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Statistical Fluctuations of Energy Spectra in ^{32}A Nuclei using the Proton-Neutron Formalism Interaction

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Abstract: ***Statistical fluctuations of nuclear energy spectra in ^{32}A (^{32}S, ^{32}P and ^{32}Si) nuclei are probed by the framework of the nuclear shell model. Energy levels of considered nuclei are evaluated via performing sd-shell model calculations using the OXBASH computer code with the realistic effective interaction of WPN in the proton-neutron formalism. Here, we assume the ^{32}A nuclei consists of an inert ^{16}O core with 16 nucleons move in the $1d_{5/2}$, $2s_{1/2}$ and $1d_{3/2}$ orbitals. For full hamiltonian calculations of ^{32}A nuclei, we have found an intermediate behavior between Wigner and Poisson limits for both the $P(s)$ distributions and Δ_3 statistics as a result of the absence of mixing and repulsion between levels with different isospin. Further, they transfer gradually toward the GOE limit when going over ^{32}S, ^{32}P and ^{32}Si nuclei, individually (i.e. through changing the numbers of valence protons and neutrons in ^{32}A). Moreover, they are independent of the spin J (universal for different spins). For unperturbed hamiltonian (the non-interacting particles case) calculations, we have found a regular behavior for both the $P(s)$ distributions and Δ_3 statistics due to the absence of mixing and repulsion between levels with different isospin (as a result of using the proton-neutron formalism interaction) and also due to the nonexistence of the off-diagonal residual interaction.***

Keywords: Quantum chaos; Random matrix theory; Statistical fluctuations of nuclear energy spectra; Shell model calculations

1. Introduction

Chaotic properties of many body quantum systems were probed deeply during the previous three decades [1]. Bohigas et al. [2] proposed a relation between chaos in a classical system and the spectral fluctuations of the analogous quantum system, where an analytical proof of the Bohigas et al. conjecture is found in [3]. It is now typically known that quantum analogs of most classically chaotic systems demonstrate spectral fluctuations that agree with the random matrix theory (RMT) [4,5] while quantum analogs of classically regular systems reveal spectral fluctuations that agree with a Poisson distribution. For time-reversal-invariant systems, the suitable form of RMT is the Gaussian orthogonal ensemble (GOE). RMT was firstly utilized to characterize the statistical fluctuations of neutron resonances in compound nuclei [6]. RMT has become a standard tool for analyzing the universal statistical fluctuations in chaotic systems [7-10].

The chaotic manners of the single particle dynamics in the atomic nuclei can be investigated via the mean field approximation. On the other hand, the nuclear residual interaction mixes different mean field configurations and affects the statistical fluctuations of the many particle spectrum and wave functions. These fluctuations may be investigated with different nuclear structure models. The statistics of the low-lying collective part of the nuclear spectrum were studied in the framework of the interacting boson model [11, 12], in which the nuclear fermionic space is mapped onto a much smaller space of bosonic degrees of freedom. Because of the relatively small number of degrees of freedom in this model, it was also possible to relate the statistics to the underlying mean field collective dynamic. At higher excitations, additional degrees of freedom (such as broken pair) become important [13], and the effects of interactions on the statistics must be studied in larger model spaces. The nuclear shell model offers an attractive framework for such studies. In this model, realistic effective interactions are available and the basis states are labeled by exact quantum numbers of angular momentum (J), isospin (T) and parity (π) [14].

In the articles [15-19], the distribution of eigenvector components was studied by the context of the shell model. Brown and Bertsch [17] found that the basis vector amplitudes are consistent with Gaussian distribution (which is the GOE prediction) in regions of high level density but deviated from Gaussian behavior in other regions unless the calculation employs degenerate single particle energies. Later studies [19] also suggested that calculations with degenerate single particle energies are chaotic at lower energies than more realistic calculations.

Hamoudi et al [20] performed the fp-shell model calculations to analyze the statistical fluctuations of energy spectrum and electromagnetic transition intensities in A=60 nuclei using the F5P [21] interaction. The calculated results were in agreement with RMT and with the previous finding of a Gaussian distribution for the eigenvector components [15-19]. Hamoudi [22] investigated the effect of the one-body hamiltonian on the fluctuation properties of energy spectrum and electromagnetic transition intensities in ^{136}Xe using a realistic effective interaction for the N82-model space defined by $2d_{5/2}$, $1g_{7/2}$ $1h_{11/2}$, $3s_{1/2}$ and $2d_{3/2}$ orbitals. A clear quantum signature of breaking the chaoticity was observed as the values of the single particle energies are increased. Later, Hamoudi et al [23] carried out full fp-shell model calculations to investigate the regular to chaos transition of the energy spectrum and electromagnetic transition intensities in ^{44}V using the interaction of FPD6 [24] as a realistic interaction in the isospin formalism. The spectral fluctuations and the distribution of electromagnetic transition intensities were found to have a regular dynamic at $\beta = 0$, (β is the strength of the off-diagonal residual interaction), a chaotic dynamic at $\beta \geq 0.3$ and intermediate situations at $0 < \beta < 3$. Recently, Hamoudi et al [25] have carried out sd-shell model calculations to analyze the chaotic properties of energy spectra in ^{32}A nuclei using the interaction of W [26] as a realistic interaction in the isospin formalism. The results have been well described by the Gaussian orthogonal ensemble of random matrices and they show no dependency on the spin J and isospin T.

All above studies performed shell model calculations using interactions in the isospin formalism for nuclei have no more than 12 valence nucleons. There has been no detailed study for spectral fluctuations through performing shell model calculations using interaction in the proton-neutron formalism for nuclei have more than 12 valence nucleons. We thus, in this study, perform sd-shell model calculations using the realistic effective interaction of WPN [26] in the proton-neutron formalism to investigate the statistical fluctuations of nuclear energy spectra in ^{32}A (^{32}S, ^{32}P and ^{32}Si) nuclei. For full hamiltonian calculations, the $P(s)$ and Δ_3 statistics are found to have an intermediate behavior between Wigner and Poisson limits. Besides, they move gradually toward the GOE limit when going over ^{32}S, ^{32}P and ^{32}Si nuclei, individually. Moreover, they are independent of the spin J. For unperturbed hamiltonian calculations, the $P(s)$ and Δ_3 statistics are found to have a regular behavior (in agreement with the Poisson distribution).

2. Theory

The many-body system can be described by an effective shell-model hamiltonian [14]

$$H = H_0 + H', \tag{1}$$

where H_0 and H' are the independent particle (one body) part and the residual two-body interaction of H. The unperturbed hamiltonian

$$H_0 = \sum_{\lambda} e_\lambda a_\lambda^+ a_\lambda \tag{2}$$

characterizes non-interacting fermions in the mean field of the appropriate spherical core. The single-particle orbitals $|\lambda\rangle$ have quantum numbers $\lambda = (ljm\tau)$ of orbital (l) and total angular momentum (j), projection $j_z = m$ and isospin projection τ. The antisymmetrized two-body interaction H' of the valence particles is written as

$$H' = \frac{1}{4} \sum V_{\lambda\mu;\nu\rho} a_\lambda^+ a_\mu^+ a_\nu a_\rho. \tag{3}$$

The many-body wave functions with good spin J and isospin T quantum numbers are constructed via the $m-$scheme determinants which have, for given J and T, the maximum spin and isospin projection [14],

$$|M = J, T = T_3; m\rangle, \tag{4}$$

where m span the $m-$scheme subspace of states with $M = J$ and $T_3 = T$.

The matrix of the many-body hamiltonian

$$H_{kk'}^{JT} = \sum_k \langle JT;k|H|JT;k'\rangle \tag{5}$$

is eventually diagonalized to obtain the eigenvalues E_α and the eigenvectors

$$|JT;\alpha\rangle = \sum_k C_k^\alpha |JT;k\rangle \tag{6}$$

Here, the eigenvalues E_α are considered as the main object of the present investigation.

The fluctuation properties of nuclear energy spectrum are obtained via two statistical measures: the nearest-neighbors level spacing distribution $P(s)$ and the Dyson-Mehta or Δ_3 statistics [4, 27]. The staircase function of the nuclear shell model spectrum $N(E)$ is firstly build. Here, $N(E)$ is defined as the number of levels with excitation energies less than or equal to E. In this study, a smooth fit to the staircase function is performed with polynomial fit. The unfolded spectrum is then defined by the mapping [12]

$$\tilde{E}_i = \tilde{N}(E_i). \tag{7}$$

The real spacings reveal strong fluctuations whereas the unfolded levels $\tilde{E}_i$ have a constant average spacing. The level spacing distribution (which exemplifies the fluctuations of the short-range correlations between energy levels) is defined as the probability of two neighboring levels to be a distance s apart. The spacings s_i are determined from the unfolded levels by $s_i = \tilde{E}_{i+1} - \tilde{E}_i$. A regular system is forecasted to perform by the Poisson statistics

$$P(s) = \exp(-s). \tag{8}$$

If the system is classically chaotic, we foresee to get the Wigner distribution

$$P(s) = (\pi/2) s \exp(-\pi s^2/4), \tag{9}$$

which is consistent with the GOE statistics.

The Δ_3 statistic (which characterizes the fluctuations of the long-range correlations between energy levels) is utilized to measure the rigidity of the nuclear spectrum and defined by [4]

$$\Delta_3(\alpha, L) = \min_{A,B} \frac{1}{L} \int_\alpha^{\alpha+L} \left[N(\tilde{E}) - (A\tilde{E} + B)\right]^2 d\tilde{E}. \tag{10}$$

It measures the deviation of the staircase function (of the unfolded spectrum) from a straight line. A rigid spectrum corresponds to smaller values of Δ_3 whereas a soft spectrum has a larger Δ_3. To get a smoother function $\bar{\Delta}_3(L)$, we average $\Delta_3(L)$ over several n_α intervals $(\alpha, \alpha+L)$

$$\bar{\Delta}_3(L) = \frac{1}{n_\alpha}\sum_\alpha \Delta_3(\alpha, L). \qquad (11)$$

The successive intervals are taken to overlap by $L/2$. In the Poisson limit, $\Delta_3(L) = L/15$. In the GOE limit, $\Delta_3 \approx L/15$ for small L, while $\Delta_3 \approx \pi^{-2}\ln L$ for large L.

3. Results and Discussion

Calculations of the shell model are performed, via the OXBASH code [28], for ^{32}A (^{32}S, ^{32}P and ^{32}Si) nuclei. These nuclei are assumed to have an inert core of ^{16}O with 16 valence nucleons move in the sd-shell ($1d_{5/2}$, $2s_{1/2}$ and $1d_{3/2}$ orbitals) model space.

To exemplify the role of hidden integrals of motion in the spectral fluctuations, we provide in the following some samples of calculations in the proton-neutron formalism. Here we do not discriminate the states by isospin, i.e. they are only discriminated by its spin J and parity π. The interaction of proton-neutron formalism of Wildenthal (WPN) [26] is taken in this study as a realistic effective interaction together with realistic single particle energies.

The spectral fluctuations in ^{32}A nuclei are investigated for states, which have the same parity (π) and good spin (J). These fluctuations are analyzed by two statistical measures: the nearest neighbor level spacing distribution $P(s)$ and the Dyson-Mehta statistics (Δ_3 statistics).

Table 1 shows the dimensions of all J^π states considered in this study for ^{32}S, ^{32}P and ^{32}Si nuclei.

Fig. 1 exhibits the calculated $P(s)$ distributions (histograms), obtained with full hamiltonian calculations, for unfolded even $J^\pi = 0^+, 2^+, 4^+, 6^+$ and 8^+ states in ^{32}S (left column), ^{32}P (middle column) and ^{32}Si (right column) nuclei. The GOE distribution (which characterizes chaotic systems) is presented by the solid line. The Poisson distribution (which corresponds to a random sequence of levels and characterizes regular systems) is presented by the dashed line. The calculated histograms in ^{32}S (left column) show no dependency on the spin J (universal for different spins). Similar notice is found for histograms in ^{32}P (middle column) and ^{32}Si (right column) nuclei. The histograms of ^{32}S (with 8 valence protons and 8 valence neutrons), computed in the proton-neutron formalism for the above even positive parity states, show the nearest neighbor level spacing distributions intermediate between Wigner and Poisson distributions due to the nonexistence of mixing and repulsion between levels with different isospin. However, these histograms (left column) are closer to the Poisson limit giving enlarged values of $P(s)$ at small spacings. The picture is qualitatively slightly different for the $P(s)$ distributions of ^{32}P (with 7 valence protons and 9 valence neutrons) because of reducing the values of $P(s)$ at small spacings and consequently these histograms begin to grow in the direction of the Wigner limit. However, these histograms (middle column) are still nearer to the Poisson limit. The histograms (right column) of ^{32}Si (with 6 valence protons and 10 valence neutrons) are nearer to the Wigner limit giving more reduced values of $P(s)$ at small spacings. It is apparent from this figure that the level repulsion at small spacings (which is a distinctive feature of chaotic level statistics) increases gradually with going through ^{32}S (left column) to ^{32}Si (right column) nuclei, consequently the $P(s)$ distributions (histograms) move progressively toward the GOE limit. It is found that the level repulsion at small spacings is regularly increased with reducing the valence protons and rising the valence neutrons in A=32 as seen in the histograms of ^{32}S (left column), ^{32}P (middle column) and ^{32}Si (right column) nuclei. Similar argument is found in Fig. 2 for the unfolded odd $J^\pi = 1^+, 3^+, 5^+, 7^+$ and 9^+ states.

Fig.3 illustrates the spectral rigidity (Dyson's Δ_3 statistics), obtained with full hamiltonian calculations, in ^{32}S (left column), ^{32}P (middle column) and ^{32}Si (right column) nuclei. The calculated average $\Delta_3(L)$ statistic (denoted by open circles) is plotted as a function of the length of the spectrum (L) for unfolded even $J^\pi = 0^+, 2^+, 4^+, 6^+$ and 8^+ states. The Poisson distribution (denoted by the dashed line) and the GOE distribution (denoted by the solid line) are also displayed for comparison. It is obvious that the calculated Δ_3 statistics (open circles) in ^{32}S (left column), ^{32}P (middle column) and ^{32}Si (right column) nuclei have no dependency on the spin J (common for dissimilar spins). The open circles in ^{32}S, ^{32}P and ^{32}Si nuclei show the calculated Δ_3 statistics intermediate between GOE and Poisson predictions due to the absence of the isospin. Going through left, middle and right columns of this figure shows the rigidity (open circles) distributions shift progressively in the direction of the GOE limit. This shift is gradually increased with reducing the valence protons and rising the valence neutrons in A=32 as seen in the distributions of ^{32}S (left column), ^{32}P (middle column) and ^{32}Si (right column) nuclei. It is so clear that the rigidity distributions in ^{32}Si, for different spins, are very close to the GOE prediction. The same result is obtained in Fig. 4 for the unfolded odd $J^\pi = 1^+, 3^+, 5^+, 7^+$ and 9^+ states. It is important to denote that the calculated results for the Δ_3

statistics in Figs. 3 and 4 confirm the outcome that we have obtained in Figs. 1 and 2 for the $P(s)$ distributions.

Fig. 5 demonstrates the outcomes of the unperturbed hamiltonian (the situation of the non-interacting particles) for the $P(s)$ distribution of states with $J^{\pi}=0^{+}$ (upper panel), 1^{+} (middle panel) and 2^{+} (lower panel) in ^{32}A nuclei. The left, middle and right columns correspond to ^{32}S, ^{32}P and ^{32}Si nuclei, respectively. Here, the solid and dashed lines are the GOE and Poisson distributions, respectively. It is obvious that all computed $P(s)$ (histograms) displayed in this figure shows regular behavior because of the absence of mixing and repulsion between levels with different isospin (due to using the proton-neutron formalism of WPN interaction) and also due to the nonexistence of the off-diagonal residual interaction. The calculated histograms displayed in the left column for ^{32}S (with 8 valence protons and 8 valence neutrons) expose Poisson-like distribution, where enormous number of $P(s)$ accumulates at small spacing. However, Changing the number of valence protons and neutrons in ^{32}A nuclei, as in ^{32}P (with 7 valence protons and 9 valence neutrons) and ^{32}Si (with 6 valence protons and 10 valence neutrons), leads to reduce the values of $P(s)$ at small spacing and consequently makes the calculated histograms, presented in middle and right columns, to be in agreement with the Poisson limit.

Fig. 6 shows the unperturbed hamiltonian results (the non-interacting particles case) for the calculated Δ_3 statistics (open circles) of states with $J^{\pi}=0^{+}$ (upper panel), 1^{+} (middle panel) and 2^{+} (lower panel) in ^{32}A nuclei. The left, middle and right columns exemplify the outcomes for ^{32}S, ^{32}P and ^{32}Si nuclei, respectively. The GOE limit (which describes disordered systems) is shown by the solid line. The Poisson limit (which relates to an arbitrary sequence of levels and describes ordered systems) is shown by the dashed line. It is apparent that all calculated Δ_3 statistics (open circles) presented in this figure reveals ordered manners as a result of the absence of mixing and repulsion between levels with dissimilar isospin and also through the nonexistence of the off-diagonal residual interaction. Moreover, the calculated Δ_3 statistics (open circles) shown in the left column (right column) for ^{32}S (^{32}Si) overestimate (underestimate) slightly the Poisson limit at about $L>30$ while those shown in the middle column agree well with Poisson limit throughout all values of L.

4. Conclusions

The chaotic properties of energy spectra in ^{32}A nuclei are investigated through the nuclear shell model. The sd-shell model calculations are accomplished by the OXBASH computer code using the proton-neutron formalism interaction of WPN. For full hamiltonian calculations of ^{32}A nuclei, we find an intermediate behavior between Wigner and Poisson distributions for both the $P(s)$ distributions and Δ_3 statistics due to the absence of mixing and repulsion between levels with different isospin. Besides, they move progressively toward the GOE limit with going through ^{32}S, ^{32}P and ^{32}Si nuclei (i.e. through changing the numbers of valence protons and neutrons in ^{32}A). Moreover, they have no dependency on the spin J (universal for different spins). For unperturbed hamiltonian (the non-interacting particles case) calculations, we find a regular behavior for both the $P(s)$ distributions and Δ_3 statistics because of the absence of mixing and repulsion between levels with different isospin (due to using the proton-neutron formalism interaction) and also due to the nonexistence of the off-diagonal residual interaction.

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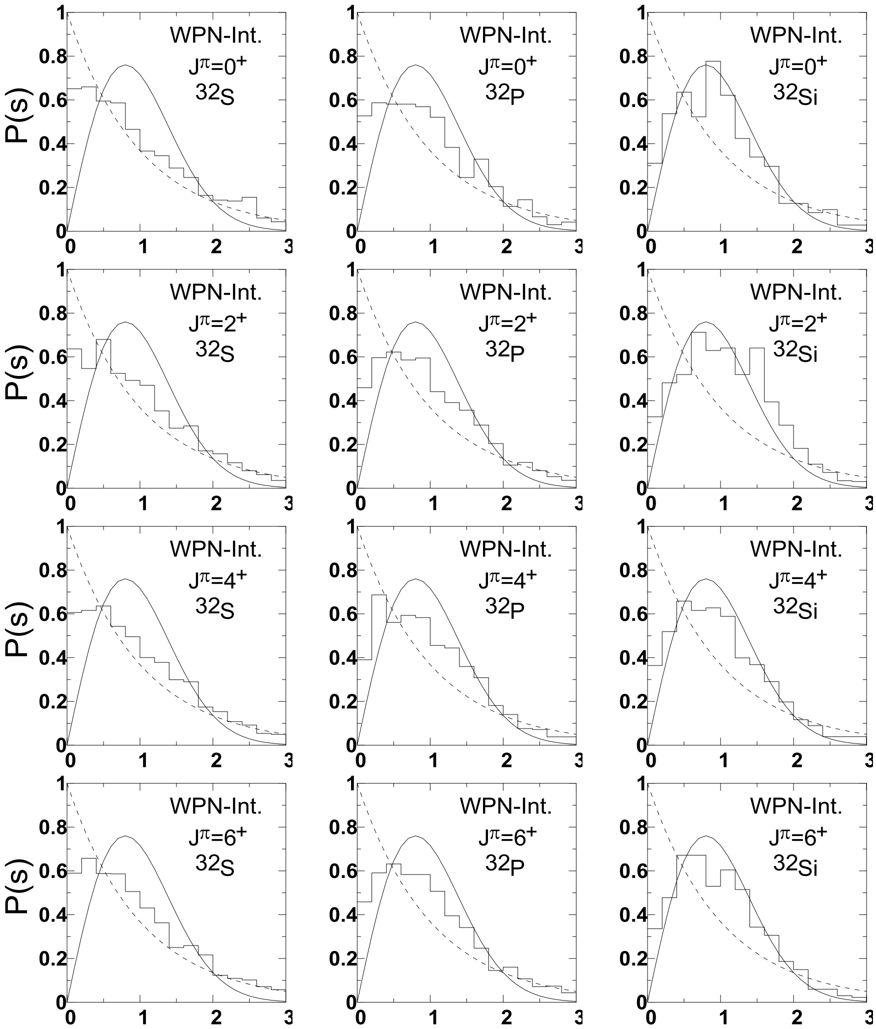
Table 1: Dimensions of J^{π} states obtained by WPN interaction for ^{32}S, ^{32}P and ^{32}Si nuclei.

J^{π}	^{32}S	^{32}P	^{32}Si
0^+	1161	836	355
1^+	3096	2317	904
2^+	4518	3312	1320
3^+	4968	3664	1396
4^+	4734	3423	1292
5^+	3843	2773	982
6^+	2799	1964	671
7^+	1746	1215	372
8^+	972	643	183
9^+	441	287	65



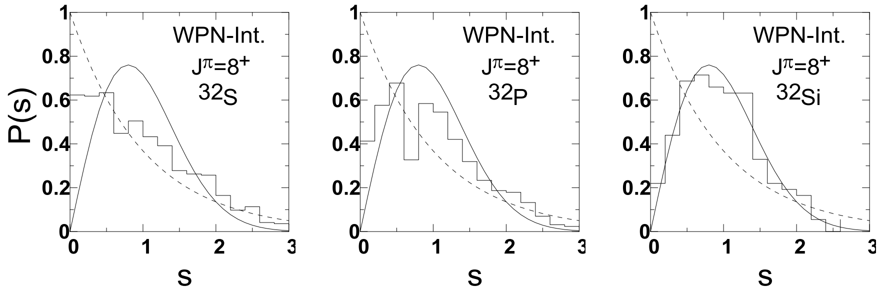


Figure 1: The nearest neighbor level spacing distributions $P(s)$ in ^{32}A [^{32}S $(T=0)$, ^{32}P $(T=1)$ and ^{32}Si $(T=2)$] nuclei for various even $J^{\pi}=0^{+}, 2^{+}, 4^{+}, 6^{+}$ and 8^{+} states. The histograms are the calculated $P(s)$ with full hamiltonian. The solid and dashed lines are the GOE and Poisson distributions, respectively.

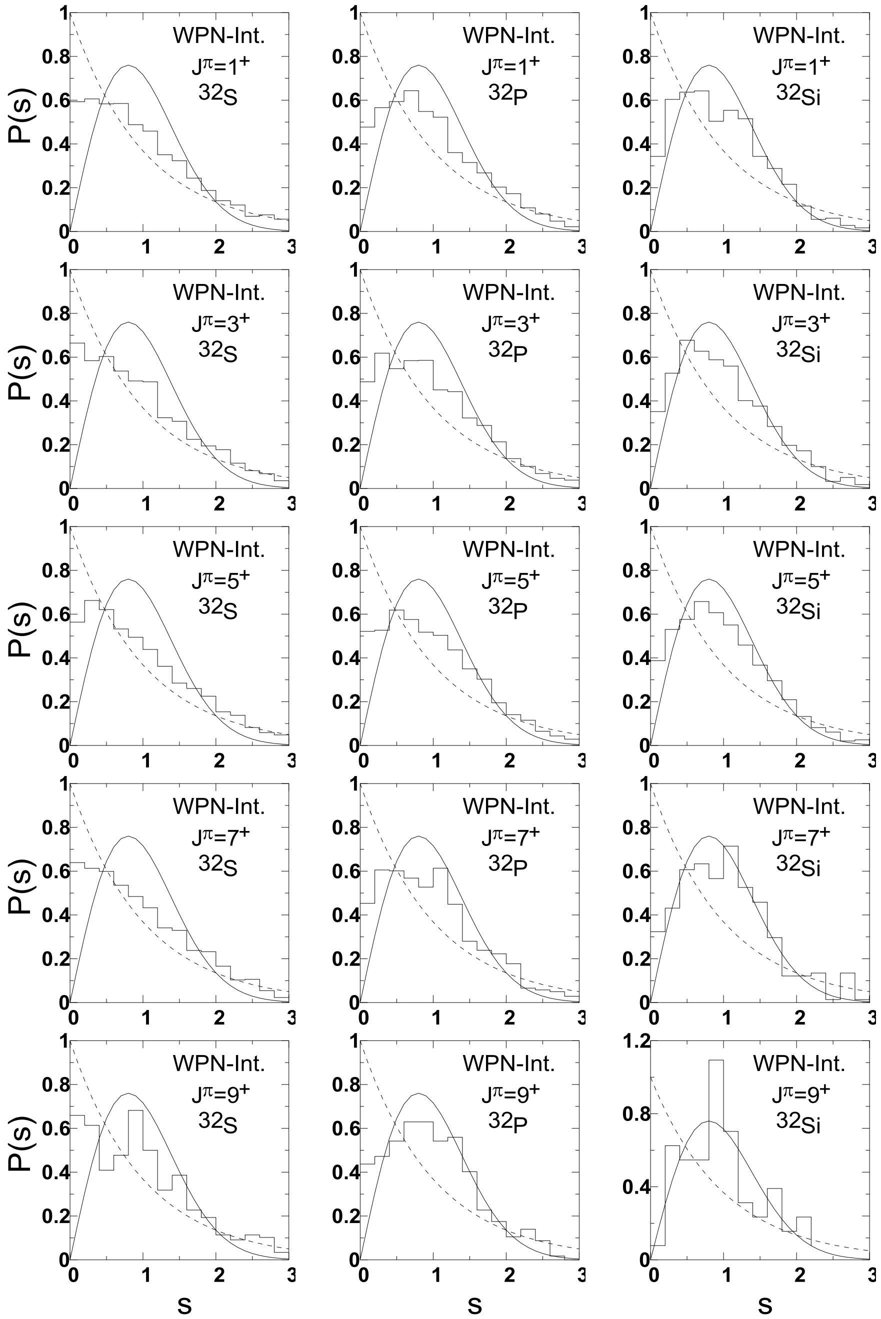


Figure 2: As in figure 1 but for various odd $J^{\pi}=1^{+}, 3^{+}, 5^{+}, 7^{+}$ and 9^{+} states.

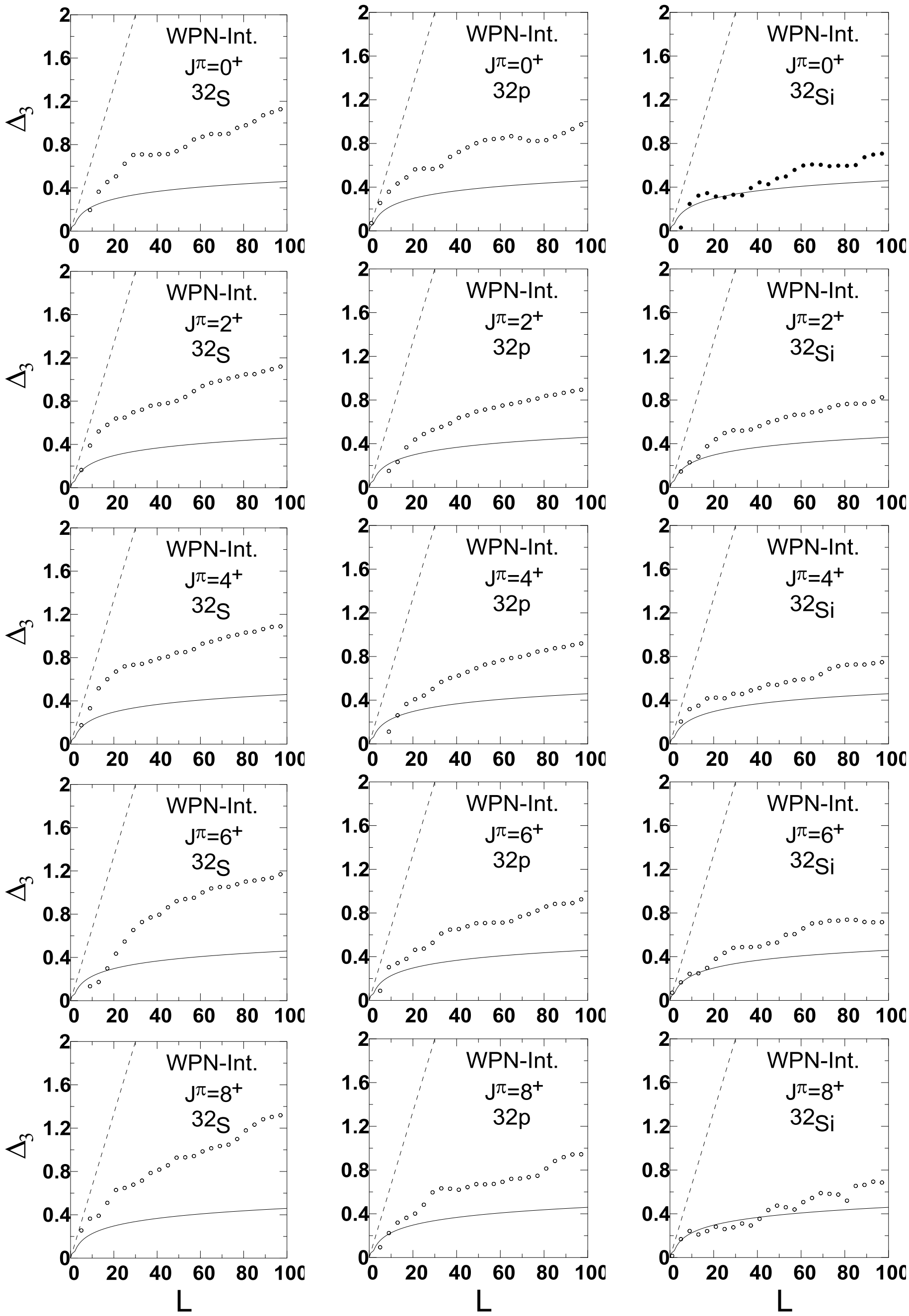
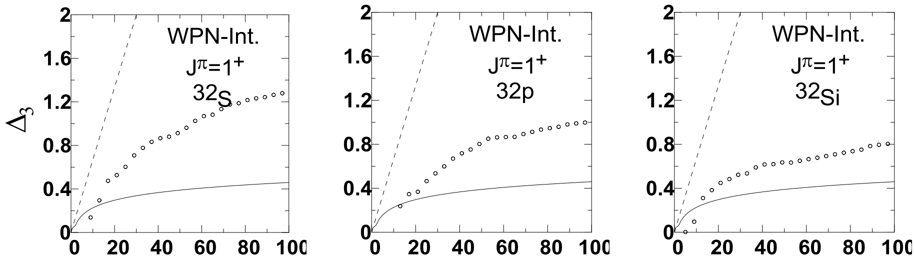


Figure 3: The average Δ_3 statistics in ^{32}A [$^{32}S\ (T=0)$, $^{32}P\ (T=1)$ and $^{32}Si\ (T=2)$] nuclei for various even $J^{\pi}=0^{+}, 2^{+}, 4^{+}, 6^{+}$ and 8^{+} states. The open circles are the calculated results obtained with full Hamiltonian. The solid and dashed lines are the GOE and Poisson distributions, respectively.



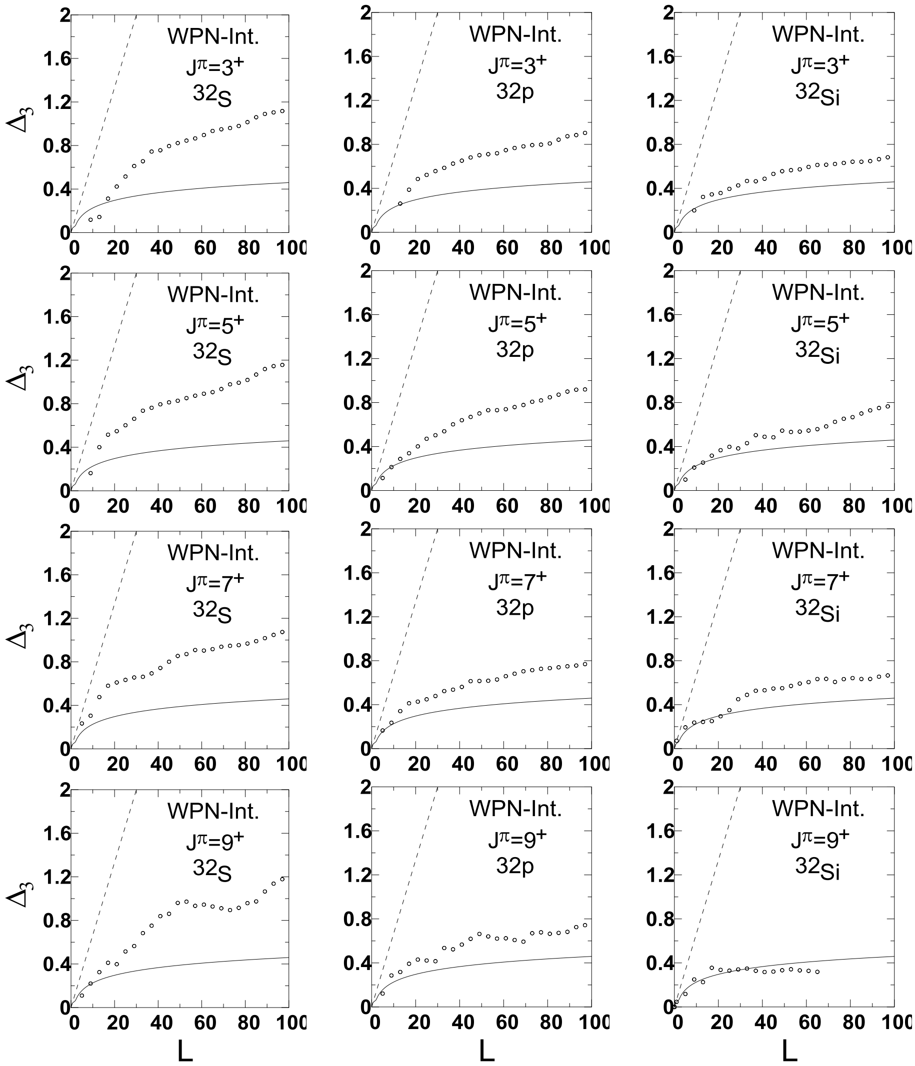
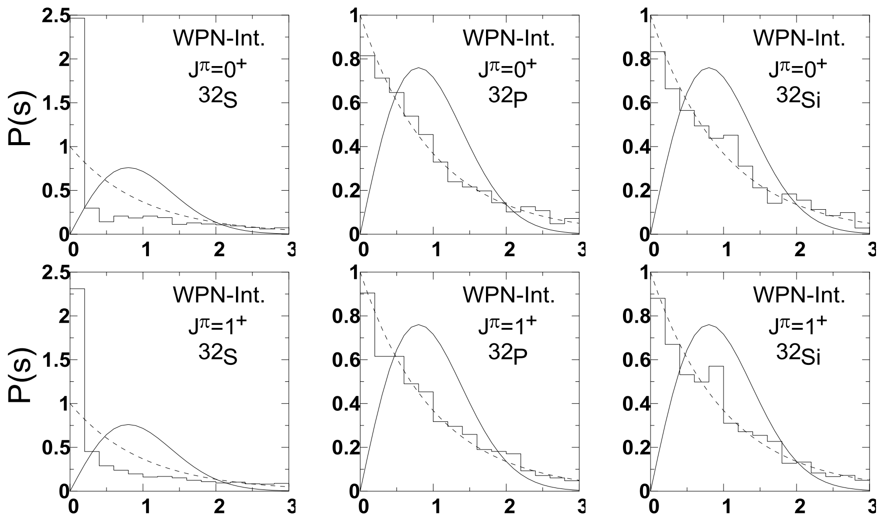


Figure 4: As in figure 3 but for various odd $J^{\pi} = 1^{+}, 3^{+}, 5^{+}, 7^{+}$ and 9^{+} states.



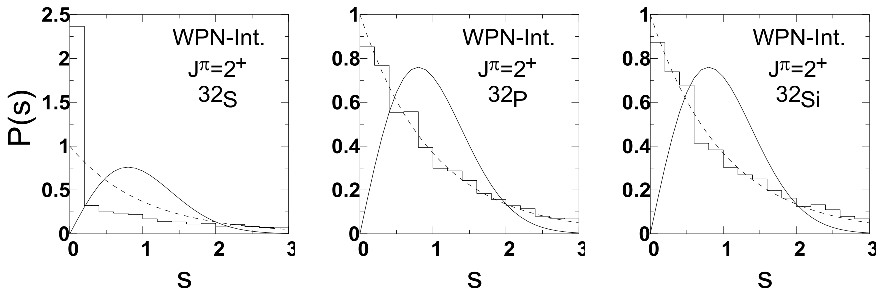


Figure 5: The nearest neighbor level spacing distributions $P(s)$ for $J^{\pi}=0^{+}, 1^{+}$ and 2^{+} states in ^{32}A nuclei. The calculated distributions (histograms) are obtained with the absence of the off-diagonal residual interaction. The solid and dashed lines are the GOE and Poisson distributions, respectively.

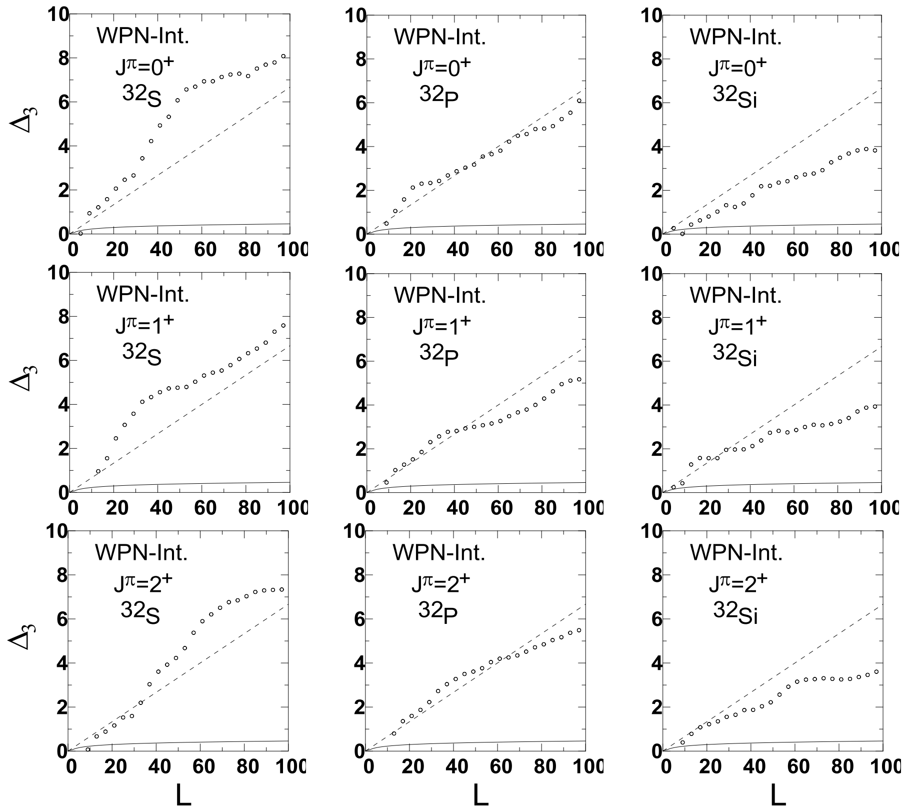


Figure 6: The average Δ_3 statistics for $J^{\pi}=0^{+}, 1^{+}$ and 2^{+} states in ^{32}A nuclei. The calculated distributions (open circles) are obtained with the absence of the off-diagonal residual interaction. The solid and dashed lines are the GOE and Poisson distributions, respectively.