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Quantum Evaporation of Black Holes and Hawking Radiation

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Abstract: ***Black Holes are the most mysterious objects in our universe. In this paper I will use Schwarzschild solution, Quantum Field Theory and Thermodynamics to show that a Black Hole emits as though it were a Black body with a temperature called the Hawking Temperature. I will derive an expression for the Hawking Temperature. I will also calculate the entropy of a Black Hole. Finally I will show why small Black Holes should evaporate from Hawking Radiation in finite Time.***

Keywords: Black Holes, Hawking Radiation, Entropy, Temperature

1. Introduction

The fascinating link between the physics of black holes and thermodynamics has come as a big surprise for twentieth century physicists. How does an idea of black hole dynamics relate to laws of thermodynamics and fate of the universe will be discussed in this paper.

Black holes are formed when the core of a supernova is more than 2.5 times the mass of the Sun. This is the Landau-Oppenheimer-Volkov limit,[4] above which neutron degeneracy pressure is insufficient to support the star against unlimited gravitational collapse. Such a core collapse to form a black hole [5]. Karl Schwarzschild was the first to find exact solutions of Einstein's equations of general relativity. The Schwarzschild solution gives equations for the space time geometry outside a spherical non-rotating mass .These equations describe how the curvature of space time changes from point to point. The greater the curvature the more significant non-Euclidean effects like gravitational time dilation, light deflection become. We shall use Schwarzschild solution to understand black holes and Hawking radiation.

Black holes of three different origins, with three different mass scales are found in our universe they are
(i)Black Holes in X-Ray Binaries-The collapse of massive stars in supernova explosions can result in black holes with mass ranging up to of order 10 solar masses. When these are members of binary systems they can be detected by their influence on orbit of the companion star and by radiation from accretion disks that may form around them.
(ii)Black Holes in Galaxy Centers-The deep gravitational potential wells at the center of galaxies are natural sites of gravitational collapse. The resulting super massive black holes at the center of galaxies range from millions to billions of solar masses.

(iii)Exploding Primordial Black Holes-The evidence of cosmic background radiation is that the distribution of matter in the early universe was remarkably smooth with tiny fluctuations in the density that seeded the formation of today's galaxies. Some early fluctuations might have grown and collapsed under the action of gravitational attraction and produced small black holes. Small primordial black holes would be exploding today via the quantum mechanical hawking process.

Black holes have been detected in X-ray binaries and in the centers of galaxies. If exploding black holes is detected it will shed light on the union of gravity and quantum theory.

2. Origin of Hawking Radiation

It was the union of special theory and quantum mechanics that gave birth to quantum field theory. In quantum field theory the vacuum is a boiling sea of fluctuating pairs of particles and antiparticles. If you have enough energy in vacuum special relativity allows electron and positron to be created out of vacuum if the process occurs within the time Δt <h/ΔE according to Heisenberg's uncertainty principle. Although this idea is very strange it has real physical consequences. One of these is the Casmir effect [6] a small but measurable force of attraction between two parallel metal plates in a vacuum.

These incessant but ephemeral fluctuations were largely of interest only to particle physicists until Hawking came along. Hawking asked what will happen to virtual pairs of particles and anti-particles which are created and annihilated all the time in empty space near a black hole. If one member of pairs falls into a black hole, the other radiates away. Hawking showed that the radiation has a black-body spectrum leading to the idea that black holes have a definable temperature.

3. Determining the Hawking Temperature

There is a mysterious correspondence between quantum statistical mechanics and quantum field theory. In the Heisenberg's formulation of quantum mechanics, the evolution of a quantum state after time δ is governed by the evolution operator $e^{-iH\delta}$,with H the Hamiltonian.The probability amplitude for an initial state |I> to end up in the final state |F> is given by

$$Z = \langle F|e^{-iH\delta}|I\rangle \quad (1)$$

However Boltzmann showed that at temperature T the relative probability of a state |n> of energy E_n occurring is given by
$e^{-\beta E_n}$,where β=1/T .We define the partition function of a quantum mechanical system with Hamiltonian H by

$$Z = \sum \langle n|e^{-\beta H}|n\rangle = \sum e^{-\beta E_n} = \mathrm{Tr}e^{-\beta H} \quad (2)$$

There is a correspondence between equations (1) and (2).To go from equation (1) to (2) we simply replace time δ by $-i\beta$,set |I>=|F>=|n>,and sum over |n> .The inverse temperature β is equal to the recurrence period in this strange world with imaginary time.

Consider a quantum field, be it the field of photon, an electron, or whatever propagating in space time. Suppose it discovers that time is actually imaginary and cyclic. The field is fooled into thinking that it is living in a temperature bath with the temperature determined by the inverse of the recurrence period β of this bizarre imaginary time.

We can use this strange observation to determine the temperature of the Hawking radiation from a Schwarzschild black hole. Consider the electromagnetic field governed by the action

$$S=\int d^4x\sqrt{-g}(-1/4g^{\mu\rho}g^{\nu\sigma}F_{\mu\nu}F_{\rho\sigma}) \quad (3)$$

This field is propagating in the Schwarzschild space time described by

$$ds^2= -(1-r_s/r)dt^2+(1-r_s/r)^{-1}dr^2+r^2d\theta^2+\sin^2\theta\, d\varnothing^2 \quad (4)$$

with r_s=2GM.Near the horizon

$$ds^2=- r-r_s/r_s dt^2+r_s/r-r_s dr^2+r_s^2d\Omega^2 \quad (5)$$

Change variables from r to ρ given by $\rho^2=4r_s(r-r_s)$.Then $\rho d\rho=2r_s dr$,so that $\rho^2 d\rho^2=4r_s^2dr^2$ or $(r-r_s)d\rho^2=r_s dr^2$. Plugging this in equation(5) we get

$$ds^2=-\rho^2/4r_s^2dt^2+d\rho^2+r_s^2d\Omega^2 \quad (6)$$

If we set $t=-it_E$ as per the procedure outlined above, we get

$$ds^2=\rho^2/4r_s^2dt_E^2+d\rho^2+r_s^2d\Omega^2 \quad (7)$$

If we now change variable setting $t_E=2r_s\Psi$ we obtain

$$ds^2=d\rho^2+\rho^2d\Psi^2+r_s^2d\Omega^2 \quad (8)$$

We recognize the first two terms describe a plane with polar radius ρ and polar angle Ψ. The (3+1) dimensional spacetime has been analytically continued into a 4 dimensional Euclidean space consisting of a plane at every point which is attached to a sphere of radius r_s. More importantly since Ψ is an angular variable we se that the imaginary time $t_E=2r_s\Psi$ has recurrence period of $2r_s(2\pi)=4\pi r_s$.Thus according to the preceding discussion, The electromagnetic field propagating near the horizon of a Schwarzschild black hole thinks that it is living in a heat bath with temperature

$$T_H = 1/4\pi r_s=1/8\pi GM=\hbar c^3/8\pi GM \quad (9)$$

This is the Hawking temperature of a black hole.

4. Calculation of Entropy

We have derived the precise expression for the Hawking temperature. Let us derive an expression for the entropy

$$d(Mc^2)= T_H dS= \hbar c^3dS/8\pi GM \quad (10)$$

which implies that the entropy S is equal to

$$S = 4\pi GM^2/\hbar c \quad (11)$$

Using $A=4\pi r_s^2$ and $r_s=2GM/c^2$

Defining l_p as the Planck length, $l_p=\sqrt{\hbar G/c^3}$

We obtain a very important result

$$S = A/4l_p^2 \quad (12)$$

5. Conclusions

For a black hole entropy is related to the area, this is a shocking result because for any normal physical system entropy is proportional to its volume. It is as if entropy of a black hole were to reside completely on its surface. This mysterious property of black holes which represent one of the deepest puzzles in theoretical physics, led 't Hooft and Susskind separately to formulate the so called holographic principle.

When the Planck constant is set equal to zero the Hawking temperature T=0, meaning that the black hole will not radiate, Hawking radiation is a quantum effect.

As the black hole radiates, it loses mass. As the mass decreases the temperature rises. The black body therefore radiates at an increasingly rapid rate as it shrinks, resulting in an explosive end. A black hole evaporates from Hawking radiation in finite time. The detection of the explosion of one of these black holes would be a significant confirmation of quantum black hole physics as well as information about early universe where primordial black holes were formed.

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