}^{\infty}e^{(k+i\lambda)Ъ_1}df_{¥}^{\zeta}(\lambda)}\right]\left[\int_{-\infty}^{\infty}e^{(k+i\lambda)Ъ_2}df_{¥}^{\zeta}(\lambda)\right]\right\}$$

$$\le\int_{-\infty}^{\infty}\overline{e^{(k+i\lambda)Ъ_1}}e^{(k+i\lambda)Ъ_2}E\left[\overline{df_{¥}^{\zeta}(\lambda)}df_{¥}^{\zeta}(\lambda)\right]$$

$$\le\int_{-\infty}^{\infty}\overline{e^{(k+i\lambda)Ъ_1}}e^{(k+i\lambda)Ъ_2}f_{¥}^{\zeta}(\lambda)d\lambda$$

$$\le\int_{-\infty}^{\infty}e^{(k-i\lambda)Ъ_1}e^{(k+i\lambda)Ъ_2}f_{¥}^{\zeta}(\lambda)d\lambda$$

$$\le\int_{-\infty}^{\infty}e^{kЪ_1+kЪ_2}e^{i\lambda(Ъ_2-Ъ_1)}f_{¥}^{\zeta}(\lambda)d\lambda$$

$$\le\int_{-\infty}^{\infty}e^{k(Ъ_1+Ъ_2)}e^{i\lambda(Ъ_2-Ъ_1)}f_{¥}^{\zeta}(\lambda)d\lambda$$

For $\tau=Ъ_2-Ъ_1$, the fuzzy linkage function of the resolution of the F.C.S.D.D. (19) will become

$$B_{Д_p^{\zeta}}^{\zeta}(\tau,\zeta)\le\int_{-\infty}^{\infty}e^{k(Ъ_1+Ъ_2)}e^{i\lambda\tau}f_{¥}^{\zeta}(\lambda)d\lambda \tag{35}$$

or via equation (6)

$$B_{Д_p^{\zeta}}^{\zeta}(\tau,\zeta)\le e^{k(Ъ_1+Ъ_2)}¥^{\zeta}(\tau,\zeta) \tag{36}$$

6.4 The Fuzzy Ghostly Intensity Function of $Д_p^{\zeta}(Ъ,\zeta)$:

For Case One:

Via equation (6), the F.C.S.D.D. (33) can be written as

$$\int_{-\infty}^{\infty}e^{i\lambda\tau}f_{Д_p^{\zeta}}^{\zeta}(\lambda)d\lambda\le\int_{-\infty}^{\infty}(\overline{\sum_{j=0}^{m}c_j Ъ_1^{m-j})(\sum_{j=0}^{m}c_j Ъ_2^{m-j}})e^{i\lambda\tau}f_{¥}^{\zeta}(\lambda)d\lambda$$

Crop that the fuzzy ghostly intensity function of the resolution $Д_p^{\zeta}(Ъ,\zeta)$ of the F.C.S.D.D. (19) will have the style

$$f_{Д_p^{\zeta}}^{\zeta}(\lambda)\le(\overline{\sum_{j=0}^{m}c_j Ъ_1^{m-j})(\sum_{j=0}^{m}c_j Ъ_2^{m-j}})f_{¥}^{\zeta}(\lambda) \tag{37}$$

And for $f_{¥}^{\zeta}(Ъ,\zeta)$, the fuzzy ghostly intensity function $f_{Д_p^{\zeta}}^{\zeta}(\lambda)$ can be completely defined.

For Case Two:

Via equation (6), the F.C.S.D.D. (35) can be written as

$$\int_{-\infty}^{\infty}e^{i\lambda\tau}f_{Д_p^{\zeta}}^{\zeta}(\lambda)d\lambda=\int_{-\infty}^{\infty}e^{k(Ъ_1+Ъ_2)}e^{i\lambda\tau}f_{¥}^{\zeta}(\lambda)d\lambda$$

Crop that, the fuzzy ghostly intensity function of the resolution $Д_p^{\zeta}(Ъ,\zeta)$ of the F.C.L.D.I (23) will have the style

$$f_{Д_p^{\zeta}}^{\zeta}(\lambda)\le e^{k(Ъ_1+Ъ_2)}f_{¥}^{\zeta}(\lambda) \tag{38}$$

And for $f_{¥}^{\zeta}(Ъ,\zeta)$, the fuzzy ghostly intensity function $f_{Д_p^{\zeta}}^{\zeta}(\lambda)$ can be completely defined.

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