New Implementation of Paired Triple Connected Domination Number of a Graph

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Abstract: ***A set $S \subseteq V$ is a paired triple connected dominating set if S is a triple connected dominating set of G and the induced subgraph<S> has a perfect matching. The paired triple connected domination number $\gamma_{ptc}(G)$ is the minimum cardinality taken over all paired triple connected dominating sets in G. The minimum number of colours required to colour all the vertices so that adjacent vertices do not receive the same colour and is denoted by $\chi(G)$. In [5], Mahadevan G et. al., characterized the classes of the graphs whose sum of paired triple connected domination number and chromatic number equals 2n – 1. In this paper we characterize the classes of all graphs whose sum of paired triple connected domination number and chromatic number equalsto 2n − 2, 2n – 3, 2n – 4 , for any n ≥ 5.***

Keywords: Paired triple connected domination number, Chromatic number

AMS (2010): 05C69

1. Introduction

Throughout this paper, by a graph we mean a finite, simple, connected and undirected graph G(V, E). For notations and terminology, we follow [2]. The number of vertices in G is denoted by n. Degree of a vertex v is denoted by deg(v). We denote a cycle on n vertices by C_n, a path of nvertices by P_n, complete graph on n vertices by K_n. The *friendship graph*, denoted by F_n can be constructed by identifying *n* copies of the cycle C_3 at a common vertex.A spider is a tree which has atmost one vertex of degree ≥ 3. A wounded spider $S^*(K_{1,n-1})$, is the graph formed by subdividing (exactly once) atmost n -1 of the edges of a star $K_{1,n-1}$. If S is a subset of V, then <S> denotes the vertex induced subgraph of G induced by S. A subset S of V is called a dominating set of G if every vertex in V-S is adjacent to at least one vertex in S. The domination numberγ(G) of G is the minimum cardinality of all such dominating sets in G. One can get a comprehensive survey of results on various types of domination number of a graph in [16]. The chromatic number χ(G) is defined as the minimum number of colors required to color all the vertices such that adjacent vertices do not receive the same color. Many authors have introduced different types of domination parameters by imposing conditions on the dominating set [14, 15]. Recently, the concept of triple connected graphs has been introduced by Paulraj Joseph J. et. al.,[13]by considering the existence of a path containing any three vertices of G. They have studied the properties of triple connected graphs and established many results on them. A graph *G* is said to be *triple connected* if any three vertices lie on a path in *G*. All paths, cycles, complete graphs and wheels are some standard examples of triple connected graphs. In[4] Mahadevan G. et. al., introduced triple connected domination number of a graph and found many results on them. A subset *S* of *V* of a nontrivial connected graph *G* is said to be *triple connecteddominating set*, if *S* is a dominating set and the induced sub graph <*S*> is triple connected. The minimum cardinality taken over all triple connected dominating sets is called the *triple connected domination number*of G and is denoted by $\gamma_{tc}(G)$. In [5] Mahadevan G. et. al., introduced paired triple connected domination number of a graph and found many results on them. A subset *S* of *V* of a nontrivial connected graph *G* is said to be *paired triple connected dominating set*, if *S* is a triple connected dominating set and the induced sub graph <*S*> has perfect matching. The minimum cardinality taken over all paired triple connected dominating sets is called the *paired triple connected domination number*of G and is denoted by $\gamma_{ptc}(G)$.

Several authors have studied the problem of obtaining an upper bound for the sum of adomination parameter and a graph theoretic parameter and characterized the corresponding extremal graphs. In [12], Paulraj Joseph J and Arumugam S proved that $\gamma + \kappa \le p$, where κ denotes the vertex connectivity of the graph. They also proved that $\gamma_c + \chi \le p + 1$ and characterized the corresponding extremal graphs. They also proved similar results for γ and γ_t. In [10], Mahadevan G Selvam A, IravithulBasira A characterized the extremal of graphs for which the sum of the complementary connected domination number and chromaticnumber. In [5], Mahadevan G proved that$\gamma_{ptc} + \chi \le 2n - 1$, and characterized the corresponding extremal graph. Motivated by the above results, inthis paper, we characterize all graphs for whichthe sum of paired triple connected domination number and chromatic number equals 2n−2, 2n – 3, 2n – 4 for any n ≥ 5.

2. Previous Results

Theorem2.1 [5] For any connected graph G with n ≥ 5, we have $4 \le \gamma_{ptc}(G) \le n - 1$.

Notation 2.2 Let *G* be a connected graph with *m* vertices v_1, v_2, ….,v_m. The graph obtained from *G* by attaching n_1 times a pendant vertex of P_{l_1} on the vertex v_1, n_2 times a pendant vertex of P_{l_2} on the vertex v_2 and so on,is denoted by

$G(n_1P_{l_1}, n_2P_{l_2}, n_3P_{l_3}, \ldots, n_mP_{l_m})$ where n_i, $l_i \geq 0$ and $1 \leq i \leq m$.

Example 2.3 Let v_1, v_2, v_3, v_4, be the vertices of C_4. The graph $C_4(P_2, 2P_2, 3P_2, P_3)$ is obtained from C_4 by attaching 1 time a pendant vertex of P_2 on v_1, 2 times a pendant vertex of P_2 on v_2, 3 times a pendant vertex of P_2 on v_3 and 1 time a pendant vertex of P_3 on v_4 and is shown in Figure 2.1.

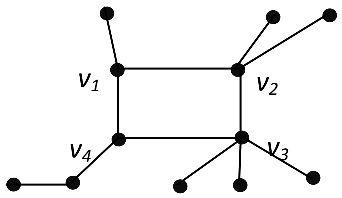


Figure 2.1 $C_4(P_2, 2P_2, 3P_2, P_3)$

Notation2.4$C_3(u(P_{m1}, P_{m2}))$ is a graph obtained from C_3 by attaching the pendent vertex of P_{m1}(Path on m_1 vertices) and the pendent vertex of P_{m2}(Paths on m_2 vertices) to any vertex u of C_3.

Example 2.5

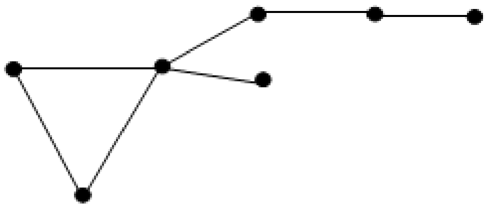


Figure 2.2: C3(u(P4, P2))

Notation 2.6 For $m \leq n$, $K_n(m)$ is the graph obtained from K_n by adding a new vertex and joint it with m vertices of K_n.

Example 2.7

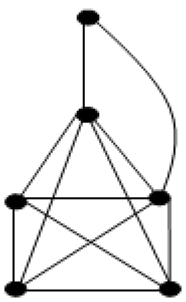


Figure 2.3: $K_5(2)$

3. Main Results

Theorem 3.1 For any connected graph G with $n \geq 5$ vertices, $\gamma_{ptc}(G) + \chi(G) = 2n - 2$ if and only if G is isomorphic to $K_4(P_2)$, K_6or any one of the graphs shown in Figure 3.1.

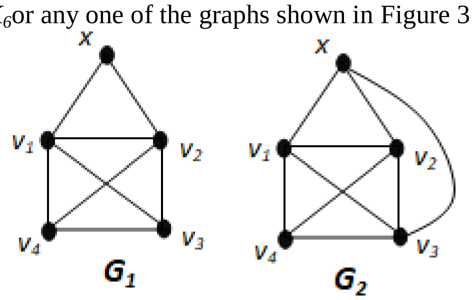


Figure 3.1

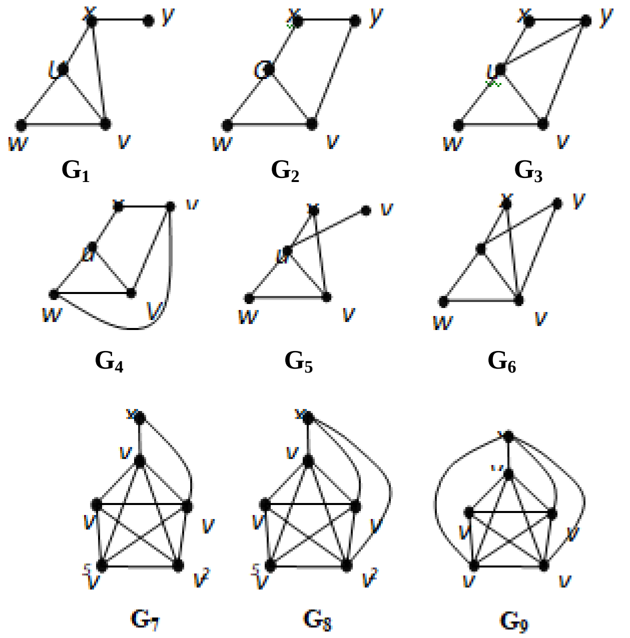
ProofLet G be a connected graph with $n \geq 5$ vertices. Suppose G is isomorphic to $K_4(P_2)$, K_6, or the graphs given in Figure 2.1, then clearly $\gamma_{ptc}(G) + \chi(G) = 2n - 2$.

Conversely, Let $\gamma_{ptc}(G) + \chi(G) = 2n - 2$. This is possible if $\gamma_{ptc}(G) = n - 1$ and$\chi(G) = n - 1$ or if $\gamma_{ptc}(G) = n - 2$ and$\chi(G) = n$.

Case(i)$\gamma_{ptc}(G) = n - 1$ and$\chi(G) = n - 1$.
Since $\chi(G) = n - 1$, G contains a clique K_{n-1}on $n - 1$ vertices. Let x be the vertex other than the $n - 1$ vertices in K_{n-1}. Since G is connected x is adjacent to a vertex $v_i$in K_{n-1}.
Now $S = \{x, v_i, v_j, v_k\}$ (for $i \neq j \neq k$) is a paired triple connected dominating setof G. Since $\gamma_{ctc}(G) = n - 1$, so that $p = 5$. Hence $K_{n-1} = K_4 = \langle v_1, v_2, v_3, v_4 \rangle$.
Let x be adjacent to v_1 in K_4. If $d(x) = 1$, then $G \cong K_4(P_2)$. If x is adjacent to v_1, v_2 in K_4.If $d(x) = 2$, then $G \cong G_1$. If x is adjacent to v_1, v_2 and v_3 in K_4.If $d(x) = 3$, then $G \cong G_2$. In all the other cases, no graph exists.

Case (ii)$\gamma_{ctc}(G) = n - 2$ and$\chi(G) = n$.
But $\chi(G) = n$, we have G is isomorphic to K_n. For K_n, $\gamma_{ptc}(K_n) = 4$ so that $n = 6$. Hence $G \cong K_6$.

Theorem 3.2 For any connected graph G with $n \geq 5$ vertices, $\gamma_{ptc}(G) + \chi(G) = 2n - 3$ if and only if G is isomorphic to W_5, F_2, K_7, $K_3(P_3)$, $K_3(2P_2)$, $K_5(P_2)$, $K_3(P_2, P_2, 0)$ or any one of the graphs shown in Figure 3.2.



Proof: Let G be a connected graph with $n \geq 5$ vertices. Suppose G is isomorphic to W_5, F_2, K_7, $K_3(P_3)$, $K_3(2P_2)$, $K_5(P_2)$, $K_3(P_2, P_2, 0)$ or any one of the graphs given in Figure 2.2, then clearly $\gamma_{ptc}(G) + \chi(G) = 2n - 3$.

Conversely, Let $\gamma_{ptc}(G) + \chi(G) = 2n - 3$. This is possible if $\gamma_{ptc}(G) = n - 1$ and$\chi(G) = n - 2$ or if $\gamma_{ptc}(G) = n - 2$ and$\chi(G) = n - 1$ or if $\gamma_{ptc}(G) = n - 3$ and$\chi(G) = n$.

Case (i)$\gamma_{ptc}(G) = n - 1$ and$\chi(G) = n - 2$.

Since $\chi(G) = n - 2$, G contains a clique K_{n-2} on $n - 2$ vertices. Let $T = \{x, y\}$ be the vertices other than the $n - 2$ vertices in K_{n-2}. Then the induced subgraph of T may be K_2 or $\overline{K}_2$.

Subcase (i) $<T> = K_2$.
Since G is connected, there exists a vertex x in T is adjacent to a vertex v_i in K_{n-2}. Now $S = \{x, y, v_i, v_j\}$ (for $i \neq j$) forms a paired triple connected dominating set of G. Since $\gamma_{ptc}(G) = n - 1$, so that $n = 5$. Hence $K_{n-2} = K_3 = uvwu$.

Let x be adjacent to u in K_3. If $d(x) = 2$, $d(y) = 1$, then $G \cong K_3(P_3)$. Let x be adjacent to u and v in K_3. If $d(x) = 3$, $d(y) = 1$, then $G \cong G_1$. Let x be adjacent to u and let y be adjacent to u in K_3. If $d(x) = 2$, $d(y) = 2$, then $G \cong F_2$. Let x be adjacent to u in K_3 and y be adjacent to w in K_3. If $d(x) = d(y) = 2$, then $G \cong G_2$. Let x be adjacent to u in K_3 and y be adjacent to u and v in K_3. If $d(x) = 2$, $d(y) = 3$, then $G \cong G_3$. Let x be adjacent to u in K_3 and y be adjacent to v and w in K_3. If $d(x) = 2$, $d(y) = 3$, then $G \cong G_4$. Let x be adjacent to u and v in K_3 and y be adjacent to u and w in K_3. If $d(x) = d(y) = 3$, then $G \cong W_5$. In all the other cases, no new graph exists.

Subcase (ii) $<T> = \overline{K}_2$.
Since G is connected, x and y in T are adjacent to a vertex v_i in K_{n-2}. Then $S = \{x, v_i, v_j, v_k\}$ (for $i \neq j \neq k$) forms a paired triple connected dominating set of G. Since $\gamma_{ptc}(G) = n - 1$, so that $n = 5$. Hence $K_{n-2} = K_3 = uvwu$.

Let x and y be adjacent to u in K_3. If $d(x) = d(y) = 2$, then $G \cong K_3(2P_2)$. Let x be adjacent to u and v in K_3 and y be adjacent to u in K_3. If $d(x) = 2$, $d(y) = 1$, then $G \cong G_5$. Let x be adjacent to u and v in K_3 and y be adjacent to u and v in K_3. If $d(x) = 2$, $d(y) = 1$, then $G \cong G_6$. In all the other cases, no graph exists.

Since G is connected, x in T is adjacent to v_i in K_{n-2} and y in T is adjacent to v_j (for $i \neq j$) in K_{n-2}. Then $S = \{x, v_i, v_j, y\}$ (for $i \neq j$) forms a paired triple connected dominating set of G. Since $\gamma_{ptc}(G) = n - 1$, so that $n = 5$. Hence $K_{n-2} = K_3 = uvwu$.

Let x be adjacent to u in K_3 and y be adjacent to v in K_3. If $d(x) = d(y) = 1$, then $G \cong K_3(P_2, P_2, 0)$. In all the other cases, no new graph exists.

Case (ii) $\gamma_{ptc}(G) = n - 2$ and $\chi(G) = n - 1$.
Since $\chi(G) = n - 1$, G contains a clique K_{n-1} on $n - 1$ vertices. Let x be the vertex other than the $n - 1$ vertices in K_{n-1}.

Since G is connected x is adjacent to a vertex v_i in K_{n-1}. Then $S = \{x, v_i, v_j, v_k\}$ (for $i \neq j \neq k$) forms a paired triple connected dominating set of G. Since $\gamma_{ptc}(G) = n - 2$, so that $n = 6$. Hence $K_{n-1} = K_5 = <v_1, v_2, v_3, v_4, v_5>$.

Let x be adjacent to v_1 in K_5. If $d(x) = 1$, then $G \cong K_5(P_2)$. Let x be adjacent to v_1 and v_2 in K_5. If $d(x) = 2$, then $G \cong G_7$. Let x be adjacent to v_1, v_2 and v_3 in K_5. If $d(x) = 3$, then $G \cong G_8$. Let x be adjacent to v_1, v_2, v_3 and v_4 in K_5. If $d(x) = 4$, then $G \cong G_9$. In all the other cases, no graph exists.

Case (iii) $\gamma_{ptc}(G) = n - 3$ and $\chi(G) = n$.

Since $\chi(G) = n$, we have G is isomorphic to K_n. But For K_n, $\gamma_{ptc}(K_n) = 4$ so that $n = 7$. Hence $G \cong K_7$.

Theorem 3.3 For any connected graph G with $n \geq 5$ vertices $\gamma_{ptc}(G) + \chi(G) = 2n - 4$ if and only if $G \cong S^*(K_{1,4})$, P_5, $C_4(P_2)$, $K_4(P_3)$, $K_4(2P_2)$, $K_4(P_2, P_2, 0, 0)$, $K_6(P_2)$, $K_6(2)$, $K_6(3)$, $K_6(4)$, $K_6(5)$, K_8 or any one of the following graphs in Figure 3.3.

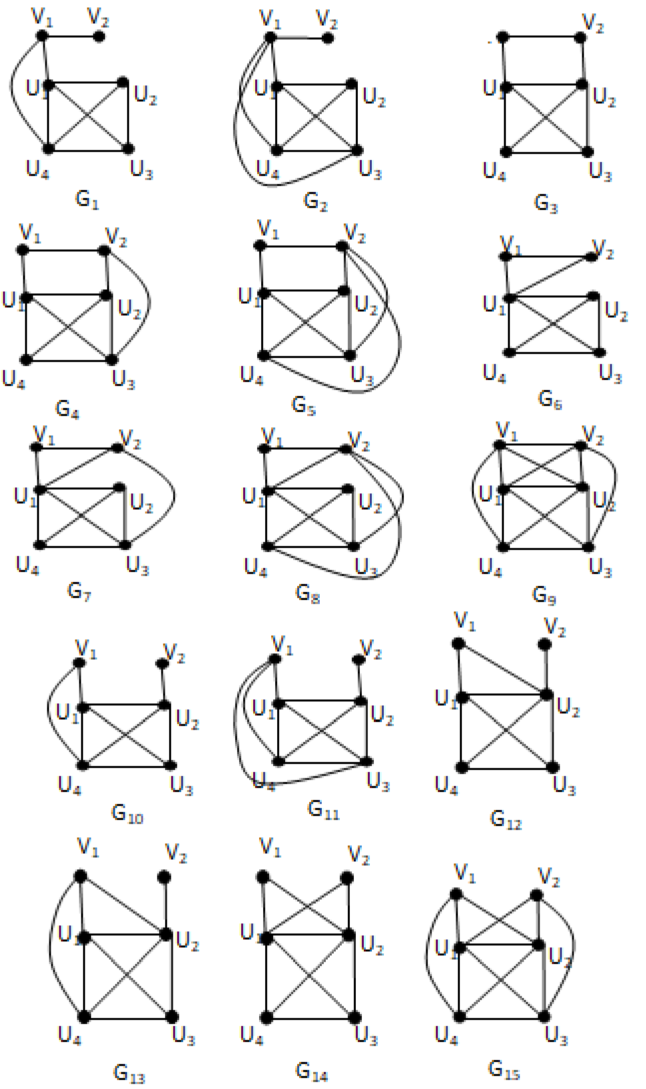


Figure 3.3

Proof: Let G be a connected graph with $n \geq 5$ vertices. Suppose G is isomorphic to P_5, $S^*(K_{1,4})$, $C_4(P_2)$, $K_4(P_3)$, $K_4(2P_2)$, $K_4(P_2, P_2, 0, 0)$, $K_6(P_2)$, $K_6(2)$, $K_6(3)$, $K_6(4)$, $K_6(5)$, K_8 or any one of the graphs given in Figure 2.3, then clearly $\gamma_{ptc}(G) + \chi(G) = 2n - 4$.

Conversely, let $\gamma_{ptc}(G) + \chi(G) = 2n - 4$. This is possible if $\gamma_{ptc}(G) = n-1$ and $\chi(G) = n-3$ or if $\gamma_{ptc}(G) = n-2$ and $\chi(G) = n-2$ or if $\gamma_{ptc}(G) = n-3$ and $\chi(G) = n-1$ or if $\gamma_{ptc}(G) = n-4$ and $\chi(G) = n$.
Case: (i) $\gamma_{ptc}(G) = n-1$ and $\chi(G) = n-3$

Since $\chi(G)$ = n-3, G contains a clique K on n-3 vertices or does not contain a clique K on n-3 vertices.

Let G contains a clique K on n - 3 vertices.

Let S = V(G) - V(K) = $\{v_1, v_2, v_3\}$. Then the inducedsubgraph<S> has the following possible cases. <S> = K_3 ,$\overline{K}_3$, P_3, $K_2 \cup K_1$.

Subcase i.<S> = K_3.
Let v_1,v_2,$v_3$be the vertices of K_3. Since G is connected, there exists a vertex u_i in K_{n-3} which is adjacent to any one of $\{v_1, v_2, v_3\}$. Let u_i be adjacent to v_2, then $\{u_i, v_2, v_3, v_1\}$ is a γ_{ptc} set of G, so that γ_{ptc}= 4 and n = 5, which is a contradiction. Hence no graph exists.

Subcase ii.<S> = $\overline{K}_3$.
Let v_1,v_2,$v_3$be the vertices of $\overline{K}_3$. Since G is connected, there exits a vertex u_i be adjacent to v_1, v_2 ,v_3 and u_j for $(i \neq j)$ and u_k for $i \neq j \neq k$ .In this case$\{v_1, u_i, u_j, u_k\}$ is a γ_{ptc} set of G, so that γ_{ptc}= 4 and n = 5 . Hence K=K_2 , which is a contradiction. Hence no graph exists.

If u_i is adjacent to v_1 and u_j for $i \neq j$ is adjacent to v_2 and v_3, and u_k for $i \neq j \neq k$, then $\{v_1, u_i, u_j, u_k\}$ is an γ_{ptc} set of G, so that γ_{ptc}= 4 and p = 5. Hence K = K_2 = u_1u_2. If u_1 is adjacent to v_1 and u_2 is adjacent to v_2 and v_3. If $deg(v_1)=1=deg(v_2)=deg(v_3)$, then G ≅ $S^*(K_{1,4})$.

Since G is connected, there exists a vertex $u_i$in K_{n-3} which is adjacent to v_1 and u_j for $i \neq j$ in K_{n-3} is adjacent to v_2 and u_k for $i \neq j \neq k$ in K_{n-3}, which is adjacent to v_3. In this case $\{u_i, u_j, u_k, v\}$ for some v in K_{p-3} is a γ_{ptc} set of G, so that γ_{ptc} = 4 and n = 5, which is a contradiction. Hence no graph exists.

Subcase iii.<P_3> = $v_1v_2v_3$.
Let v_1,v_2,v_3 be the vertices of P_3 .Since G is connected, there exists a vertex u_i in K_{n-3} which is adjacent to v_1 (or equivalently v_3) or v_2. If u_i is adjacent to v_2 and $u_j$for $i \neq j$ then $\{v_1, v_2, u_i, u_j\}$ is a γ_{ptc} set of G, so that γ_{ptc}= 4 and n = 5. Hence K = K_2 = u_1u_2.

If u_1 is adjacent to v_2 .If $deg(v_1)=1=deg(v_3)$,$deg(v_2)=3$, then G ≅ $S^*(K_{1,4})$. If $u_1$is adjacent to v_2 and u_2 is adjacent to v_1. If deg (v_1) = 2, $deg(v_2)$ = 3, $deg(v_3)$ = 1, then G ≅ $C_4(P_2)$.

Since G is connected, there exists a vertex u_i in K_{n-3} which is adjacent to v_1, then $\{u_i, v_1 v_2, v_3\}$ for some i, is a γ_{ptc} set of G, so that γ_{ptc}= 4 and n = 5 and hence K = K_2 = < u_1, u_2>.

Let u_1 be adjacent to v_1, then G ≅ P_5. Let u_1 be adjacent to v_1 and v_3. If $deg(v_1)$ = 2= $deg(v_2)$, $deg(v_3)$ = 2, then G ≅ $C_4(P_2)$. Let u_1 be adjacent to v_1 and u_2 be adjacent to v_2. If $deg(v_1)$ = 2, $deg(v_2)$ = 3, $deg(v_3)$ = 1, then G ≅ $C_4(P_2)$.

Subcase iv.<S> = $K_2 \cup K_1$.
Let v_1, v_2 be the vertices of K_2 and v_3 be the isolated vertex. Since G is connected, there exists a vertex u_i in K_{p-3} which is adjacent to v_1 and u_j for $i \neq j$ is adjacent to v_3. In this case $\{v_2, v_1, u_i, u_j\}$ is a γ_{ptc} set of G, so that γ_{ptc}= 4 and p = 5 and hence K = K_2 = <u_1,u_2>.

Let u_1 be adjacent to v_1 and u_2 be adjacent to v_3. If $deg(v_1)$ = 2, $deg(v_2)$ = 1 = $deg(v_3)$, then G ≅ P_5. Let u_1 be adjacent to v_1 and u_2 be adjacent to v_3 and v_2. If $deg(v_1)$ = 2= $deg(v_2)$, $deg(v_3)$=1 , then G ≅ $C_4(P_2)$.

Since G is connected, there exists a vertex u_i in K_{n-3} which is adjacent to v_1 and v_3. In this case $\{v_2, v_1, u_i, v_3\}$ is a γ_{ptc} set of G, so that γ_{ptc}= 4 and n = 5 and hence K = K_2 = <u_1,u_2>.

Let u_1 be adjacent to v_1 and v_3. If $deg(v_1)$ = 2, $deg(v_2)$ = 1 = $deg(v_3)$, then G ≅ $S^*(K_{1,4})$. Let u_1 be adjacent to v_1 and v_3 and u_2 be adjacent v_2. If $deg(v_1)$ = 2= $deg(v_2)$, $deg(v_3)$=1, then G ≅ $C_4(P_2)$.

Case (ii).γ_{ptc}= n - 2 and χ = n – 2.

Since χ = n - 2, G contains a clique K on n - 2 vertices or does not contain a clique K on n - 2 vertices.

Let G contains a clique K on n - 2 vertices.

Let S = V(G) - V(K) = $\{v_1, v_2\}$. Then <S> = K_2, $\overline{K}_2$.

Subcase i.<S> = K_2.
Let v_1, v_2 be the vertices of K_2. Since G is connected, there exists a vertex u_i in K_{p-2} is adjacent to v_1 and uj for $i \neq j$ then $\{v_2, v_1, u_i, u_j\}$ is a γ_{ptc} set of G, so that γ_{ptc} = 4 and n = 6 and hence K = K_4 = <u_1, u_2, u_3, u_4>.

Let u_1 be adjacent to v_1. If $deg(v_1)$ = 2, $deg(v_2)$ = 1, then G ≅ $K_4(P_3)$. Let u_1 be adjacent tov$_1$ and u_4 be adjacent to v_1. If $deg(v_1)$ = 3, $deg(v_2)$ = 1, then G ≅ G_1.

Let u_1 be adjacent to v_1 and u_4 be adjacent to v_1 and u_3 be adjacent to v_1. If $deg(v_1)$ = 4, $deg(v_2)$ = 1 then G ≅ G_2. Let u_1 be adjacent to v_1 and u_2 be adjacent to v_1. If $deg(v_1)$ = 3, $deg(v_2)$=1, then G ≅ G_1. Let $u_1$be adjacent to v_1 and u_2 be adjacent to v_1 and u_4 be adjacent to v_1. If $deg(v_1)$ = 4, $deg(v_2)$ = 1, then G ≅ G_2. Let u_1 be adjacent to v_1 and u_4 be adjacent to v_1 and u_3 be adjacent to v_1 . If $deg(v_1)$ = 4, $deg(v_2)$ = 1, then G ≅ G_2. Let u_1 be adjacent to v_1 and u_2 be adjacent to v_2. If $deg(v_1)$ = 2= $deg(v_2)$, then G ≅ G_3. Let u_1 be adjacent to v_1 and u_2 be adjacent to v_2 and u_3 be adjacent to v_2. If $deg(v_1)$=2, $deg(v_2)$=3, then G ≅ G_4. Let u_1 be adjacent to v_1 and u_2 be adjacent to v_2 and u_3 be adjacent to v_2 and u_4 be adjacent to v_2. If $deg(v_1)$=2, $deg(v_2)$=4, then G ≅ G_5. Let u_1 be adjacent to v_1 and v_2. If $deg(v_1)=2=deg(v_2)$ then G ≅ G_6. Let u_1 be adjacent to v_1 and v_2 and u_3 be adjacent to v_2 if $deg(v_1)$=2, $deg(v_2)$=3, then G ≅ G_7. Let u_1 be adjacent to v_1 and v_2 and u_3 be adjacent to v_2 and u_4 be adjacent to v_2. If $deg(v_1)$=2, $deg(v_2)$=4, then G ≅ G_8. Let u_1 be adjacent to v_1 and u_2 be adjacent to v_2 and u_3 be adjacent to v_2 and u_4 be adjacent to v_1 . If $deg(v_1)$=4, $deg(v_2)$=4 then G ≅ G_9.

Subcase ii. Let <S> = $\overline{K}_2$.
Let v_1, v_2 be the vertices of $\overline{K}_2$. Since G is connected, v_1 and v_2 are adjacent to a common vertex say u_i of K_{n-2} (or) v_1 is adjacent to u_i for some i and v_2 is adjacent to u_j for some $i \neq j$ in K_{n-2}. In both cases $\{v_1, u_i, u_j u_k\}$ is a γ_{ptc} set of G, so that γ_{ptc} = 4 and n = 6 and hence K = K_4 = <u_1, u_2, u_3, u_4>.

Let u_1 be adjacent to v_1 and u_2 be adjacent to v_2. If $\deg(v_1) = 1 = \deg(v_2)$, then $G \cong K_4(P_2,P_2,0,0)$. Let u_1 be adjacent to v_1 and u_2 be adjacent to v_2 and u_4 be adjacent to v_1. If $\deg(v_1) = 2$, $\deg(v_2) = 1$, then $G \cong G_{10}$. Let u_1 be adjacent to v_1 and u_2 be adjacent to v_2 and u_3 be adjacent to v_1 and u_4 be adjacent to v_1. If $\deg(v_1) = 3$, $\deg(v_2) = 1$, then $G \cong G_{11}$. Let u_1 be adjacent to v_1 and u_2 be adjacent to v_1 and v_2. If $\deg(v_1) = 2$, $\deg(v_2) = 1$, then $G \cong G_{12}$. Let u_1 be adjacent to v_1 and u2 be adjacent to v_2 and u_4 be adjacent to v_1. If $\deg(v_1) = 3$, $\deg(v_2) = 1$, then $G \cong G_{13}$. Let u_1 be adjacent to v_1 and v_2 and u_2 be adjacent to v_2. If $\deg(v_1) = 1$, $\deg(v_2) = 2$, then $G \cong G_{12}$. Let u_1 be adjacent to v_1 and u_2 be adjacent to v_2 and u_3 be adjacent to v_2. If $\deg(v_1) = 1$, $\deg(v_2) = 3$, then $G \cong G_{13}$. Let u_1 be adjacent to v_1 and v_2 and u_2 be adjacent to v_1 and v_2. If $\deg(v_1) = 2 = \deg(v_2)$, then $G \cong G_{14}$. Let u_1 be adjacent to v_1 and u_2 be adjacent to v_2 and u_3 be adjacent to v_2 and u_4 be adjacent to v_1. If $\deg(v_1) = 3$, $\deg(v_2) = 3$, then $G \cong G_{15}$. Let u_1 be adjacent to v_1 and v_2. If $\deg(v_1) = 1 = \deg(v_2)$, then $G \cong K_4(2P_2)$. Let u_1 be adjacent to v_1 and v_2 and u_4 be adjacent to v_1. If $\deg(v_1) = 2$, $\deg(v_2) = 1$, then $G \cong G_{12}$. Let u_1 be adjacent to v_1 and v_2 and u_3 be adjacent to v_1 and u_4 be adjacent to v_1. If $\deg(v_1) = 3$, $\deg(v_2) = 1$, then $G \cong G_{13}$. Let u_1 be adjacent to v_1 and v_2 and u_3 be adjacent to v_2. If $\deg(v_1) = 1$, $\deg(v_2) = 2$, then $G \cong G_{12}$. Let u_1 be adjacent to v_1 and v_2 and u_3 be adjacent to v_2 and u_4 be adjacent to v_2. If $\deg(v_1) = 1$, $\deg(v_2) = 3$, then $G \cong G_{13}$. Let u_1 be adjacent to v_1 and v_2 and u_2 be adjacent to v_2. If $\deg(v_1) = 1$, $\deg(v_2) = 2$, then $G \cong G_{12}$. Let u_1 be adjacent to v_1 and v_2 and u_2 be adjacent to v_2 and u_3 be adjacent to v_2. If $\deg(v_1) = 1$, $\deg(v_2) = 3$, then $G \cong G_{13}$.

Case iii. $\gamma_{ptc} = n - 3$ and $\chi = n - 1$.

Since $\chi = n - 1$, G contains a clique K on n - 1 vertices.

Let v_1 be the vertex not on K_{n-1}. Since G is connected, there exists a vertex v_1 is adjacent toone vertex u_i of K_{n-1} and u_j for $i \neq j$ and u_k for $i \neq j \neq k$. In this case $\{v_1, u_i, u_j, u_k\}$ is a γ_{ptc} set of G, so that $\gamma_{ptc} = 4$ and $n = 7$ and hence $K = K_6 = \langle u_1,u_2,u_3,u_4,u_5,u_6 \rangle$.

Let u_1 be adjacent to v_1. If $\deg(v_1) = 1$, then $G \cong K_6(P_2)$. Let u_1 be adjacent to v_1 and u_2 be adjacent to v_1. If $\deg(v_1) = 2$, then $G \cong K_6(2)$. Let u_1 be adjacent to v_1 and u_2 be adjacent to v_1 and u_3 be adjacent to v_1. If $\deg(v_1) = 3$, then $G \cong K_6(3)$. Let u_1 be adjacent to v_1 and u_2 be adjacent to v_1 and u_3 be adjacent to v_1 and u_4 be adjacent to v_1. If $\deg(v_1) = 4$, then $G \cong K_6(4)$. Let u_1 be adjacent to v_1 and u_2 be adjacent to v_1 and u_3 be adjacent to v_1 and u_4 be adjacent to v_1 and u_5 be adjacent to v_1. If $\deg(v_1) = 5$, then $G \cong K_6(5)$.

Case iv. $\gamma_{ptc} = n - 4$ and $\chi = n$.
Since $\chi = n$, we have G is ismorphic to K_n, But for K_n, $\gamma_{ptc}(K_n) = 4$, so that $n = 8$. Hence $G \cong K_8$

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4. Conclusion and Future Scope

The authors obtained a large classes of graphs whose sum of paired triple connected number and chromatic number equals to $2n - 5$ for $n \geq 5$, which will be reported later.

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