

# Nuclear Structure Study of Zn <sup>72-74</sup> Isotopes

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**Abstract:** In this research the nuclear structure for isotope (<sup>74-72</sup>Zn) have been studied using the Interaction Boson Model (IBM-1), this nucleus was determined depending on the practical available values and calculating energy levels. In addition to potential Energy Surface using IBMP-Code. The results obtained from this study showed a good agreements with practical values. This nucleus was confirmed as a transitional nucleus between vibrational limit and  $\gamma$ -unstable limit depending on the results of energy levels. Contour lines and axillary symmetries have been drawn Which confirmed that there is no nuclear deformation but vibrations around the position of balance.

## 1. Introduction

Physics as yet lack a comprehensive, interconnected theory through which we can explain all the nuclear phenomena. Since there is no comprehensive theory of nuclear structure, attempts have been made to link nuclear data through a number of different nuclear models. To describe the interaction between nucleons, several basic models have been proposed. One of these models is the liquid-drop model, Von Weisker and Niles (1935) [1] based on the similarity of nuclear material with a drop of liquid.

This model showed a description of the aggregate properties but failed to explain the stability of the nucleus and the nature of the nuclear forces that bind the nucleons represented by protons and neutrons. This was followed by the appearance of the Shell model by W. Elasser (1974).[2] This model is used to study the "ground state" nuclei or those in the low -lying excited state, where the interaction is weak. The shell model studies the light (spherical) nuclei that are usually close to the closed shells and provided an explanation for the magical numbers and other nuclear characteristics in terms of the effect of the nucleus as a whole on the nucleons. Of the Shell Model failures is that it assumed the nuclear to be a spherical shape where the value of the quadruple moment is zero. In fact, the heavy nuclei are practically found not to be spherical. In addition, this model is expected to have no monopole transitions. These transitions are of great significant through which one can know the exact structure of the nucleus [3].

Following the failure of the shell model, another model, which included the manifestations of both the shell model and the liquid droplet model, called the collective model, emerged. One of the most important problems of the collective model, although its success in studying the nuclear spectra, is being very complex in mathematical calculations. The Hamilton solution for the nuclear system with eight nucleons outside the closed shell needs to account for  $10^{15} \times 7$  matrix elements. After the collective models showed a weak agreement with the experimental results of transitional nuclei, Iachello and Arima (1974) proposed a nuclear model combining Geometric models and shell mode. This model was able to study the properties of low aggregate levels in even- even nuclei.[4] The Interacting Boson model depends on group symmetricity to describe the

vibrational and periodic spectra of the nuclei. There are several forms of this model (IBM-1, IBM-2). The other formulations of the reactive forms of bosons were created by adding the isospin to the Hamilton system so as to increase degrees of freedom and the possibility of studying different types of nuclei. There is the first and second models of the Interaction Boson-Fermion Model, where they describe odd-even and even-odd nuclei. The first reactive boson model is based on algebraic grouping, namely U (6), which is divided into three sub-strings (U (5), SU (3), O (6)) which corresponds to the series of group (Group Chain), the dynamic of the following sub-equations [5].

$$U(6) \supset \begin{cases} U(5) \supset O(5) \supset O(3) \supset O(2) \\ SU(3) \supset O(3) \supset O(2) \dots\dots(1) \\ O(6) \supset O(5) \supset O(3) \supset O(2) \end{cases}$$

Hamilton's indicator can be written in terms of the subunits of the subunits, which are divided into three parts[6]:

### 1) Vibrational Dynamical Symmetry (vibrational limit)

This is described as the subset U (5) and its quantitative numbers are written as

$$\text{follows[7]:} \left\{ \begin{array}{cccccc} U(6) \supset U(5) \supset O(5) \supset O(3) \supset O(2) \\ \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\ [N] \quad n_d \quad v, n_\Delta \quad L \quad M_L \end{array} \right\} \dots\dots(2)$$

and Hamilton is written in polar form in this limit with the following formula [8]

$$\hat{H}^I = \varepsilon \hat{n}_d + a_1 \hat{L} \cdot \hat{L} + a_3 \hat{T}_3 \cdot \hat{T}_3 + a_4 \hat{T}_4 \cdot \hat{T}_4 \dots\dots\dots(3)$$

where L is the angular momentum operator and T<sub>3</sub> and T<sub>4</sub> are the operator of the eight-pole and hexagonal pole respectively. (a<sub>1</sub>, a<sub>2</sub>, a<sub>3</sub>) are the parameters associated with the accompanying operator. Note that the effectors (Q and P) are not effective in this limit and the energy of the boson is much greater than the interaction voltage between the bosons and the eigenvalue of Hamilton U (5) which gives the following equation: [9]

$$E|N, n_d, v, n_\Delta, L, M\rangle = \varepsilon n_d + \alpha \frac{1}{2} n_d (n_d - 1) + \beta (n_d - v)(n_d + v + 3) + \gamma [L(L + 1) - 6n_d] \dots\dots\dots(4)$$

### 2) $\gamma$ -Unstable Dynamical Symmetry

This symmetry is described by subgroup O (6) and is written as[10]:

$$\left. \begin{array}{cccccc} U(6) \supset O(6) \supset O(5) \supset O(3) \supset O(2) \\ \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\ [N] \quad \sigma \quad \tau, v_{\Delta} \quad L \quad M_L \end{array} \right\} \dots\dots\dots(5)$$

[N] the number of total bosons and  $(\tau, \sigma, v_{\Delta})$  is quantum numbers. In this limit, the interaction of pairing is dominant of the bosons energy and Hamilton can writes for this limit as[11]:

$$\hat{H}^{III} = a_0 \hat{p} \cdot \hat{p} + a_1 \hat{L} \cdot \hat{L} + a_3 \hat{T}_3 \cdot \hat{T}_3 \dots\dots\dots(6)$$

The eigenvalues of the Hamilton cases can be written as follows: [12]

$$E|N, \sigma, \tau, v_{\Delta}, L, M_L\rangle = A(N - \sigma)(N + \sigma + 4) + B\tau(\tau + 3) + CL(L + 1) \dots\dots\dots(7)$$

**3)The transition region between U (5) and O (6)**

The function of this type is determined by the following [13]:

$$\hat{H}^{I-III} = \epsilon n_d + a_0 \hat{p} \cdot \hat{p} + a_1 \hat{L} \cdot \hat{L} + a_3 \hat{T}_3 \cdot \hat{T}_3 \dots\dots\dots(8)$$

The ratio of  $(\epsilon / a_0)$  determines the properties of this region. When this ratio is large, this means that the nucleus is close

to its properties of U (5) and when it is small, it approaches the nucleus O (6).

**Potential EnergySurface**

From the operator of the Hamilton function can be obtain the surface potential energy and written as [14]:

$$E(N; \beta, \gamma) = E_0 + \frac{N}{(1+\beta^2)} (\epsilon_s + \epsilon_d \beta^2) + \frac{N(N-1)}{(1+\beta^2)^2} (f_1 \beta^4 + f_2 \beta^3 \cos 3\gamma + f_3 \beta^2 + f_4) \dots\dots\dots(9)$$

where f1, f2, f3, f4 represent the parameters associated with the Hamilton function parameters.

**2. Results and Discussion**

The behavior of isotopes <sup>72-74</sup>Zn was determined based on the values of the energy levels of the process [15] by calculating the ratio of the practical energies that showed the nuclei belonging to the transition region between the vibrations and gamma unstable limits. Table (1) shows the values of Hamilton Which gave the best match with the practical values.

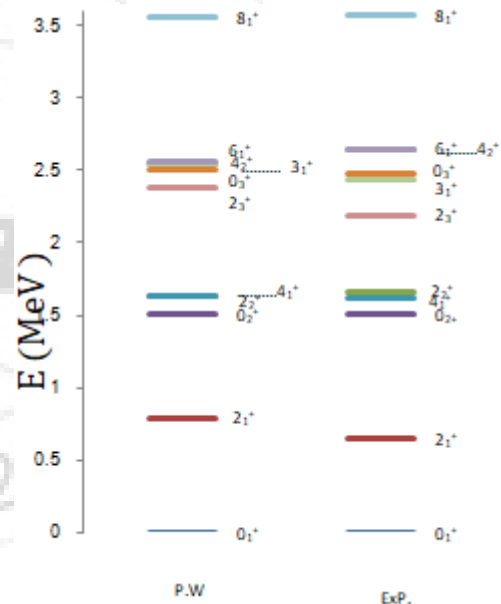
**Table 1:** Represents the values of the Hamilton operator parameters used in IBM-1 model calculations.

Isotope	N	ε	a <sub>0</sub>	a <sub>1</sub>	a <sub>3</sub>
<sup>72</sup> <sub>30</sub> Zn <sub>42</sub>	5	0.6257	0.4126	0.06701	0.4040
<sup>74</sup> <sub>30</sub> Zn <sub>44</sub>	4	0.6059	0.5315	0.05530	0.3606

The theoretical energy values calculated using the IBM-1 model and the experimental available for the energy levels are listed in Table (2), (3) and Figure (1) and (2) represent the comparison between the theatrical and experimental value

**Table 2:** Shows the comparison between the practical [15] and theatrical <sup>72</sup>Zn energy levels with the correlation coefficient value (0.994).

J <sup>P</sup>	p.w	Exp	Notes
0 <sup>+</sup> <sub>1</sub>	.000	.000	
2 <sup>+</sup> <sub>1</sub>	.781	.652	
0 <sup>+</sup> <sub>2</sub>	1.514	1.511	
4 <sup>+</sup> <sub>1</sub>	1.636	1.612	The spen and parity for (4 <sup>+</sup> <sub>1</sub> ) are confirmed
2 <sup>+</sup> <sub>2</sub>	1.63	1.657	
2 <sup>+</sup> <sub>3</sub>	2.382	2.192	The energy level (2.192) don't have spin and parity which confirmed with spin and parity (2 <sup>+</sup> <sub>3</sub> )
3 <sup>+</sup> <sub>1</sub>	2.549	(2.441)	The energy level (2.441) was confirmed at the spin and parity (3 <sup>+</sup> <sub>1</sub> ), which was uncertain in (3, 4)
0 <sup>+</sup> <sub>3</sub>	2.544	2.476	
4 <sup>+</sup> <sub>2</sub>	2.552	2.645	The energy level (2.645) was confirmed at the spin and parity (4 <sup>+</sup> <sub>2</sub> ), which was uncertain in (3, 4)
6 <sup>+</sup> <sub>1</sub>	2.561	(2.653)	The value (2.653) uncertainpractically and confirmed at the spin and parity (6 <sup>+</sup> )
6 <sup>+</sup> <sub>2</sub>	3.54424	3.395	The energy level (3.395) don't have spin and parity which confirmed with spin and parity (6 <sup>+</sup> <sub>2</sub> )
8 <sup>+</sup> <sub>1</sub>	3.556	3.569	The value (3. 569) uncertainpractically and confirmed at the spin and parity (8 <sup>+</sup> )

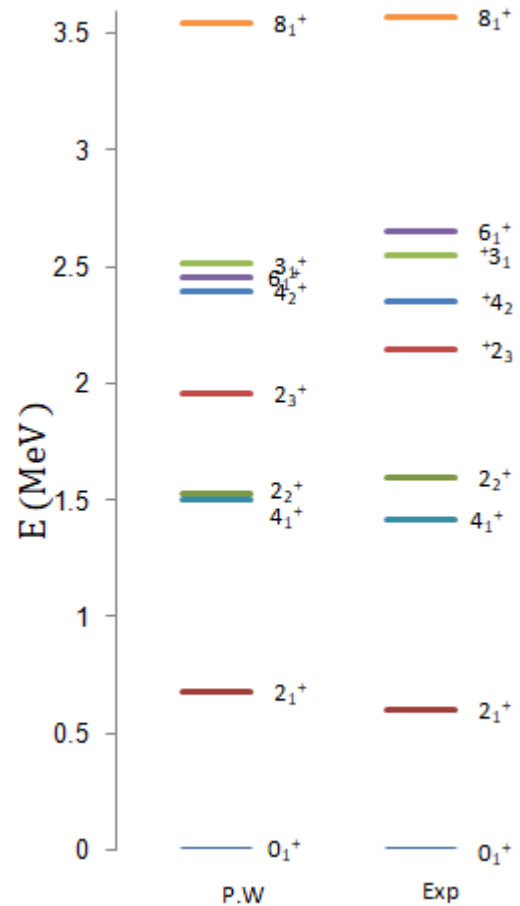


**Figure 1:** Shows the comparison of energy levels of the current study with the practical values of <sup>72</sup>Zn

**Table 3:** Shows the comparison between the practical [15] and theatrical <sup>74</sup>Zn energy levels with the correlation coefficient value (0.990)

J <sup>P</sup>	p.w	Exp	Notes
0 <sup>+</sup> <sub>1</sub>	.000	.000	
2 <sup>+</sup> <sub>1</sub>	.680	0.605	
4 <sup>+</sup> <sub>1</sub>	1.501	(1.418)	The energy level (1.41856) was confirmed at the spin and parity (4 <sup>+</sup> <sub>1</sub> ), which was uncertain in (4 <sup>+</sup> , 0 <sup>+</sup> )
2 <sup>+</sup> <sub>2</sub>	1.527	1.67	The spin and parity for (2 <sup>+</sup> ) are confirmed
0 <sup>+</sup> <sub>2</sub>	1.148	—	Expected
0 <sup>+</sup> <sub>3</sub>	2.518	—	Expected
2 <sup>+</sup> <sub>3</sub>	2.518	(2.148)	The energy level (2.1482) was confirmed at the spin and parity (2 <sup>+</sup> <sub>3</sub> ), which was

			uncertain in ( $2^+, 1$ )
$4_2^+$	2.498	2.353	The energy level (2.355) don't have spin and parity which confirmed with spin and parity ( $4_2^+$ )
$3_1^+$	2.513	2.551	The energy level (2.5518) don't have spin and parity which confirmed with spin and parity ( $3_1^+$ )
$6_1^+$	2.458	2.657	The energy level (2.6576) don't have spin and parity which confirmed with spin and parity ( $6_1^+$ )
$6_2^+$	3.601	—	Expected
$8_1^+$	3.5461	3.571	The energy level (3.571) don't have spin and parity which confirmed with spin and parity ( $8_1^+$ )

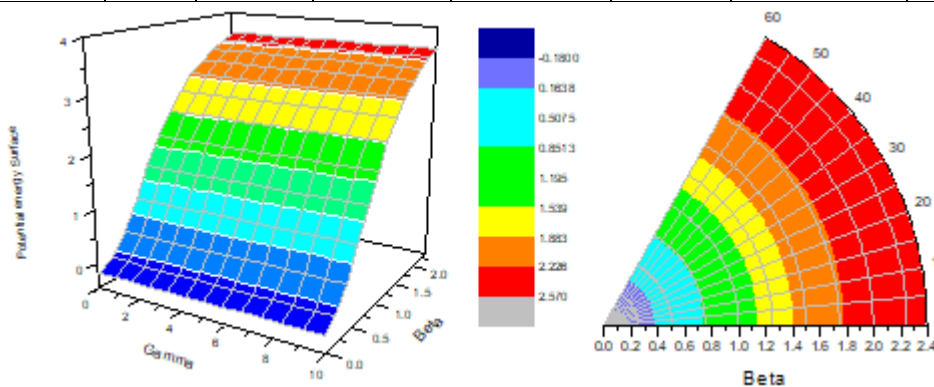


**Figure 1:** Shows the comparison of energy levels of the current study with the practical values of  $^{74}\text{Zn}$

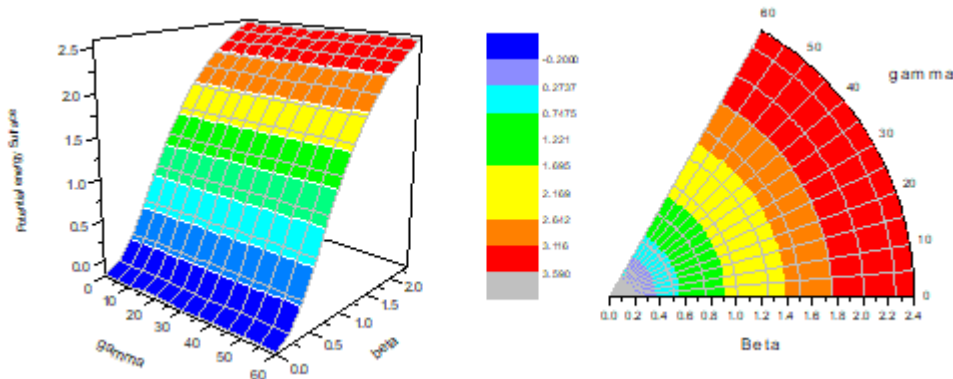
The parameter of the Hamilton function was determined to calculate the surface energy of the nucleus, which gave the final form of the nucleus in terms of two variables ( $\gamma, \beta$ ) where the minimum  $\beta$  values indicating that the nuclei belong to the transition zone between U (5) -O (6), Table (4) shows the values of the parameter used in the program and Figure (3) and Figure (4) shows the contour lines and axial symmetries of the  $^{72}\text{Zn}$  and  $^{74}\text{Zn}$  nuclei, respectively.

**Table 4:** Shows the values of the parameters used in the IBMP-code program for measured isotopes

parameters	N	$\epsilon_s$	$\epsilon_d$	$F_1$	$F_2$	$F_3$	$F_4$
<b>Isotope</b>							
$^{72}\text{Zn}_{42}$	5	0.00	0.8640	0.011	0.00	-0.0201	0.00
$^{74}\text{Zn}_{44}$	4	0.00	0.7740	0.018	0.00	-0.035	0.00



**Figure 3** shows the contour lines of the axial asymmetric angles of the calculated potential energy surface of the nucleus  $^{72}\text{Zn}$



**Figure 4:** Shows the contour lines of the axial asymmetric angles of the calculated potential energy surface of the nucleus  $^{74}\text{Zn}$

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