Effects of P-Delta and Concrete Cracking on Modal Analysis for the Seismic Response of High Rise Reinforced Concrete Buildings

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Abstract: ***This paper aims to study the effect of concrete cracking and the second-order geometric effect of P-Delta on the modal analysis for the seismic response of high rise reinforced concrete buildings. High rise reinforced concrete building models with different heights up to 50 stories are employed. To achieve this goal, the finite element code ETABS is used to analyze the structural dynamic behavior of the reinforced concrete building models when P-Delta and concrete cracking is accounted for. Different aspects of modes of vibration from modal analysis are examined including time-properties, participating factors and participation mass ratios. The study shows that time periods of vibration elongated when concrete cracking and second order effects of P-Delta are accounted for in the analyses. Generally, results show that modal analysis are significantly affected by concrete cracking and to a lesser extent by P-Delta analyses. P-Delta effects on periods and frequencies of modes of vibration are related to building height, unlike concrete cracking effect, and for buildings with less than 20 story height P-Delta effect can be neglected. Furthermore, results of the study indicate that buildings with cracked concrete sections require additional number of modes than buildings with uncracked section properties to achieve modal participation mass ratio not less than 90%, and thus sufficient number of vibration modes are included in the analyses, whereas P-Delta analysis has negligible effect on the minimum number of modes required. The paper recommends that the P-Delta and concrete cracking effects be evaluated for tall-building systems as part of an overall assessment of their free vibration response.***

Keywords: Modal Analysis, P-Delta effect, Cracked Concrete, Seismic Analysis, High-rise Buildings.

1. Introduction

High-rise buildings are becoming one of the most impressive reflection of today's civilization. High-rise buildings respond to seismic motion differently than low-rise buildings. The magnitude of inertia forces induced in an earthquake depends on the building mass, ground acceleration, nature of the foundation, and the dynamic characteristics of the building [1]. High-rise buildings experience much lower accelerations than low-rise buildings. But a flexible building subjected to ground motions for a prolonged period may experience much larger forces if its natural period is near that of the ground waves. Thus, the magnitude of the lateral force is influenced to a great extent by the type of response of the structure itself and its foundation as well. This interrelationship of building behavior and seismic ground motion also depends on the building period [1].

Generally, to estimate the seismic loading, there are two general approaches which tack into account the properties of the structure and the past record of earthquakes in the region. The first approach termed the equivalent lateral force procedure and the second approach, more refined is a modal analysis in which the modal frequencies of the structure are analyzed then used in conjunction with earthquake design spectra to estimate the maximum modal response [2]. The use of dynamic analysis will produce structural designs that are more earthquake resistant than structures designed using static loads [3].

The most important criteria to determine the vibration modes of a structure is the modal analysis. These modes are useful to understand the behavior of the structure. They can also be used as the basis for modal superposition in response spectrum and modal time-history load cases [4] where the modal superposition method is a general procedure for linear analysis of the dynamic response of structures. In various forms, modal analysis has been widely used in the earthquake-resistant design of special structures such as high-rise buildings, offshore drilling platforms, dams, and nuclear power plants for a number of years; however, its use has become more common for ordinary structures as well because of the advent of high-speed computers and the availability of relatively inexpensive structural analysis software capable of performing 3D modal analyses [1].

All structures deflect transversely under seismic loading. The effects of building loading acting on the deformed geometry of the structure creates what is referred to as the second order effects or P-Delta effect. Normally, vertical loads are concentric with the members' bases. When the structure is acted upon by a lateral load such as seismic load, it begins to be laterally displaced and the vertical load that applied on the structure become eccentric with the respect to the bases. The overturning moment is referred to as primary moment (Mp) when the total vertical loads are concentric with the structure's base. The magnitude of this moment is Fh as shown in Figure 1, where F is the lateral load and h is the height of the structure. When the vertical load becomes eccentric with respect to the base, the overturning moment adds an eccentric bending stress to the members, this additional moment is referred to as secondary moment (Ms) and the magnitude of it is $P\Delta$ where P is a function of the weight of the building include the vertical load such as dead load and live load and Δ is the drift [5]. Therefore, P-Delta is the additional sway and overturning moments due to effect of vertical loads acting through the relative transverse displacement of the member ends.

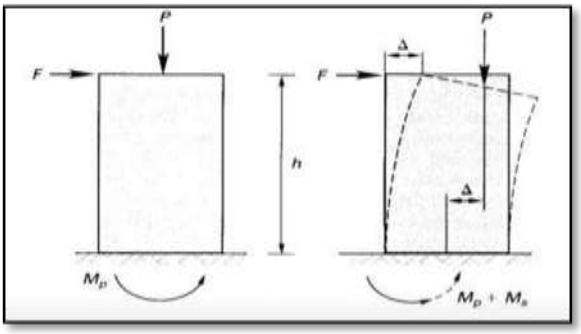

Figure 1: P-Delta effect [5]

P-Delta effect was an area extensive studies in recent years. Recently, Yousuf et al. [6] studied P-Delta effect in reinforcement concrete structure of the rigid joint and the result showed that axial, moment and displacement of the structural components from the P-Delta analysis were higher as compared with linear analysis. In other papers, Konapure and Dhanshetti [7] showed that when the number of stories increased the effect of P-Delta becomes very dominant. Prashant et al. [8] studied the P-Delta effect on high rise reinforced concrete buildings having a different number of stories, their results showed that P-Delta effect must be taken into account for buildings having 25 story or buildings having height more than or equal to 75m.

2. Statement of the Problem

The equations of motion for linear multi-degree of freedom (MDF) system without damping is [9]

$$m\ddot{u} + ku = p(t) \tag{1}$$

The simultaneous solution of these coupled equations of motion is not efficient for MDF systems, nor is it feasible for systems excited by general dynamic forces like earthquake. However, for dynamic response analysis of linear systems, a much more useful representation of the displacements is provided by the free vibration mode shapes. Hence, the displacement vector **u** of MDF system can be expanded in terms of modal contributions. Thus, the dynamic response of a system can be expressed as

$$u(t) = \sum_{r=1}^{N} \varphi_r q_r(t) = \Phi q(t) \tag{2}$$

Where **Φ** is called the modal matrix for the eigenvalue problem, **q**(t) called modal coordinated or normal coordinates. Substituting (1) in (2) gives

$$\sum_{r=1}^{N} m\varphi_r \ddot{q}_r(t) + \sum_{r=1}^{N} k\varphi_r q_r(t) = p(t) \tag{3}$$

Multiply each term in previous equation by φ_n^T gives

$$\sum_{r=1}^{N} \varphi_n^T m\varphi_r \ddot{q}_r(t) + \sum_{r=1}^{N} \varphi_n^T k\varphi_r q_r(t) = \varphi_n^T p(t) \tag{4}$$

Because of the orthogonality relations, all terms in each of the summations vanish, except the r = n term, reducing this equation to

$$(\varphi_n^T m\varphi_n)\ddot{q}_n(t) + (\varphi_n^T k\varphi_n)q_n(t) = \varphi_n^T p(t) \tag{5}$$

or

$$M_n \ddot{q}_n(t) + K_n q_n(t) = P_n(t) \tag{6}$$

where

$$M_n = \varphi_n^T m\varphi_n \quad K_n = \varphi_n^T k\varphi_n \quad P_n(t) = \varphi_n^T p(t) \tag{7}$$

(6) may be interpreted as the equation governing the response $q_n(t)$ of the single degree of freedom (SDF) system shown in Figure 2 with mass ***Mn*** , stiffness ***Kn***, and exciting force ***Pn(t)***. Therefore, ***Mn***, ***Kn***, and ***Pn(t)*** are called the generalized mass, stiffness and force for nth natural mode, respectively.

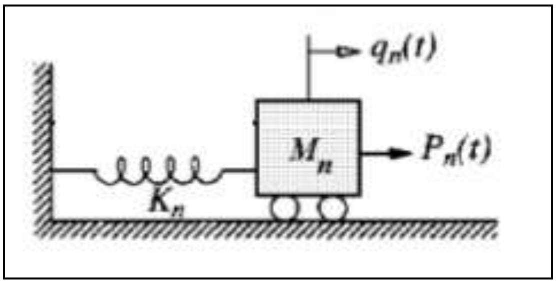

Figure 2: Generalized SDF system for the nth mode [9]

Dividing (6) by ***Mn*** and using $K_n = \omega_n^2 M_n$ gives

$$\ddot{q}_n + \omega_n^2 q_n = \frac{P_n(t)}{M_n} \tag{8}$$

The solution of the coupled equations of motion (1) are transformed to an uncoupled set of equations of the classical modal analysis given by (6) or (8). Hence, the response are provided by the free vibration mode shapes. The generalized properties (7) are influenced by the mode shapes φ_n and the response (8) is influenced by circular frequencies ω_n of the independent modes of the free vibration of the system.

P-Delta and concrete cracking effects are usually detrimental to the strength and stability of high rise concrete buildings. Including concrete members cracking in the analysis model will result in a reduced member stiffness, and thus a more deformable structure. P-Delta or second order effects contribute to reduce effective lateral resistance and ratcheting residual deformations. Moreover, the second order effects usually increase displacements, member forces and alter dynamic response characteristics [10]. The aim of this research is to assess P-delta and concrete cracking effects on the properties of the free vibration characteristics of high rise reinforced concrete buildings with different heights. Various properties of the vibration modes from modal analysis including time period, circular frequency, eigenvalues and participation mass ratios are presented and discussed.

3. Problem Assessment

In order to assess the effects of P-Delta and concrete cracking on the dynamic characteristics of high-rise reinforced concrete buildings, five building models with different heights are investigated. All building models have the same plan view shown in Figure 3 for typical building story. The structural system has been assumed as a dual system consists of a central core of shear wall structure and interior and exterior columns arranged in a rectangular 6x6 meter grid with the exterior columns are connected by edge beam to form moment resisting frames in the two orthogonal directions. The plan of the multi-story reinforced concrete building is square 36 by 36 meter. The floor system for the building models has been assumed to be a reinforced concrete flat slab of 220mm thick. Table 1 listed section properties of columns and shear walls for the five building prototypes for all stories where C1 represents the square columns, C2 represents the corner columns, and C3 represents the rectangular columns. All beams have been assumed to have 30 cm by 110 cm cross-section and the coupling beams between shear walls have been assumed to have 110 cm depth and the same thickness of the shear walls that make up the central core.

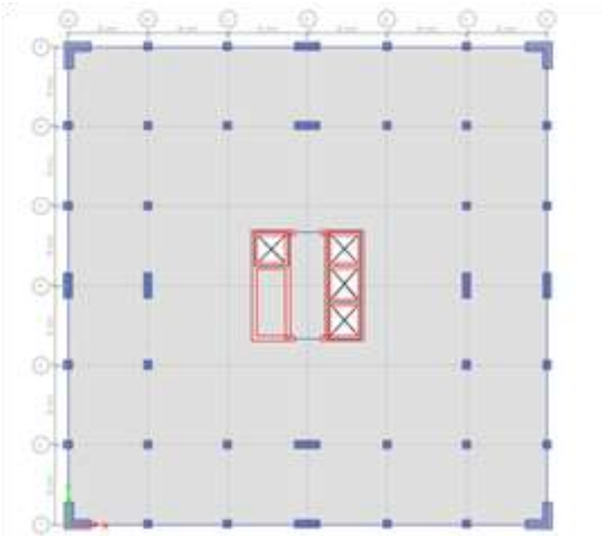

Figure 3: Plan view of typical building story

To achieve the goal of this study, the finite element code ETABS "Extended 3D Analysis of Building Systems" is used to investigate the structural dynamic behavior of the modeled reinforced concrete building prototypes when P-Delta and concrete cracking is accounted for. Figure 4 shows the three-dimensional finite element discretization for typical building model. For finite element representation, frame elements are used to model columns and beams, whereas shell elements are used to model slabs and shear walls.

Reinforced concrete structures are generally subjected to cracking as many actions including flexural stresses due to bending. To include concrete cracking effect in the structural analysis it is usually accounted for by reducing members stiffness as cracking reduces concrete member's strength. A stiffness reduction is recommended by different design standards and codes [10]. In order to take into account concrete cracked section properties in the analysis the proposed stiffness modifiers indicated in Table 2 are implemented in ETABS models. Applying stiffness modifiers in the numerical model will result in a reduced member strength, and thus a more flexible structure.

P-Delta effects are considered in this research by using second order analysis based on large displacement-small strain theory. In ETABS there are two types of modal analysis to choose from when defining a modal load case: Eigenvector analysis and Ritz-vector analysis. In this study, eigenvector analysis has been used. Eigenvector analysis determines the undamped free vibration mode shapes and frequencies of the system. These natural modes provide an excellent insight into the behavior of the structure [4]. A modal Analysis Case may be based on the stiffness of the full unstressed structure, or upon the stiffness at the end of a nonlinear Analysis Case (nonlinear static or nonlinear direct integration time-history). By using the stiffness at the end of a nonlinear case, one can evaluate the modes under P-delta or geometric stiffening conditions

Table 2: Stiffness modifiers for concrete elements [4], [11]

Element	*ACI*	*ETABS*
Columns	0.70 I_g	$I_{22}=I_{33}=0.7$
Beams	0.35 I_g	$I_{22}=I_{33}=0.35$
Walls-uncracked	0.70 I_g	$f_{11}=f_{22}=0.70$
Walls-cracked	0.35 I_g	$f_{11}=f_{22}=0.35$
Slabs	0.25 I_g	$f_{11}=f_{22}=f_{12}=0.25$, and $m_{11}=m_{22}=m_{12}=0.25$

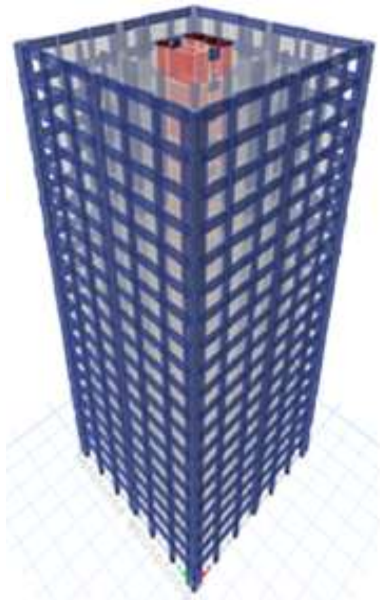

Figure 4: Three-Dimensional ETABE model for typical building layout

Table 1: Structural system section properties for building models

Building Model	Story	Dimension of columns (cm)			shear wall thickness (cm)	Concrete strength*
		C1	C2	C3		
G+9	G to 9	70x70	L 200x70	200x70	40	C40
G+19	G to 9	80x80	L 200x80	200x80	45	C50
	10 to 19	70x70	L 200x70	200x70	45	C40
G+29	G to 9	90x90	L 300x50	200x80	50	C50
	10 to 19	80x80	L 300x50	200x70	50	C50
	20 to 29	70x70	L 300x50	200x60	50	C40
G+39	G to 9	100x100	L 300x60	300x60	60	C60

	10 to 19	90x90	L 300x60	300x60	60	C50
	20 to 29	80x80	L 300x50	300x50	50	C50
	30 to 39	70x70	L 300x50	300x50	50	C40
G+49	G to 9	110x110	L 300x70	300x70	70	C70
	10 to 19	100x100	L 300x70	300x70	70	C60
	20 to 29	90x90	L 300x60	300x60	60	C60
	30 t0 39	80x80	L 300x60	300x60	60	C50
	40 to 49	70x70	L 300x50	300x50	50	C40

* Letter C denotes the specified concrete compressive strength of 150mm cube at 28 days, expressed in N/mm^2

4. Analyses Results

Modal analysis by ETABS provides various properties of the vibration modes as analysis results. This information is the same regardless of whether eigenvector or Ritz-vector analysis is used [4]

- Periods and Frequencies
- Participation Factors
- Participating Mass Ratios
- Load Participation Ratios

Time-properties for each mode include: Time period, T, in units of time; cyclic frequency, *f*, in units of cycles per time; circular frequency, ω, in units of radians per time; Eigenvalue, ω^2, in units of radians per time squared.

Fundamental period or natural period is the rate at which the buildings will go back and forth if they are given a horizontal push [1]. It is impossible to make an object vibrate at anything other than its fundamental period. The time period is a function of the height of the building and there are other factors affect the period such as the building's structures system, construction materials, and geometric proportions, but the height of the building is the most important. Figure 5 shows the effect of building height on time of the period of vibration.

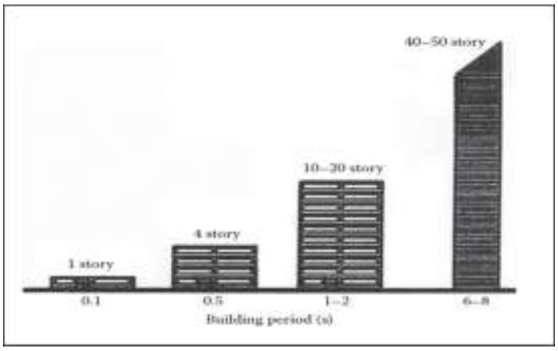

Figure 5: Effect of building height on vibration period [1]

To investigate the effect of concrete cracking on the time-properties for each mode of vibration response of the building models two values for the property/stiffness modifiers for the frame and shell elements are considered in the analysis. The values for these property/stiffness modifiers are as follows:

- Uncracked: where all property/stiffness modifiers for the analysis equal to one as the default values in ETABS.
- Cracked: where all property/stiffness modifiers for the analysis are adopted in accordance with the values shown in Table 2.

Table 3 shows time-properties for the modal analysis for all studied building models for the first three modes of vibration. Results shown for first order analysis with the effects for concrete cracking. It is observed that time periods and frequencies variation have the same trend as for the same buildings when concrete cracking effects ignored.

Table 3: Vibration time-properties for modal analysis

Bld. Model	Mode	Period, sec	Freq. cyc/sec	Circular Freq. rad/sec	Eigen value rad^2/sec^2
G+9	1	1.355	0.738	4.637	21.508
	2	1.178	0.849	5.331	28.429
	3	1.069	0.935	5.875	34.515
G+19	1	3.725	0.268	1.686	2.845
	2	3.305	0.303	1.901	3.614
	3	2.728	0.367	2.303	5.305
G+29	1	5.404	0.185	1.162	1.352
	2	5.186	0.193	1.211	1.468
	3	2.985	0.335	2.104	4.430
G+39	1	6.843	0.146	0.918	0.843
	2	6.686	0.150	0.939	0.883
	3	3.677	0.272	1.708	2.920
G+49	1	8.986	0.111	0.699	0.489
	2	8.866	0.113	0.708	0.502
	3	4.612	0.217	1.362	1.856

Table 4 shows the percentage differences for the values of fundamental time periods for different building heights due to P-Delta effects. Moreover, a comparison is presented in this table for P-Delta effects for the cases of cracked and uncracked concrete sections. On the other hand, Table 5 shows fundamental time period differences and comparison due to concrete cracking when the building models are firstly analyzed without P-Delta effect and secondly when a nonlinear second order analysis is performed with P-Delta effects. Results presented in Tables 4 and 5 are also shown schematically in Figures 6 and 7.

Results in the above-mentioned tables and figures reveal that fundamental time periods of vibration are significantly influenced by concrete cracking and P-Delta effects and, generally, time periods of vibration are elongated due to concrete cracking and P-Delta effects. A maximum increase in fundamental time periods due to cracking of concrete is about 41.5% and due to P-Delta effect is about 14.5%. It is concluded from Table 4 that P-Delta effects on time periods are increased for taller buildings, especially for cracked buildings. This behavior coincide with the fact that building sway and member forces are increased due to P-Delta effect as building height increased. Moreover, results in Table 4 show that increase in vibration time periods due to P-Delta effects is less than about 5% for building height not exceeding 20 stories, and thus can be neglected. Whereas results presented in Table 5 show that elongation in time periods due to concrete cracking is not related to building height and, generally, percentage increase in time periods is greater than 20%. Finally, Figures 8 to 12 present variation of time periods for the first 12 modes of vibration with comparison for concrete cracking and P-Delta effects. These figure reveal the difference in time periods due to concrete cracking is significantly reduced after the 3rd mode of vibration.

Table 4: Effect of P-Delta analysis on fundamental time periods for the uncracked and cracked building models

Building Model	Uncracked			Cracked		
	without P-Delta	with P-Delta	% increase	without P-Delta	with P-Delta	% increase
G+9	1.104	1.114	0.91	1.355	1.372	1.25
G+19	2.699	2.773	2.74	3.725	3.925	5.37
G+29	4.06	4.233	4.26	5.404	5.83	7.88
G+39	5.398	5.712	5.82	6.843	7.516	9.83
G+49	7.037	7.615	8.21	8.986	10.29	14.51

Table 5: Effect of concrete cracking on fundamental time periods for buildings with and without P-Delta effects

Building Model	Without P-Delta			With P-Delta		
	Uncracked	Cracked	% increase	Uncracked	Cracked	% increase
G+9	1.104	1.355	22.7	1.114	1.372	23.1
G+19	2.699	3.725	38.0	2.773	3.925	41.5
G+29	4.06	5.404	33.1	4.233	5.83	37.7
G+39	5.398	6.843	26.8	5.712	7.516	31.6
G+49	7.037	8.986	22.7	7.615	10.29	23.1

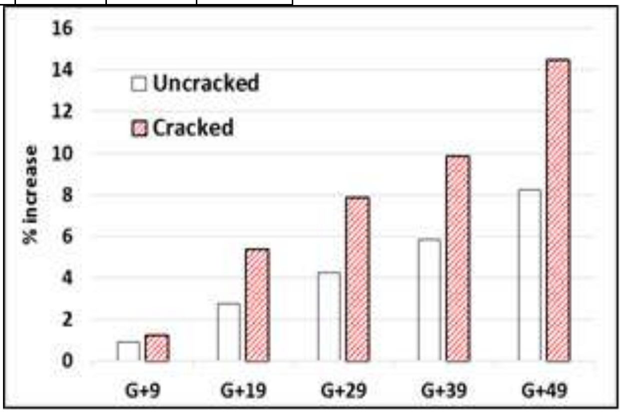

Figure 6: Increase in the fundamental time periods for the uncracked and cracked building models due to P-Delta effects

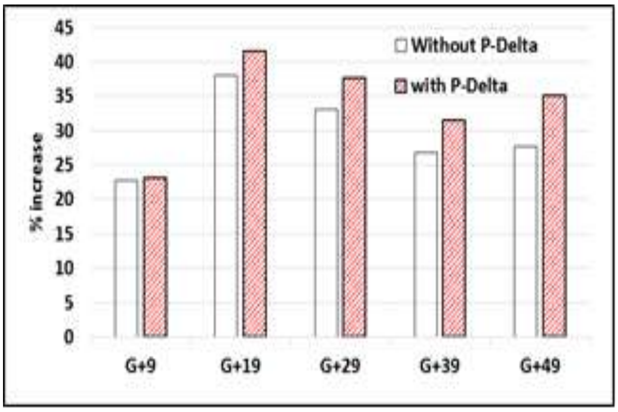

Figure 7: Increase in the fundamental time periods for the different building models due to concrete cracking

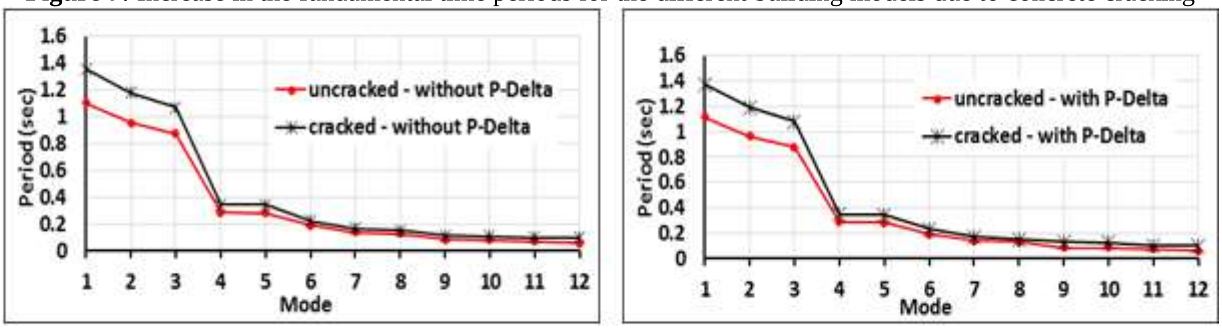

Figure 8: Periods of vibration for (G+9) building for the cracked and uncracked concrete sections and with/without P-Delta effects

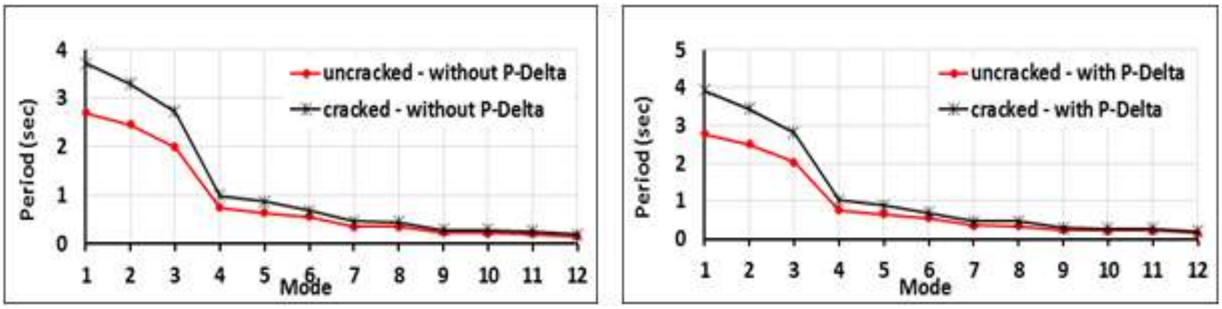

Figure 9: Periods of vibration for (G+19) building for the cracked and uncracked concrete sections and with/without P-Delta effects

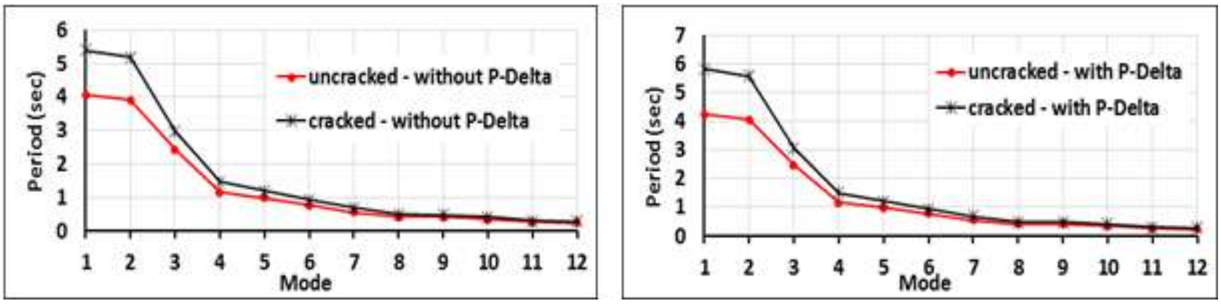

Figure 10: Periods of vibration for (G+29) building for the cracked and uncracked concrete sections and with/without P-Delta effects

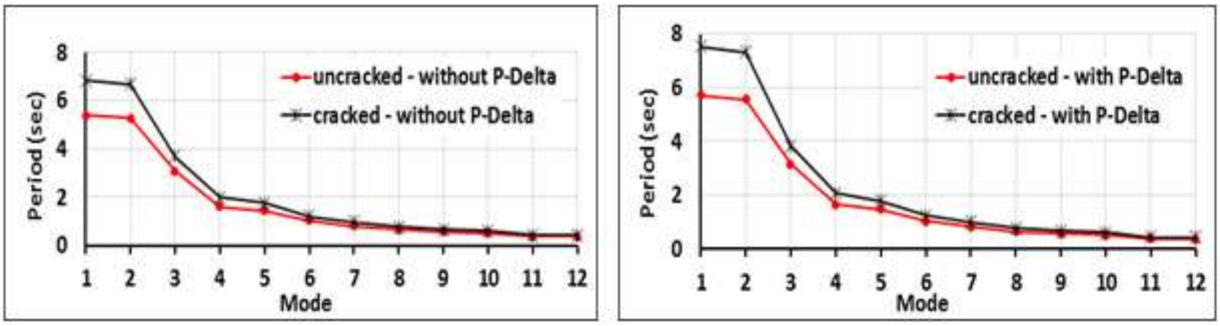

Figure 11: Periods of vibration for (G+39) building for the cracked and uncracked concrete sections and with/without P-Delta effects

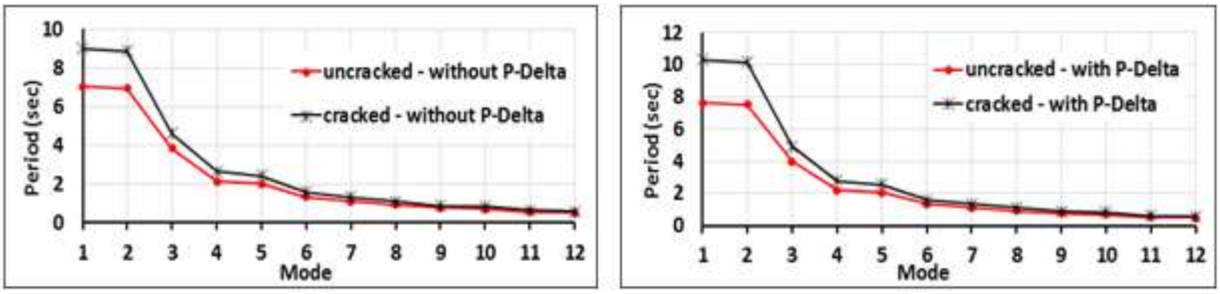

Figure 12: Periods of vibration for (G+49) building for the cracked and uncracked concrete sections and with/without P-Delta effects

Another important aspect of modal analysis to be considered is the modal participation mass ratios. The participating mass ratio for a Mode provides a measure of how important the mode is for computing the response to the Acceleration Loads in each of the three global directions. Thus it is useful for determining the accuracy of response spectrum analyses and seismic time-history analyses [4]. The participating mass ratios for Mode n corresponding to Acceleration Loads in the global X, Y, and Z directions are given by:

$$r_{xn} = \frac{(f_{xn})^2}{M_x}, \; r_{yn} = \frac{(f_{yn})^2}{M_y}, \; r_{zn} = \frac{(f_{zn})^2}{M_z} \qquad (9)$$

Where M_x, M_y and M_z are the total unrestrained masses acting in the global directions, and f_{xn}, f_{yn}, f_{zn} are the participation factors for Mode n corresponding to Acceleration Load in the global directions given by

$$f_{xn} = \phi_n^T m_x, \; f_{yn} = \phi_n^T m_y, \; f_{zn} = \phi_n^T m_z \qquad (10)$$

where ϕ_n is the mode shape and m_x, m_y, and m_z are the unit Acceleration Loads. These factors are the generalized loads acting on the Mode due to each of the Acceleration Loads.

Seismic analysis must include a sufficient number of modes to obtain a combined modal mass participation of at least 90 percent of the actual mass in each of the orthogonal horizontal directions of response considered [12]. The cumulative sums of the participating mass ratios for all Modes up to Mode n are shown in Table 6 to Table 10. This provides a simple measure of how many Modes are required to achieve a given level of accuracy for ground acceleration loading. Results presented for a sufficient number of modes of vibration to give modal mass participation of at least 90% of the actual total mass factors. In these tables the sum marked with "*", in any of the global direction, represents the smallest mode number to be included in the analyses.

Results shown reveal that buildings with cracked concrete sections require additional number of modes than buildings with uncracked sections to achieve modal participation mass ratios of at least 90%. Whereas, it is observed that including P-Delta effect in the analysis has no effect on the minimum number of modes required to achieve modal participation mass ratios of at least 90%. The same behavior for modal participation mass ratio due to concrete cracking and P-Delta effect is observed for all building heights.

Table 6: The cumulative sums of the participating mass ratios for all Modes for building (G+9)

Building Model	No. of Stories	Mode	The effective modal mass participation											
			without P-Delta						with P-Delta					
			uncracked			cracked			uncracked			cracked		
			Sum Ux	Sum Uy	Sum Rz	Sum Ux	Sum Uy	Sum Rz	Sum Ux	Sum Uy	Sum Rz	Sum Ux	Sum Uy	Sum Rz
G+9	10	1	0.7227	0.0002	0.0001	0.7184	0.0002	0.0001	0.7227	0.0002	0.0001	0.7184	0.0002	0.0001
		2	0.7228	0.0031	0.7687	0.7185	0.003	0.7642	0.7228	0.0031	0.7689	0.7184	0.003	0.7645
		3	0.723	0.689	0.7719	0.7188	0.686	0.7672	0.723	0.6889	0.7721	0.7187	0.6858	0.7675
		4	0.871	0.689	0.7793	0.8324	0.686	0.8034	0.8707	0.6889	0.7797	0.8301	0.6858	0.8054
		5	0.881	0.6891	0.8935	0.8802	0.6861	0.8912	0.881	0.689	0.8935	0.8802	0.6859	0.8912
		6	0.881	0.8949	0.8935	0.8803	0.8942	0.8913	0.881	0.8949	0.8935	0.8802	0.8942	0.8913
		7	0.8812	0.8949	**0.944**	0.8804	0.8942	**0.9425**	0.8812	0.8949	**0.9441**	0.8804	0.8942	**0.9425**
		8	**0.9403**	0.8949	0.9442	**0.9398**	0.8942	0.9427	**0.9403**	0.8949	0.9442	**0.9399**	0.8942	0.9427
		9	0.9403	**0.9527***	0.9452	0.9398	0.8942	0.9427	0.9403	**0.9524***	0.9453	0.9399	0.8942	0.9427
		10	0.9403	0.9548	0.9705	0.9407	0.8942	0.9427	0.9403	0.9548	0.9705	0.9406	0.8942	0.9427
		11	0.9689	0.9548	0.9706	0.9407	**0.953***	0.9433	0.9689	0.9548	0.9706	0.9406	**0.9529***	0.9433
		12	0.9689	0.9548	0.985	0.9408	0.9543	0.9693	0.9689	0.9548	0.985	0.9406	0.9529	0.9433

*The minimum number of modes with mass participation ≥ 90%

Table 7: The cumulative sums of the participating mass ratios for all Modes for building (G+19)

Building Model	No. of Stories	Mode	The effective modal mass participation											
			without P-Delta						with P-Delta					
			uncracked			cracked			uncracked			cracked		
			Sum Ux	Sum Uy	Sum Rz	Sum Ux	Sum Uy	Sum Rz	Sum Ux	Sum Uy	Sum Rz	Sum Ux	Sum Uy	Sum Rz
G+19	20	1	0.7065	0.0011	3.2E-06	0.6961	0.001	4.3E-06	0.7066	0.0011	3.4E-06	0.6961	0.001	4.86E-06
		2	0.7077	0.6632	0.0003	0.6972	0.6523	0.0005	0.7078	0.6628	0.0003	0.6972	0.6516	0.0005
		3	0.7077	0.6634	0.7692	0.6972	0.6526	0.76	0.7078	0.6631	0.7701	0.6972	0.6519	0.7615
		4	0.8561	0.6634	0.7692	0.8506	0.6527	0.7601	0.8561	0.6631	0.7701	0.8505	0.652	0.7616
		5	0.8561	0.6637	0.876	0.8506	0.6528	0.8693	0.8561	0.6633	0.8762	0.8506	0.6521	0.8695
		6	0.8561	0.8449	0.8763	0.8506	0.8427	0.8695	0.8561	0.8448	0.8765	0.8506	0.8426	0.8697
		7	**0.9077**	0.8449	0.8779	0.8603	0.8428	**0.9074**	**0.9075**	0.8448	0.8781	0.8595	0.8426	**0.9081**
		8	0.9097	0.845	**0.9197**	**0.9068**	0.8428	0.9155	0.9096	0.8449	**0.9198**	**0.9067**	0.8427	0.9155
		9	0.9097	**0.9134***	0.92	0.907	0.844	0.9414	0.9096	**0.9133***	0.9201	0.9069	0.8434	0.9415
		10	0.9099	0.9139	0.9446	0.907	**0.913***	0.9418	0.9098	0.9139	0.9446	0.9069	**0.9129***	0.9417
		11	0.9395	0.9139	0.9448	0.9378	0.913	0.9419	0.9395	0.9139	0.9448	0.9377	0.9129	0.9419
		12	0.9396	0.9139	0.9606	0.9378	0.913	0.9586	0.9396	0.9139	0.9606	0.9377	0.9129	0.9586

Table 8: The cumulative sums of the participating mass ratios for all Modes for building (G+29)

Building Model	No. of Stories	Mode	The effective modal mass participation											
			without P-Delta						with P-Delta					
			uncracked			cracked			uncracked			cracked		
			Sum Ux	Sum Uy	Sum Rz	Sum Ux	Sum Uy	Sum Rz	Sum Ux	Sum Uy	Sum Rz	Sum Ux	Sum Uy	Sum Rz
G+29	30	1	0.6914	0.0055	2.6E-06	0.6755	0.0072	4.7E-06	0.6913	0.0057	2.6E-06	0.6745	0.8922	4.9E-06
		2	0.6972	0.666	0.0001	0.683	0.6517	4.8E-05	0.6973	0.6656	0.0001	0.6825	0.8924	0.0001
		3	0.6972	0.6661	0.7428	0.683	0.6517	0.7293	0.6973	0.6657	0.7437	0.6825	0.8924	0.7304
		4	0.8482	0.6662	0.7428	0.8443	0.6518	0.7293	0.8483	0.6657	0.7437	0.8443	0.0077	0.7304
		5	0.8483	0.8298	0.7428	0.8444	0.8261	0.7294	0.8483	0.8297	0.7438	0.8444	0.6507	0.7305
		6	0.8483	0.8299	0.8596	0.8444	0.8262	0.8568	0.8483	0.8298	0.8597	0.8444	0.6508	0.857
		7	**0.9008**	0.8299	0.8596	0.8982	0.8262	0.8568	**0.9007**	0.8298	0.8597	0.8981	0.6508	0.857
		8	0.9008	0.8454	0.8953	0.8982	0.8612	0.8793	0.9007	0.8441	0.8962	0.8981	0.8259	0.8807
		9	0.9008	0.8953	**0.9068**	0.8982	0.8924	**0.9054**	0.9007	0.8952	**0.9068**	0.8981	0.826	**0.9054**
		10	0.9289	0.8953	0.9068	**0.9269**	0.8924	0.9054	0.9288	0.8952	0.9068	**0.9268**	0.826	0.9054
		11	0.9289	0.8954	0.9323	0.9269	0.8926	0.9312	0.9288	0.8953	0.9323	0.9268	0.8592	0.9311
		12	0.929	**0.9299***	0.9324	0.945	0.8926	0.9312	0.929	**0.9296***	0.9324	0.9449	0.8922	0.9311
		13	0.9467	0.93	0.9324	0.945	**0.9274***	0.9313	0.9466	0.93	0.9324	0.9449	**0.9274***	0.9313

*The minimum number of modes with mass participation ≥ 90%

Table 9: The cumulative sums of the participating mass ratios for all Modes for building (G+39)

Building Model	No. of Stories	Mode	The effective modal mass participation											
			without P-Delta						with P-Delta					
			uncracked			cracked			uncracked			cracked		
			Sum Ux	Sum Uy	Sum Rz	Sum Ux	Sum Uy	Sum Rz	Sum Ux	Sum Uy	Sum Rz	Sum Ux	Sum Uy	Sum Rz
G+39	40	1	0.6824	0.0071	0	0.6705	0.0074	0	0.6821	0.0077	0	0.6689	0.0087	0
		2	0.6897	0.6664	5.8E-06	0.6781	0.6543	4.3E-06	0.69	0.6662	6.4E-06	0.6779	0.6535	4.9E-06
		3	0.6897	0.6664	0.7381	0.6781	0.6543	0.7232	0.69	0.6662	0.74	0.6779	0.6535	0.7254
		4	0.8377	0.6665	0.7381	0.8352	0.6544	0.7232	0.8378	0.6663	0.74	0.8353	0.6535	0.7254
		5	0.8378	0.8171	0.7381	0.8353	0.8143	0.7232	0.8379	0.8168	0.74	0.8353	0.8139	0.7254
		6	0.8378	0.8171	0.8522	0.8353	0.8143	0.8495	0.8379	0.8168	0.8525	0.8353	0.8139	0.8498
		7	0.891	0.8171	0.8522	0.8895	0.8143	0.8495	0.8909	0.8169	0.8525	0.8894	0.8139	0.8498
		8	0.891	0.8802	0.8523	0.8895	0.8784	0.8495	0.891	0.88	0.8526	0.8894	0.8781	0.8499
		9	0.891	0.8802	0.898	0.8895	0.8784	0.8974	0.891	0.8801	0.8981	0.8894	0.8782	0.8975
		10	**0.9197**	0.8802	0.898	**0.9185**	0.8784	0.8974	**0.9196**	0.8801	0.8981	**0.9183**	0.8782	0.8975
		11	0.9197	0.8825	**0.9223**	0.9185	**0.9037**	**0.9045***	0.9196	0.8822	**0.9224**	0.9183	**0.9024**	**0.9054***
		12	0.9197	**0.9151***	0.924	0.9185	0.9132	0.9239	0.9196	**0.9149***	0.924	0.9183	0.9131	0.9238

*The minimum number of modes with mass participation ≥ 90%

Table 10: The cumulative sums of the participating mass ratios for all Modes for building (G+49)

Building Model	No. of Stories	Mode	The effective modal mass participation											
			without P-Delta						with P-Delta					
			uncracked			cracked			uncracked			cracked		
			Sum Ux	Sum Uy	Sum Rz	Sum Ux	Sum Uy	Sum Rz	Sum Ux	Sum Uy	Sum Rz	Sum Ux	Sum Uy	Sum Rz
G+49	50	1	0.6659	0.0173	0	0.6517	0.019	0	0.6616	0.0219	0	0.6411	0.0289	0
		2	0.6836	0.6659	3.3E-06	0.6712	0.653	2.4E-06	0.684	0.6659	4E-06	0.6707	0.6522	3E-06
		3	0.6836	0.6659	0.7344	0.6712	0.653	0.7182	0.684	0.6659	0.7377	0.6707	0.6522	0.722
		4	0.8303	0.6662	0.7344	0.8269	0.6532	0.7182	0.8304	0.6662	0.7377	0.8269	0.6523	0.722
		5	0.8306	0.8102	0.7344	0.8271	0.8065	0.7182	0.8307	0.8099	0.7377	0.8271	0.8059	0.722
		6	0.8306	0.8102	0.8485	0.8271	0.8065	0.8439	0.8307	0.8099	0.849	0.8271	0.8059	0.8445
		7	0.8856	0.8103	0.8485	0.8841	0.8065	0.8439	0.8855	0.8099	0.849	0.8839	0.8059	0.8445
		8	0.8856	0.8723	0.8485	0.8841	0.8707	0.8439	0.8855	0.872	0.849	0.8839	0.8703	0.8445
		9	0.8856	0.8723	0.8942	0.8841	0.8707	0.8931	0.8855	0.8721	0.8943	0.8839	0.8703	0.8933
		10	**0.9145**	0.8723	0.8942	**0.9134**	0.8707	0.8931	**0.9144**	0.8721	0.8943	**0.9132**	0.8703	0.8933
		11	0.9145	**0.9069**	0.8943	0.9134	**0.9056**	0.8932	0.9144	**0.9067**	0.8945	0.9132	**0.9054**	0.8933
		12	0.9145	0.9071	**0.9194***	0.9315	0.9056	0.8932	0.9144	0.9069	**0.9194***	0.9314	0.9054	0.8933
		13	0.9325	0.9071	0.9194	0.9316	0.9057	**0.9192***	0.9324	0.9069	0.9194	0.9314	0.9055	**0.9192***

*The minimum number of modes with mass participation ≥ 90%

5. Conclusions

In this paper, an attempt is carried out to investigate the effects of P-Delta and cracking of concrete on the modal analysis for the seismic response of high rise reinforced concrete buildings.

The study reveal that time periods of vibration are elongated when concrete cracking and second order effects of P-Delta are accounted for in the analyses. Generally, results of the study show that modal analyses are significantly affected by concrete cracking and to a lesser extent by P-Delta effects. An increase of about 20% to 40% in the fundamental time periods of vibration is observed due to cracking of the concrete elements of the investigated building models. As for P-Delta effect, results show a maximum increase in the fundamental time periods of about 15% for high rise buildings up to 50 stories, and this effect is significantly reduced and can be neglected for buildings with height less than 20 stories.

Concerning effective modes of vibration to be included in the dynamic response analyses, results of the study indicate that buildings with cracked concrete sections require additional number of modes than buildings with uncracked section properties to achieve modal participation mass ratios not less than 90% to attain a given level of accuracy for ground acceleration loading. Furthermore, it is concluded that P-Delta analysis has negligible effect on the minimum number of modes required to achieve modal participation mass ratios of at least 90%, and thus the effective number of modes of vibration.

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