

# Kinematic Analysis of Novel 3RRS Parallel Mechanism

Balaji .K<sup>1</sup>, Shahul Hamid Khan .B<sup>2</sup>

<sup>1</sup>Research Scholar, Indian Institute of Information Technology Design & Manufacturing, Kancheepuram, Chennai-600 127, India

<sup>2</sup>Assistant Professor, Indian Institute of Information Technology Design & Manufacturing, Kancheepuram, Chennai-600 127, India

**Abstract:** *In this paper, a 3 DoF (Degree of Freedom) novel RRS (Revolute-Revolute-Spherical) type parallel mechanisms has been designed and presented. The combination of straight and arc type linkages for 3 DOF parallel mechanism is introduced for the first time. The kinematic analysis and the overall reachable workspaces of the mechanisms are presented.*

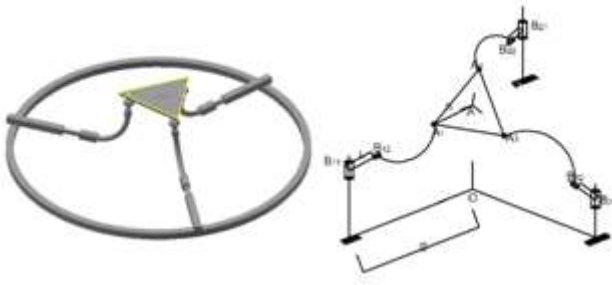
**Keywords:** RRS parallel mechanism; Forward kinematics analysis; Inverse kinematic Analysis

## 1. Introduction

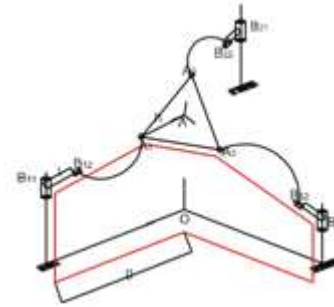
In modern world, Industrial robotic application plays a vital role in the field of automation. For the past two decades parallel robotic architectures have sought more attention, both theoretical and practical approaches. Numerous Configurations have been constituted and formulated and the same also fabricated its proto type and experimentally validated. [1, 2, 3, 5] In this study, a exhaustive list of different type of Parallel mechanisms were studied. A parallel mechanism (PM) typically consists of a moving platform that is connected to a fixed base by several limbs or legs. Due to its external heavy load can be shared by the actuators, PM tend to have a large load carrying capacity. In many applications, PM has been implemented such as airplane simulators, adjustable articulated trusses, mining machine, pointing devices, high speed machining center, and walking machines. One of the disadvantages of PMs is the difficulty of trajectory planning mainly due to singular configurations, in which the mechanism gains one or more degrees of freedom and therefore loses stiffness of the mobile platform completely. In Particular, 3 DOF RS type configuration is clearly focused and analysed. [6] The kinematic analysis of such configuration also carried out with different actuator arrangement. The reachable workspace features, dexterity characteristics such as kinematic manipulability and global dexterity also derived. [7] using homotopy continuation method also forward kinematic analysis also carried out. [12] In medical application, for CPR (Cardio Pulmonary Resuscitation) the same configuration used and its dynamic model also analysed based on lagrangian formulation. [13] For the same configuration, the work volume also determined using fuzzy logic approach. [15, 14] Based on special decomposition of reaction force, a novel approach also identified for the dynamic analysis. Experimentally these 3 DOF PMs were verified and compared with ADAMS software.

Based on its limb arrangements of 3PRS configurations were classified into seven subcategories and kinematical analysis carried out to avoid parasitic motion. A new index called Worst case Global Indices [18] (WGI) was introduced, to take into consideration the kinematic and force transmission

performance of the parallel mechanism. This new index was compared with the existing indices such as Global Conditioning Index (GCI). These comparisons were carried out in delta robot which was used in ENT surgery application. Further exploring the wide application of PMs a novel idea of spherical parallel wrist [19] was proposed. A detail kinematic analysis shows that actuator redundancy not only removes singularities but also improves the workspace by increasing the dexterity. Using parallel mechanism a new hip simulator mechanism [20] was proposed. The kinematic index is calculated to measure the force transmission ratio. The simulation results were compared and verified with the experimental works. To evaluate the performance of these mechanisms dexterity index, kinematic manipulability index and global dexterity index was derived. Another mechanisms 6URS [21] parallel mechanism, was proposed. A vast survey of the performance indices [22] which were used to evaluate the performance or characteristics of parallel mechanism was studied elaborately in kinematics, dynamics and in workspace aspects. A new 3RRPRR [23] parallel mechanism was introduced to achieve purely translation motion. Singularity analysis is carried out from the derivation of Jacobian matrices. The concept of the Jacobian matrix, manipulability index [24] and the condition no were discussed in the view of mechanisms performance. Based on the literature survey, a novel PRS configuration is proposed. The kinematic analysis and performance measure based evaluation also carried out. In this paper is organized in the following manner; the overall Design information including its construction & mobility information is presented in section II. The kinematic analysis are presented in section III, It is then evaluated with a numerical case study & presented in section 6 as results & discussion. Conclusions are drawn in section VII followed by References. 2. Proposed Mechanism



**Figure 1:** Proposed (Proposed (CAD & schematic) 3DOF PRS type Parallel Mechanism Configuration



**Figure 3:** Vector loop closure formations in 3PRS parallel mechanisms

## 2. Geometric (Constructive) Description

The proposed novel RS type PMs consist of  $n(= 7)$  links,  $j(= 9)$  joints, where DoF of revolute joint and prismatic joint is one and for the spherical joint is three. Degree of freedom of this mechanism can be calculated using Grubler Kutzbach criterion:

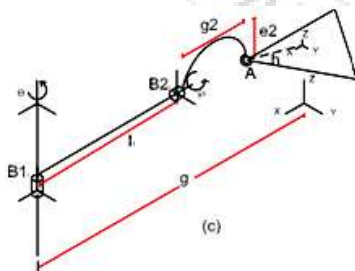
$$DoF = \lambda(n - j - 1) + \sum f_i \quad (1)$$

where  $f_i$  is degree of freedom of individual joint. Thus,  $DoF = 6(7-9-1) + (3*3+3*1+3*1) = 3$ .

**Table 1:** Quantity of Joints and linkages used in all type of 3 DoF RS type configurations

S.No	Components	Quantity
1	Number of links	7
2	Number of joints	9
3	Number of revolute/prismatic joints	3
4	Number of spherical joints	3
5	Number of actuating joints	3
6	Number of passive joints	3 spherical & 3 prismatic joints

## 3. Kinematics



**Figure 2:** Schematic Representation of RRS

### 3.1 Loop closure formulation

From the below mentioned schematic representation, in RRS mechanisms constitute of passive Revolute and spherical joint in each of its identical kinematic limb. using vector approach, the connectivity between the identical kinematic limb with its neighbour kinematic leg through the moving platform is called as loop closure. In the figure it was represented in red lines. the mathematical expression for this loop closure is called as loop closure equation.

$$B_{Ai} = B_{Bi1} + B_{i1,Ai} \quad (2)$$

Based on the vector approach,

$$\|B_{Ai} - B_{Ai+1}\| = \sqrt{3}h \quad (3)$$

Based on the above mentioned expression 3 constraint equation were framed. these 3 equations consists of all the joint variable.

$$\begin{aligned} \eta_1 &= f(\theta_1, \psi_1, \theta_2, \psi_2) \\ \eta_2 &= f(\theta_2, \psi_2, \theta_3, \psi_3) \\ \eta_3 &= f(\theta_1, \psi_1, \theta_3, \psi_3) \end{aligned}$$

In equations (4), (5) and (6) all active variables ( $\psi_1; \psi_2; \psi_3$ ) are known. Hence,

$$\begin{aligned} \eta_1 &= f(\theta_1, \theta_2) \\ \eta_2 &= f(\theta_2, \theta_3) \\ \eta_3 &= f(\theta_1, \theta_3) \end{aligned}$$

To obtain the univariate polynomial expression, Using Sylvester Dialytic elimination method in [9] can be converted in to 16 degree univariate polynomial expression of  $dl_3$ . By choosing the appropriate value for  $dl_3$  from the 16 values we can find the suitable  $dl_1$  and  $dl_2$ . Using this  $dl_1$ ,  $dl_2$  and  $dl_3$ , we can find the  $B_{A1}$ ;  $B_{A2}$  and  $B_{A3}$  respectively. Hence, we can find the centroidal position, X, by the following expression.

$$A = \left[ \frac{B_{A1} + B_{A2} + B_{A3}}{3} \right] \Rightarrow X = [x, y, z]^T$$

where,  $x = f(\theta_i, \psi_i)$ ;  $y = f(\theta_i, \psi_i)$ ;  $z = f(\theta_i, \psi_i)$ .

and orientation ( $B_{R_A}$ ) by the following expression:

$$B_{R_A} = [\hat{x}, \hat{y}, \hat{z}]$$

Where,

$$\begin{aligned} \hat{x} &= \left[ \frac{B_{A1} - B_{A2}}{|B_{A1} - B_{A2}|} \right]; \hat{y} = \left[ \frac{B_{A1} - B_{A2}}{|B_{A1} - B_{A2}|} \right] \times \left[ \frac{B_{A1} - B_{A3}}{|B_{A1} - B_{A3}|} \right]; \\ \hat{z} &= \left[ \frac{B_{A1} - B_{A2}}{|B_{A1} - B_{A2}|} \right] \times \left[ \frac{B_{A1} - B_{A3}}{|B_{A1} - B_{A3}|} \right] \end{aligned}$$

The rotational matrix  $B_{R_A}$  can also be defined based on Rodrigues formula as follows:

where,

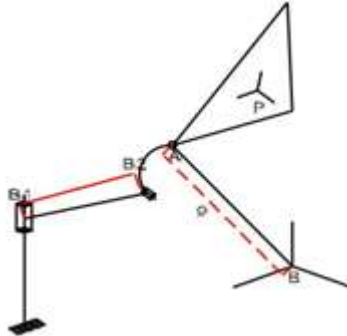
$$B_{R_A} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

$$\omega = \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} \Rightarrow \frac{1}{2} \sin(\hat{k}) \begin{bmatrix} r_{32} - r_{23} \\ r_{13} - r_{31} \\ r_{21} - r_{12} \end{bmatrix}$$

Where,  $\alpha = f(\theta_i, \psi_i)$ ;  $\beta = f(\theta_i, \psi_i)$ ;  $\gamma = f(\theta_i, \psi_i)$

### 3.2 Inverse Kinematics

In Inverse Kinematics, the pose of the moving platform is given & the corresponding joint variable (passive and active variable) is to be determined and also the length of the leg travelled also calculated from this expression.

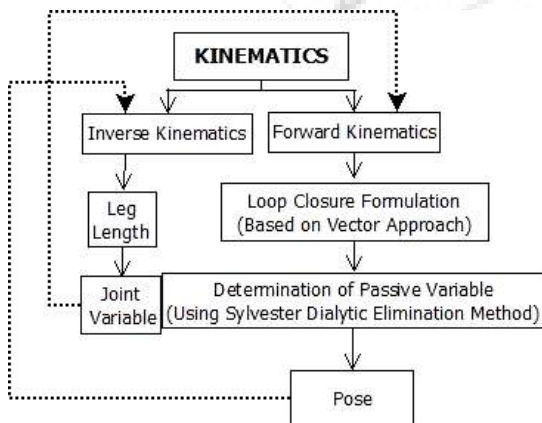


**Figure 4:** Schematic Representations for Inverse kinematics

$$q_i = P + B_{R_A} \cdot A_{A_i}$$

where Centroidal Position of the moving platform with respect to the fixed origin frame B,  $P = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$B_{R_A}$  is the rotational matrix of B to A in Euler's angle and  $A_{A_i}$  is the distance between centroidal position of the moving platform to the spherical joint. Then the distance of the limb,  $d_i^2 = (q_i - B_{B_{i1}})^T \cdot (q_i - B_{B_{i1}})$

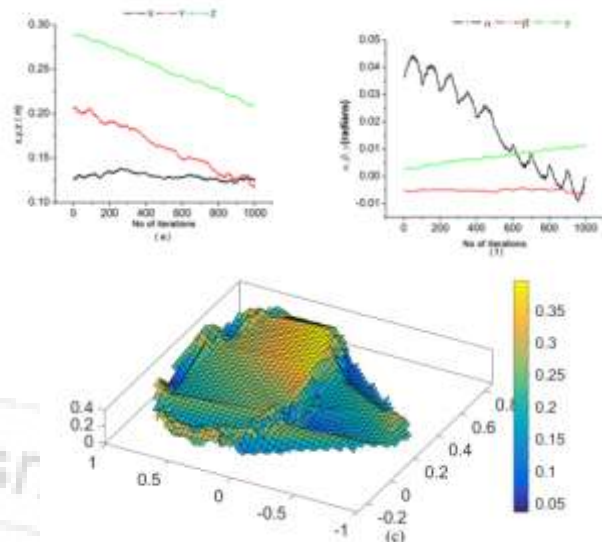


**Figure 5:** Overall Performance of 3RRS mechanism

### 4. Numerical Simulation and Results

The architectural parameter of all the RS type novel and its mechanism such as 3RRS were as follows.  $g=2.00m$ ;  $l_1=0.75m$ ;  $e_1=0.40$ ;  $g_1=0.40$ ;  $h=0.45m$ . The input (actuator) joint variables in all the mechanisms were  $\psi_1=90$  to  $0$ ;  $\psi_2=90$  to  $0$ ;  $\psi_3=90$  to  $0$ ; Using above mentioned joint

variables range and architectural parameters in the Proposed RRS type mechanism the performance are carried out. The travel of moving platform centroidal position is taken as reference. The kinematic analysis and the performance evaluation is carried out in the following manner



**Figure 3:** Graphical position, Orientation and workspace of 3RRS configurations

### 5. Results and Discussions

Based on the numerical simulation, The obtained plot are discussed as, the pose characters tics of this configurations are progressively going downwards in Y and Z axis motion whereas in X axis motion has minimum level of proportional motional when compared with the other two axis. But in tilting capabilities, the shows much variations compared with  $\beta$  &  $\gamma$ . From the overall reachable workspace, it shows that the moving platform will reaches it maximum of 0.25m in X and 0.1m in Y axis respectively and the maximum height of reach in Z axis is 0.4 m. The maximum and minimum parameters obtained during its kinematic and Indices based performance evaluations were shown in Table 2.

**Table 2:** Range of maximum parameter capabilities for 3RRS configurations

S. No	Parameter	Maximum
1	X (m)	0.869
2	Y (m)	1
3	Z (m)	0.4
4	( $\alpha$ ) $^\circ$	42
5	( $\beta$ ) $^\circ$	11
6	( $\gamma$ ) $^\circ$	10

### 6. Conclusion

This paper, a 3 DoF (Degree of Freedom) novel RRS (Revolute-Revolute- Spherical) type PMs has been designed and presented .The combination of straight and arc type linkages for 3 DOF parallel mechanism is introduced for the first time. The overall reachable workspace of the mechanism are presented.

### Appendix



$$B_{B_{i1}} = \begin{Bmatrix} g \\ 0 \\ 0 \end{Bmatrix}; B_{B_{i2}} = \begin{Bmatrix} 0.5g \\ \frac{\sqrt{3}g}{2} \\ 0 \end{Bmatrix}; B_{B_{i3}} = \begin{Bmatrix} -0.5g \\ \frac{\sqrt{3}g}{2} \\ 0 \end{Bmatrix}; B_{i1, A1} =$$

$$\begin{Bmatrix} x \\ y \\ z \end{Bmatrix} \Rightarrow \begin{Bmatrix} \cos(\theta_i)(g1 + l1) + e1 \cos(\psi_i) \sin(\theta_i) \\ \sin(\theta_i)(g1 + l1) - e1 \cos(\psi_i) \cos(\theta_i) \\ e1 \sin(\psi_i) \end{Bmatrix}$$

## References

- [1] Rao. V. Dukkupati. "Spatial Mechanisms Analysis and Synthesis". Narosha Publishing House, Delhi, 2001.
- [2] Ghosal, A. "Robotics Fundamental Concept and Analysis ". Oxford University Press, New Delhi, 2001.
- [3] Tsai L.W. "Robot Analysis-The Mechanics of Serial and Parallel Manipulator". John Wiley & Sons. Ltd., New York, 1999.
- [4] Merlet J.P. " Parallel Robots". Springer Publications, Netherland, 1999.
- [5] Kong X and Gosselin C. "Type Synthesis of Parallel Mechanisms Springer Tracts in Advanced Robotics". Springer Verlag-Heilderberg, Germany, 2007.
- [6] Li, Y. and Xu, Q., "Kinematic analysis of a 3-PRS parallel manipulator ". Robotics and Computer-Integrated Manufacturing, 23(4), pp.395-408,2007.
- [7] Yangmin.Li and Qingsong. Xu," Kinematic analysis of a 3-PRS parallel manipulator".Robotics and Computer-Integrated Manufacturing, vol. 23, 2007, pp. 395-408.
- [8] G. Abbasnejad, S. Zarkandi, and M. Imani,"Forward Kinematics Analysis of a 3-PRS Parallel Manipulator ".Engineering and Technology,pp. 329-335,2010.
- [9] Y. Li and Q. Xu." Concept Design and Dynamic Modelling of a medical Parallel Manipulator to Assist in Cardiopulmonary ". Architecture,pp. 842-845,2005.
- [10]D. Liu, Y. Xu, and R. Fei" Study of an intelligent micro-manipulator " Journal of Materials Processing Technology, vol. 139,pp. 77-80,2003.
- [11]I.A. Bonev" Singularity Analysis of Zero-Torsion Parallel Mechanisms " Manufacturing Engineering, pp. 22-26,2008.
- [12]Liu, C.H. and Ma, C.M.," Graphical Analysis for Locating Direct Singular Positions of the Parallel Manipulator MIPS". Mechanics based design of structures and machines, 38(4), pp.453-467,2010.
- [13]Li, Y. and Xu, Q., " Kinematic design and dynamic analysis of a medical parallel manipulator for chest compression task ". In Robotics and Biomimetics (ROBIO). IEEE International Conference on (pp. 693-698),2005.
- [14]Mahapatro, M. and Selvakumar, A.A.," Weight Criteria Detection to Find Work Volume of 3-PRS Parallel Manipulator Using Fuzzy Logic". Procedia Engineering, 97, pp.1929-1934,2014.
- [15]Yuan, W.H. and Tsai, M.S.," A novel approach for forward dynamic analysis of 3-PRS parallel manipulator with consideration of friction effect". Robotics and Computer-Integrated Manufacturing, 30(3), pp.315-325,2014.
- [16]Selvakumar, A.A. and Kumar, M.A.," Experimental Investigation on Position Analysis of 3-DOF Parallel Manipulators", Procedia Engineering, 97, pp.1126-1134,2014.
- [17]Li, Q., Chen, Z., Chen, Q., Wu, C. and Hu, X., " Parasitic motion comparison of 3-PRS parallel mechanism with different limb arrangements"Robotics and Computer-Integrated Manufacturing, 27(2), pp.389-396,2011.
- [18] K. C. Olds, " Global indices for kinematic and force transmission performance in parallel robots"IEEE Transactions on Robotics, 31(2), pp.494--500, 2015.
- [19] S. Leguay-Durand and C. Reboulet, " Optimal design of a redundant spherical parallel manipulator" Robotica 15(4), pp.399--405 1997.
- [20]B. R. So and B.-J. Yi," Design of a redundantly actuated hip mechanism" JSME International Journal Series C, 49(1), pp.253--259 (2006).
- [21]Y. Li and Q. Xu, " Kinematic analysis of a 3-prs parallel manipulator" Robotics and Computer-Integrated Manufacturing,23(4), pp395--408, 2007.
- [22]K. E. Zanganeh, R. Sinatra, and J. Angeles" Kinematics and dynamics of a six-degree-of-freedom parallel manipulator with revolute legs",Robotica, 15(4),pp.385--394 1997.
- [23]S. Patel and T. Sobh," Manipulator performance measures-a comprehensive literature survey" Journal of Intelligent & Robotic Systems, 77(3-4), pp.547--570,2015.
- [24]R. Di Gregorio, " A new family of spherical parallel manipulators"Robotica, 20(4), pp.353--358 2002.
- [25]J.-P. Merlet, " Jacobian, manipulability, condition number, and accuracy of parallel robots" ,Journal of Mechanical Design,128(1), pp.199--206, 2006.

## Author Profile



**K. Balaji** is a research scholar in the Dept. of Mechanical Engineering, Indian Institute of Information Technology Design and Manufacturing (IIITD&M) Kancheepuram, INDIA.. He received his B.E. degree (2007) in Mechanical Engineering from Anna University and M.E Degree in Engineering Design (2009) from Government college of Engineering, Tirunelveli (GCE,Tly). His research focus is on **Parallel Mechanisms, its Kinematics and Dynamics, Multi objective optimization**



**Dr. Shahul Hamid Khan B.** Received the B.E in Mechanical Engineering and M.E in Production Engineering at Thiagarajar College of Engineering, Madurai. He completed his PhD at National Institute of Technology – Tiruchirappalli, India. His current Research includes **flow shop scheduling, multi-objective optimization, Green supply chain management and Reverse Logistics**. He is currently working in the Department of Mechanical Engineering at Indian Institute of Information Technology, Design and Manufacturing - Kancheepuram, Chennai, India.