

Optimum Location of Phasor Measurement Units in Smart Grid Environment Using Binary Integer Linear Programming

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Abstract: In this thesis novel optimal placement of phasor measurement unit (PMU) is approached for applications such as state estimation and fault detection. In this thesis, the PMU placement is with full network observability under different contingency conditions. The IEEE 14, 30, and 57 standard test systems will be used to exam the proposed approach adequately and the result will be compared to existing methods. In this thesis, it is demonstrated that the proposed methods are very effective in determining the minimum number of PMU and the results are comparable to the best methods presented in the past literature. In addition, System Observability Redundancy Index is calculated and according to which PMUs are placed to make only the important buses of power system observable even if a contingency occurs in the system.

Keywords: Binary Integer Linear Programming (BILP), Optimal Phasor Placement (OPP), Phasor Measurement Unit (PMU), Observability Analysis, Zero Injection Buses (ZIB), Global Positioning Systems (GPS), Wide Area monitoring Protection and Control (WAMPAC)

1. Introduction

Current Electric Grid was conceived more than a hundred years ago, when electricity needs were simple, power generation was localized and built around the communities. Most houses had small energy demands such as a few light bulbs or a radio. In previously designed Grid, the limited one way interaction makes it difficult to the ever-changing and rising energy demand of twenty-first century. The Smart Grid introduces two way dialogs, where electricity and system monitoring comes information were exchanged to the utility and their customers. That's where into play.

The idea of PMUs was introduced since the 1980's, but the need for their deployment was not recognized until the industry saw how these devices could help in the analysis and mitigation of the occurrence of major blackouts in the power systems. Synchro-phasor Measurement Units (PMUs) are devices that can measure, time stamp voltage, current, frequency, among others. PMUs take these synchronized measurements as fast as 60 times per second; compared with the traditional 2-4 second SCADA measurements, PMUs create a much clear and real-time picture of what is happening in the power system. PMUs have been increasingly deployed across transmission power grids worldwide.

Based on the GPS synchronized clock, the phasor measurement unit can measure vast amount of critical power network information, which includes bus voltage, bus current, generator speed and power angle. By receiving the real-time PMU measurement information over wide locations, the operators in the control room can monitor and analyze the quality of distribution network under statistic and dynamic operating conditions. Phadke A. G. [1] suggested that the installation of PMUs in all substations can significantly improve the power network reliability. Nevertheless, the investment of PMU device in all locations is unprofitable due to high cost of device. To reduce maintenance fee and unit costs, Optimal PMU Placement

(OPP) is implemented to minimize the amount of PMU placement and to achieve the entire degree of observability. As a result, the problem of optimal PMU placement in power network has been focused on in recent years. In general the OPP algorithm can be categorized into three groups, namely: Mathematical Programming Method, Heuristic Method and Meta-Heuristic Method.

2. Integer Linear Programming

A linear programming (LP) problem in which all the design variable must take integer values is called linear integer programming problem. Minimization of strategically located PMUs that eliminate measurement criticality in the entire system was addressed by Chen J. and Abur A. [3]. The placement problem was then extended to incorporate conventional measurements as candidates for placement. Furthermore, the same formulation could be used to determine optimal locations when a desired level of local redundancy is considered. This allows design of measurement systems with different degrees of vulnerability against loss of measurements and bad data. Xu B. et al. proposed the proper placement of PMUs for a given budget [4]. This issue was addressed via a special case of ILP, known as binary integer programming (BIP), considering the presence of injection and power flow measurements. Furthermore, loss of single PMUs was taken into account to minimize the vulnerability of state estimation to PMU failures [5]. Devesh Dua et al. addressed various aspects of PMU placement problem [6]. A procedure for multi-staging of PMU placement was proposed in a given time horizon using an integer linear programming (ILP) framework. The paper showed that zero injection constraints can also be modeled as linear constraints in an ILP framework. Minimum PMU placement problem has multiple solutions. It proposed two indices, via, BOI and SORI, to further rank these multiple solutions, where BOI is Bus Observability Index giving a measure of number of PMUs observing a given bus and SORI is System Observability Redundancy Index giving sum of all BOI for a system. Optimal PMU

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placement was done by using Bus Observability and Zero Injection constraints and placement quality was improved using BOI and SORI.

3. Project Description

The present thesis is concerned with the optimal placement of PMUs, so that these devices can provide the maximum benefit to the state estimation function. It is understood that these devices have multiple uses and therefore their placement may have to be based on considerations related to several other applications, however this study limits its scope to the specific application of state estimation. Hence, the objective of the placement problem is to ensure that the entire system remains a single observable island for the given measurement set. In this report a novel binary integer programming method is proposed to solve the PMU placement problem.

Problem Formulation:

The installation cost $F(X)$ is directly linked to a minimal number of PMUs to be placed. The objective function for allocation of PMUs is formulated as

The minimization function may take the following form:

$$\text{Min} \sum_{k=1}^n F_k x_k$$

Subjected to observability constraints

$$GX \geq B$$

G is the observability constraint vector function, whose entries are nonzero if the corresponding individual buses are observable with respect to a given measurement set and zero otherwise. Where

$X = [x_1, x_2, x_3, \dots, x_n]$, n is number of buses in system, x_k is a binary decision variable which is defined as

$$x_k = \begin{cases} 1 & \text{if PMU is installed at bus } k \\ 0 & \text{otherwise} \end{cases}$$

and G is bus incidence matrix formed from line connectivity data and is represented as

$$G = \begin{cases} 1 & \text{if } k = p \text{ or connected to each other} \\ 0 & \text{otherwise} \end{cases}$$

F is cost vector and is represented as diagonal unit matrix of order $n \times n$. B is vector of observability constraints defined as $[1 \ 1 \ 1 \ 1 \dots n]$

Constraint vector function ensures full network observability. A solution i.e. a set of minimum x_i is to be found which will satisfy the constraint. The constraint vector function is formed using binary connectivity matrix (G) of power system. The binary connectivity matrix (G) represents the bus connectivity information of a power system, which can be formed using line-data of the power system network.

Proposed Binary Integer Linear Programming Method:-

The idea of the new binary integer linear programming method is that the placement of PMUs is equivalent to a problem that minimizes the number of PMUs in order to allow each bus

In the system to be measured at least once by the set of PMUs. Thus the objective functions the minimum number of PMUs which are able to make the entire system observed. The constraints enable that each bus should be reached by the PMU at least once.

Consider the simple 8-bus system shown in figure 1

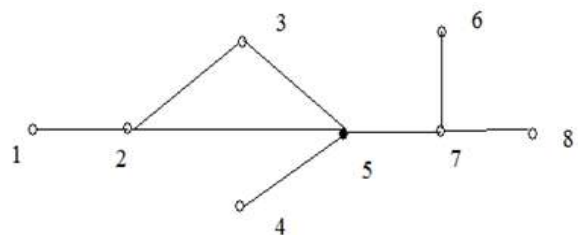


Figure 1: Single line diagram of 8-Bus system

Let x_i be a binary decision variable associated with the bus i . Variable x_i is set to one if a PMU is installed at bus i , else it is set to zero. Then minimum PMU placement problem for this 8-bus system can be formulated as follows

Objective $\text{Min} \sum X_i$

Subject to:

Bus-1: $X_1 + X_2 \geq 1$

Bus-2: $X_1 + X_2 + X_3 + X_5 \geq 1$

Bus-3: $X_2 + X_3 + X_5 \geq 1$

Bus-4: $X_4 + X_5 \geq 1$

Bus-5: $X_2 + X_3 + X_4 + X_5 + X_7 \geq 1$

Bus-6: $X_6 + X_7 \geq 1$

Bus-7: $X_5 + X_6 + X_7 + X_8 \geq 1$

Bus-8: $X_7 + X_8 \geq 1$

$x_i \in (0,1)$

In matrix form the above equations can be written as

$$\text{Min} \sum_{i=1}^8 x_i$$

Subjected to

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \end{bmatrix} \geq \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$x_i \in (0,1)$

Observing the 0-1 matrix multiplied by the placement variable vector x , it is almost the same as the bus adjacency

matrix except for the diagonal elements in this matrix. If bus i is associated with bus j , the elements (i, j) and (j, i) in this matrix equal to 1. The diagonal elements in this matrix are all equal to 1; otherwise, the other elements in this matrix are zero. The unit vector on the right side of the formula indicates that each bus should be directly observed or indirectly observed by PMUs at least once. If considering the loss of PMU in a sudden urgent case, all the elements in the vector on the right side of the formula could be set as 2. This means if a PMU is out of service in the system, a backup PMU is still available to provide the measurements instead of the PMU out of service.

For any N -bus power system, the generalized modeling of the Integer Linear

Programming in PMU placement is shown below:

$$\text{Min} \sum_{i=1}^N x_i$$

Subjected to $T_{PMU}X \geq b_{PMU}$

$$X = [x_1, x_2, x_3, \dots, x_n]$$

$$x_i \in (0,1)$$

where N is the total number of buses in the network. The matrix T_{pmu} is defined as

follow:

$$t_{i,j} = \begin{cases} 1 & \text{if } i = j \\ 1 & \text{if } i \text{ and } j \text{ are connected} \\ 0 & \text{otherwise} \end{cases}$$

Because each bus in the system should be reached by the set of PMUs at least once, the vector b_{pmu} is defined as:

$$b_{pmu} = \begin{bmatrix} 1 \\ 1 \\ \cdot \\ \cdot \\ 1 \end{bmatrix}_{N \times 1}$$

The MATLAB Integer Programming function *binprog* is used to solve this optimization problem. For this simple 8-bus system case, an optimal solution is $X = [0 \ 1 \ 0 \ 1 \ 0 \ 0 \ 1 \ 0]^T$, which means the PMUs should be installed at bus 2, 4 and 7.

The optimal number of PMUs is 3.

4. Modelling of Zero Injection Buses

Zero injection busses are the busses from which no current is being injected into the system. If zero injection busses are also modeled in the PMU placement problem, the total number of PMUs can be further reduced. To understand this issue, consider the four bus example shown in Fig. 3.3. Fig. 3.3(a) depicts the system with injections in all the busses. Fig. 3.3(b) shows a similar system with zero injection in bus 2 and injections in bus 1, 3 and 4. For system (a), it can easily be seen that a minimum of two PMUs are required to make the system completely observable. These can be placed on any two of the four busses. For example, if a PMU

is placed on bus 1, another PMU is required to make observable bus 4. In contrast consider system 2 (b). For a PMU at bus 1, the current in branch 2-4 becomes known as bus 2 is a zero injection bus. i.e. $I_{24} = I_{12}$. Hence knowing the line parameters, the voltage at bus 4 can be calculated 1 as:

$$V_4 = V_2 - I_{12}Z_{24}$$

Hence a separate PMU is not required at bus 4 for 2 (b). Therefore it is seen that presence of zero injections can help in reducing total number of PMUs required to observe the system.

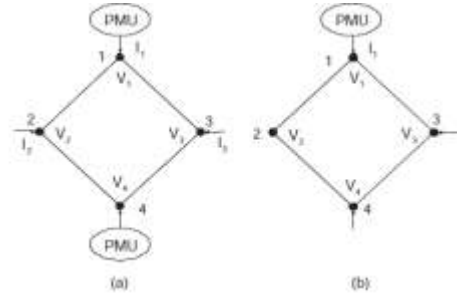


Figure 2: Single line diagram of Four Bus System

Modeling of zero injection busses in ILP framework has remained a challenge. We now propose a method to model these constraints within a linear framework. Consider a zero injection bus as shown in figure. If busses 1 to $(m - 1)$ are observable, i.e. their voltage phasors are known, then either current $I_{i,1}$ is available directly from a PMU or it can be calculated as follows:

$$I_{i,1} = y_{i,1}[V_i - V_1]$$

Where $y_{i,1}$ is the line admittance between bus 1 and bus i .

Consequently, bus m can also be made observable by calculating the bus voltage as follows:

$$V_m = V_1 - z_{1,m} \sum_{i=2}^{m-1} I_{i,1}$$

Where $z_{1,m}$ is the line impedance between busses 1 and m .

Every zero injection node leads to one additional constraint. Hence, in the best case, the minimum number of PMUs required to observe the system can be reduced by the total number of zero injection busses in the system.

Proposed method with zero injection modeling:-

When considering the conventional measurements, the optimal placement of PMUs can be formulated as a problem of a new binary *integer linear programming* as follows:

$$\text{Min} \sum_{i=1}^N x_i$$

Subjected to $T_{con}PT_{PMU}X \geq b_{con}$

$$X = [x_1, x_2, x_3, \dots, x_n]^T$$

$$x_i \in (0,1)$$

Where the matrix; $T_{con} = \begin{bmatrix} T_{M \times M} & 0 \\ 0 & T_{meas} \end{bmatrix}$ the matrix is a

permutation matrix and is the number of buses not associated to conventional measurements. The details of forming these matrices are given in the following examples.

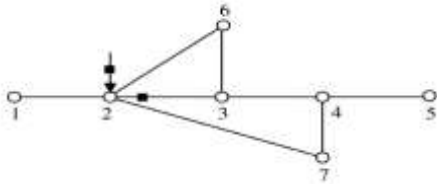


Figure 3: Single Line diagram Seven Bus System

In order to clearly explain the above formulation of integer linear programming for the case of full observability, suppose that a branch flow measurement be on line 2-3 and an injection measurement at bus 2. Bus 1, 2, 3, 6 and 7 are associated to these two conventional measurements. According to the definition given above, we have

$$T_{PMU} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Buses 4 and 5 are not associated to these two conventional measurements. The two inequality constraints corresponding to these two conventional measurements are

$$y_2 + y_3 \geq 1, \quad y_1 + y_6 + y_7 \geq 2$$

The measurement matrix, constraint matrix, and Permutation matrix are designed as follows

$$T_{meas} = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \end{bmatrix}$$

$$T_{con} = \begin{bmatrix} I_{M \times M} & 0 \\ 0 & T_{meas} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 \end{bmatrix}$$

$$P = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

The problem is formulated as follows

$$\text{Min} \sum_{i=1}^7 x_i$$

$$\text{Subjected to} \quad \begin{bmatrix} 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 2 & 2 & 1 & 0 & 2 & 1 \\ 1 & 3 & 1 & 1 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} \geq \begin{bmatrix} 1 \\ 1 \\ 1 \\ 2 \end{bmatrix}$$

$$X = [0 \ 1 \ 0 \ 0 \ 1 \ 0 \ 0]$$

The optimal solution of this integer linear programming is which means the PMU needs to be placed as buses 2 and 5.

This example shows that the conventional measurements do not affect the decision of optimal placement because of the system configuration and the locations of conventional measurements.

Proposed approach with single line outage or PMU loss:-

To enhance the reliability of system monitoring, if a bus is observed by at least two PMUs instead of one, the loss of one will still keep the system observable. This can be modeled by modifying the constraints given by equation as follows

$$\text{Min} \sum_{k=1}^n F_k x_k$$

Subjected to observability constraints

$$GX \geq 2b_{con}$$

Proposed approach with single line outage or PMU loss considering ZIB modeling:-

With considering Zero injection modeling in a new binary integer linear programming method the PMU to be placed can be decreased with the formulation as shown

$$\text{Min} \sum_{i=1}^N x_i$$

Subjected to $T_{con} P T_{PMU} X \geq 2b_{con}$

$$X = [x_1, x_2, x_3, \dots, x_n]^T$$

$$x_i \in (0,1)$$

5. Observability Analysis

If the minimum PMU placement problem defined by formulation has multiple number of optimal solutions, then the question of superiority of a particular solution or other optimal solution arises. In this section, we propose *Bus Observability Index* (BOI) as a performance indicator on quality of the optimization. Let us define BOI for bus- as the number of PMUs which are able to observe a given bus. Consequently, maximum bus observability index is limited to *maximum connectivity* of a bus plus one, i.e.

$$\beta_i \leq \eta_i + 1$$

Now we define SORI as the sum of bus observability for all the buses of a system. Then

$$\gamma = \sum_{i=1}^n \beta_i$$

Now where γ represents SORI. Consider a six-bus system shown in Fig. 4. It is seen that a minimum of two PMUs are required to ascertain system observability. Consider two such optimal solutions shown in Fig. 4.

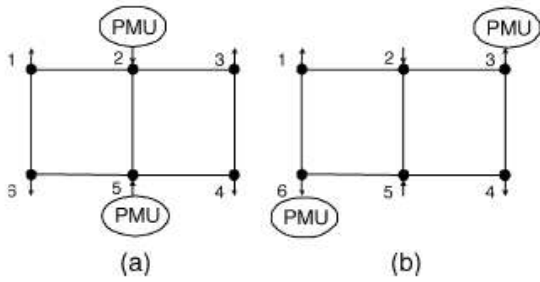


Figure 4: Single line diagram of four bus with PMU allocation

For the PMU placement as given in figure, BOI for busses 1 to 6 are 1, 2, 1, 1, 2, and 1, respectively. This makes SORI, Alternatively, for PMU placement in Fig. BOI for busses 1 to 6 are unity, making. Hence, the PMU placement with maximum SORI in figure (a) should be chosen for final placement. Maximizing SORI has the advantage that a larger portion of system will remain observable in case of a PMU outage. For example, in figure (a), one PMU outage will result in loss of observability of two busses, as against three busses remaining unobservable for loss of single PMU for system in figure (b). After the solution of the minimum PMU placement problem given by formulation OPP, index SORI can be maximized by solving a *slave* ILP problem where we maximize subject to constraints of optimization problem and additional linear equality constraint that number of PMUs in the solution should be restricted to number where is the minimum number of PMUs obtained for complete observability. This formulation, referred as maximum observability.

Formulation of Maximum observability:-

Index SORI which measures the redundancy in system observability can be expressed by a linear equation as follows:

$$\gamma = b^T AX$$

To solve the problem of maximizing SORI, while guaranteeing system observability, with minimum number of PMUs, we solve the following *slave* problem.

Formulation of maximum observability:

$$\max b^T AX$$

Subject to the following constraints

$$\sum_{i=1}^N x_i = \alpha_0$$

$$AX \geq b_{con}$$

Where α_0 is the minimum number of PMUs obtained for complete observability.

6. MATLAB Simulation and Results

Simulation results are organized in four parts in order to demonstrate the efficiency of the proposed method. First, for IEEE 14-bus, 30-bus, and 57-bus Test systems with and without considering zero injection buses is executed. Next, The OPP for single Line or PMU outage is executed. Finally, the observability is studied without and with zero injection modeling. All simulations have been carried out by

a 2.30 GHz Intel i3-2350M processor with 4 GB RAM. For simulations Binary Integer Programming of MATLAB/Simulations 2010 has been used. Single line diagram of 14 bus system is shown in figure 5 and system information is mentioned in Table 1

IEEE 14-Bus system

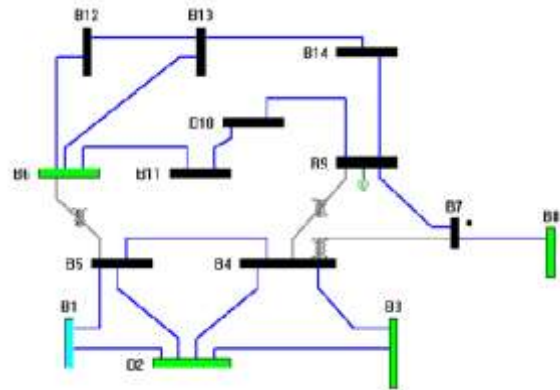


Figure 5: Single Line diagram of 14-Bus System

Table 1: 14 -Bus System Information

System	No. of Branches	Total no of Zero Injections	Zero Injection Bus number	Total number of radial buses	Radial Bus numbers
14-Bus System	20	1	7	1	8

IEEE 30 Bus system

IEEE 30-bus system is shown in figure 6. The information of the system and zero injection are given in the table 2.

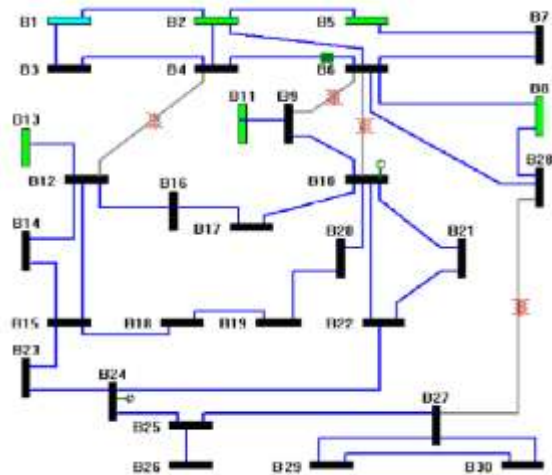


Figure 6: Single Line diagram of 30-Bus System

Table 2: 30-Bus system Information

System	No. of Branches	Total no of Zero Injections	Zero Injection Bus number	Total number of radial buses	Radial Bus numbers
30-Bus System	41	6	6,9,22,25,27,28	3	11,13,26

IEEE-57 Bus System:

IEEE 57-bus system is shown in figure 7. The information of the system and zero injection are given in the table 3

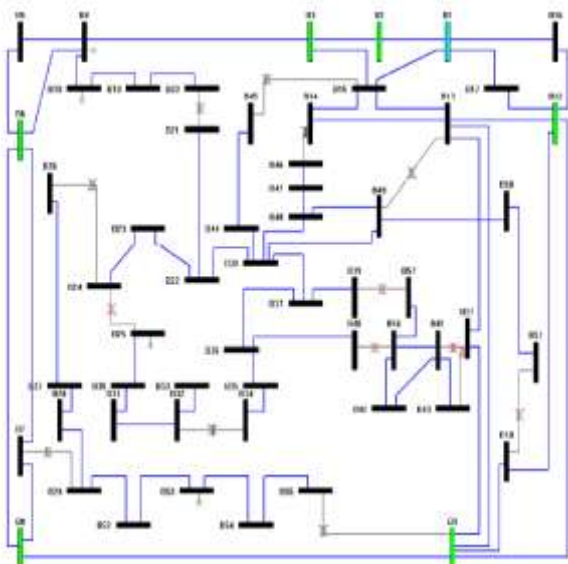


Figure 7: Single Line diagram of 57-Bus System

Table 3: 57-Bus system Information

System	No. of Branches	Total no Of Zero Injection	Zero Injection Bus	Total number of radial buses	Radial Bus numbers
30-Bus System	78	15	4,7,11,21,24,26,34,36,37,39,40,45,46,48	3	11,13,26

Table 4: Location of PMUs with ZIB modeling under no line outage

IEEE Test System	No of PMUs	Location of PMUs with ZIB modeling
14	3	2,6,9
30	7	1,7,10,12,18,23,27,
57	11	1,4,10,19,25,29,32,37,41,49,54

This study determines the optimal PMU locations that will maintain complete system observability under the loss of a single PMU loss or a line outage. Binary integer linear programming is used with considering zero injections. If a PMU at any bus mentioned in the table lost, the system remains observable by the rest of PMUs similarly as described in the previous section. The result is shown in the Table 4

Table 5: Location of PMUs with ZIB modeling under Single line outage

IEEE Test System	No of PMUs	Location of PMUs with single line outage considering ZIB modeling
14	7	2,3,5,6,9,11,13
30	16	2,3,4,7,10,12,13,15,17,19,20,21,24,25,27,29
57	29	1,2,4,6,9,12,15,19,20,22,24,25,28,29,30,32,33,35,36,38,41,45,48,49,50,51,53,54,56

System Observability Redundancy Index (SORI) is a performance indicator of the quality of optimization. Comparison of SORI with and without ZIB modeling for the single line contingency and no line contingency is shown in Table 5. The PMU placement with maximum SORI is chosen for final placement with a particular number of

allocations. The bus with maximum BOI is considered in optimization subjected to observability constraints of the system.

Table 6: SORI with and without ZIB modeling for no line contingency and single line contingency

IEEE Test systems	No Line Contingency		Single Line Contingency	
	SORI with Zero Injection Modeling	SORI without Zero Injection Modeling	SORI with Zero Injection Modeling	SORI without Zero Injection Modeling
14 bus	15	19	30	39
30 bus	31	43	64	74
57 bus	57	67	117	127

Comparison of Proposed BILP method with other Methods for Minimum PMU Allocations in the Bus System

Optimization Methods	14-Bus System	30-Bus System	57-Bus System
ILP[5]	3	-	14
Unified Approach BILP[22]	4	7	13
GILP[23]	4	10	17
Integer Quadratic[24]	4	10	17
Binary Search[25]	3	7	-
Proposed BILP	3	7	11

7. Conclusion

A new binary integer programming technique is proposed. Proposed technique has been applied to solve optimum PMU placement Problem. PMU placement problem has been associated with different conventional aspects of power system with the presence of ZIB, observability of important buses PMU loss and line contingency. Proposed method has been tested on IEEE standard systems and a practical system. Results are compared with the available methods in the literature and found to be satisfactory. The new binary integer programming method for Optimal PMU placement (OPP) can therefore be applied to any power system to make the system fully observable considering different operational aspects of the power system. PMU placement problem does not always have a unique solution. Depending upon weight preference criteria the optimal results can vary with same number of PMUs in different locations. On the other hand, it is not unusual to have additional considerations apart from strict observability criteria, when deciding on the location of PMUs. These considerations can be taken into account by approximately modifying the optimization problem which is formulated in this project.

Some considerations are already done in this paper. More of these considerations can be taken into account by appropriately modifying the optimization problem which is formulated in this project. This can be done as an extension to this project in the future. One of the important functions of state estimators is to detect and eliminate bad measurements in the system. Bad data processing is strongly dependent upon the measurement redundancy as well as accuracy of the measurements used. Even for fully observable systems, strategic placement of few PMUs can significantly improve bad data detection and identification capability. This aspect of PMU placement can also be

investigated in the future so that the operation of the existing state estimators can be improved via PMU placement.

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