A Ranking Method for Students of Different Socio Economic Backgrounds Based on Generalized Fuzzy Soft Sets

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Abstract: ***Student's ranking provides a clear picture to student, parents, teachers and other concerned persons. It creates a healthy competition among the students. It is a debatable topic throughout the world. A new technique for students ranking is introduced in this paper based on generalized fuzzy soft matrix theory. A problem of the students of different socio economic backgrounds has been solved in this work.***

Keywords: Student's Ranking, Fuzzy Set, Soft Set, Generalized Fuzzy Soft Set, Socio-Economic Background.

1. Introduction

Zadeh [1] introduced fuzzy sets to deal with problems of uncertainty. Molodtsov [2] pointed out the problem of inadequacy of the fuzzy set theory and gave soft set theory to overcome this problem. Maji [3] developed fuzzy soft set theory by combining soft sets and fuzzy sets. Naim Cagman introduced soft matrices and fuzzy soft matrices [4], [5].

Generalized fuzzy soft set theory was introduced by Pinaki Majumdar [6]. He applied it to a decision making problem and medical diagnosis problem. Hai Long YANG [7] pointed out some mistakes and corrected them. Pinaki Majumdar applied generalized fuzzy soft set to a student ranking problem [8]. B.K. Saikia et al. [9] defined generalized fuzzy soft matrix and studied some properties of it. They gave a decision making method based on generalized fuzzy soft matrices.

Students ranking is a measure of how an individual student's performance compares to other students in his or her class. It is very common to assign grades to measure the amount that how a student has been learned. Different letter grades are given to the students in different subjects. Rank of the students are calculated by taking grade point averages. Another method of assigning ranks is based on the total marks obtained by a student in different subjects. Many authors [10]-[14] have studied problems of educational measurement, specially student rankings and grading. Most of these methods are developed by using different statistical techniques. Pinaki Majumdar pointed out that both the methods mentioned above are not totally accurate [6]. He proposed that the standard grading and ranking systems should scrutinized. He developed a new ranking method which was based on generalized fuzzy soft set theory. The students may lose faith in their academic abilities by receiving bad ranks. It may cause give up for the rest of their studies. The present work is an extension of the work initiated by Pinaki Majumdar. We have developed a ranking method for students of different socio economic backgrounds. The rest of the paper is designed as follows: section 2 contains some basic definitions. In section 3 a selection method based on generalized fuzzy soft matrices is presented and an algorithm of the method is given in section 4. Section 5 gives a case study and section 6concludes the paper.

2. Some Basic Definitions

In this section, we recall some basic definitions of fuzzy soft set theory which would be helpful for our discussion.

2.1. Soft Set [2]

Let U be a universal set, C be set of parameters and $A \subseteq C$. A pair (F, A) is a soft set over U,where $F: A \rightarrow U$. In fact a soft set is a parameterized family of subsets of subsets over the universe U. Every set F(c), $c \in C$ represents the set of elements of the soft set (F, A).

2.2. Fuzzy Soft Set [3]

Let U be a universal set, C be set of parameters and I^U be the set of all fuzzy subsets of U. Let $A \subseteq C$. A pair (F, A) is a fuzzy soft set over U, where $F: A \rightarrow I^U$.

2.3. Generalized Fuzzy Soft Set [6]

Let U be a universal set, C be the set of parameters, F be a mapping of C to I^U, where I^U is the collection of all fuzzy subsets of U and λ be a fuzzy subset of C, i.e.
$\lambda : C \rightarrow I = [0, 1]$. Let $F_\lambda : C \rightarrow I^U \times I$ be a function, such that
$F_\lambda(c) = (F(c), \lambda(c))$,
$F(c) \in I^U$. Then F_λ is said to a generalized fuzzy soft set (GFSS in short) over (U,C).

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2.4. Generalized Fuzzy Soft Subset [6]

For two generalized fuzzy soft sets F_λ and G_μ over (U, C), F_λ is called generalized fuzzy soft subset of G_μ, denoted by $F_\lambda \subseteq G_\mu$, if $\mu \subseteq \lambda$ and F(c) ⊆ G(c), ∀c ∈ C.

2.5. Generalized Fuzzy Soft Matrix [9]

Let U be the universal set, C be the set of parameters and A ⊆ C. Suppose that (F_λ, C) be a GFSS over (U, C). A uniquely defined subset of U × C, R_A = {(u, c), c ∈ C, u ∈ F_λ (c)} is a relation form of (F_λ, C). The membership function μ_{R_A} and the function λ_{R_A} are written as μ_{R_A} : U × C → [0, 1] and λ_{R_A}: U × C → [0,1],
where R_A : (u, c) ∈ [0, 1], ∀u ∈U, c ∈ C
and λ_{R_A}: (u, c) ∈ [0, 1],
If $[\mu_{ij}, \lambda_j]_{m\times n}$ = ($\mu_{R_A}(u_i,c_j)$, λ (u_i,c_j)) then we can define a matrix as

$$[\mu_{ij}, \lambda_j]_{m\times n}=\begin{bmatrix} (\mu_{11},\lambda_1) & (\mu_{12},\lambda_2) & . & . & . & (\mu_{1n},\lambda_n) \\ (\mu_{21},\lambda_1) & (\mu_{22},\lambda_{12}) & . & . & . & (\mu_{2n},\lambda_{1n}) \\ . & . & & . & . & . \\ . & . & . & . & & . \\ . & . & & . & . & . \\ (\mu_{m1},\lambda_1) & (\mu_{m2},\lambda_2) & . & . & . & (\mu_{mn},\lambda_{1n}) \end{bmatrix}$$

which is an m × n generalized fuzzy soft matrix of GFSS over (U, E).

2.6. Generalized Resultant Matrix

Generalized resultant matrix is denoted by r_{ij} and is defined as
$[r_{ij}]_{m\times n} = [a_{ij}.\ \lambda_j]_{m\times n}$, ∀ i, j.

2.7. Generalized Choice Value Matrix

Generalized choice value matrix is denoted by C_j and is defined as
$C_i = \sum_{i=1}^{m} \sum_{j=1}^{n} r_{ij}$, ∀ i, j.

3. Mathematical Modeling of the Problem

Suppose that the authority of an institution is looking to give student of the year award to the performing students. They short listed *n* number of students after initial screening. They are tested in *m* subjects to check their performance. Maximum marks in each subject were 100. Their marks are presented in a tabular form.
Let U = {$a_1, a_2,\ldots, a_m$} be our universal set and
S = {$s_1, s_2, \ldots, s_n$} be the set of students, where $a_1, a_2,\ldots, a_m$ are the subjects in which students are to be tested.
Let λ : S→I = [0, 1] be a fuzzy subset of S, defined as

$$\lambda = \begin{array}{c} \begin{matrix} s_1 & s_2 & . & . & . & s_n \end{matrix} \\ [\begin{matrix} \lambda_1 & \lambda_2 & . & . & . & \lambda_n \end{matrix}] \end{array}$$

Here λ determines the grade of difficulties associated to each student due to different socio-economic backgrounds. To convert the marks obtained in different subjects into grades, we define *m* fuzzy sets
a_i: [0, 100] → [0, 1], i = 1, 2, …, m.
as follows :
$a_i\,(x) = (x / 100)^i$, $0 \le x \le 100$ and i = 1, 2, …, m.
We express the results in the form of generalized fuzzy soft matrix.

$$[a_{ij}, \lambda_j]_{m\times n}=\begin{bmatrix} (a_{11},\lambda_1) & (a_{12},\lambda_2) & . & . & . & (a_{1n},\lambda_n) \\ (a_{21},\lambda_1) & (a_{22},\lambda_{12}) & . & . & . & (a_{2n},\lambda_{1n}) \\ . & . & & & & . \\ . & . & . & . & & . \\ . & . & & . & & . \\ (a_{m1},\lambda_1) & (a_{m2},\lambda_2) & . & . & . & (a_{mn},\lambda_{1n}) \end{bmatrix}$$

Finally we calculate generalized resultant matrix and generalized choice value matrix respectively as follows:
$[r_{ij}]_{m\times n} = [a_{ij}.\ \lambda_j]_{m\times n}$, ∀ i, j
and
$C_i = \sum_{i=1}^{m} \sum_{j=1}^{n} r_{ij}$, ∀ i, j.

4. Algorithm

Step 1: Input the marks obtained by the students.
Step 2: Input the fuzzy sets E, M, B, H and λ.
Step 3 : Convert marks into grades.
Step 4: Express the grades in generalized fuzzy soft matrices.
Step 5: Calculate generalized resultant matrix.
Step 6: Calculate choice value matrix.
Step 7: Find maximum value.

5. Case Study

Suppose that the authority of an institution is looking to give student of the year award to the performing students. Five students are selected after initial screening. They are tested in four subjects to check their performance. Maximum marks in each subject were 100. Their marks are shown in the following table.

Table 1: Marks obtained by students

Subject /Student	S_1	S_2	S_3	S_4	S_5
English	70	78	74	66	68
Mathematics	68	73	82	74	60
Biology	73	64	65	78	78
History	80	79	72	70	80
Total Marks	291	294	293	288	278

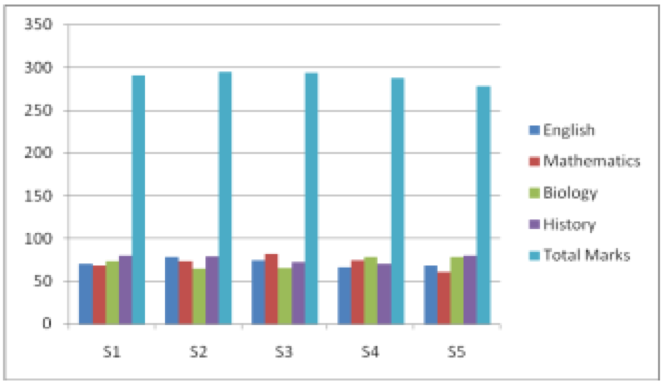

Figure 1: Marks obtained by the students

Let U = {e, m, b, h} be our universal set and

S = {s_1, s_2, s_3, s_4, s_5} be the set of students, where e, m, b and h denotes English, mathematics, biology and history respectively.

Let λ : S→I = [0, 1] be a fuzzy subset of S, defined as

$$\lambda = \begin{array}{ccccc} s_1 & s_2 & s_3 & s_4 & s_5 \\ \end{array}$$
$$\lambda = [0.5 \quad 0.6 \quad 0.8 \quad 0.7 \quad 0.9]$$

Here λ determines the grade of difficulties associated to each student due to different socio-economic backgrounds. To convert the marks obtained in different subjects into grades, we define four fuzzy sets

E : [0, 100] →[0, 1]

M : [0, 100] →[0, 1]

B : [0, 100] →[0, 1]

and H: [0, 100] →[0, 1]

as follows :

$E(x) = x / 100,\ 0 \le x \le 100$

$M(x) = (x / 100)^2,\ 0 \le x \le 100$

$B(x) = (x / 100)^3,\ 0 \le x \le 100$

and $H(x) = (x / 100)^4,\ 0 \le x \le 100$

Figures of the above functions are shown in the following figures.

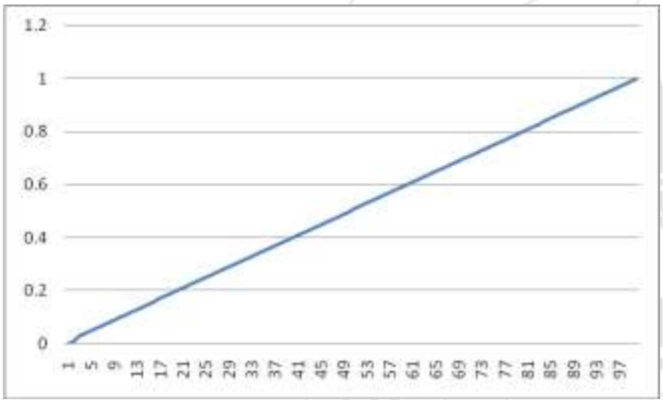

Figure 2: Membership function of English

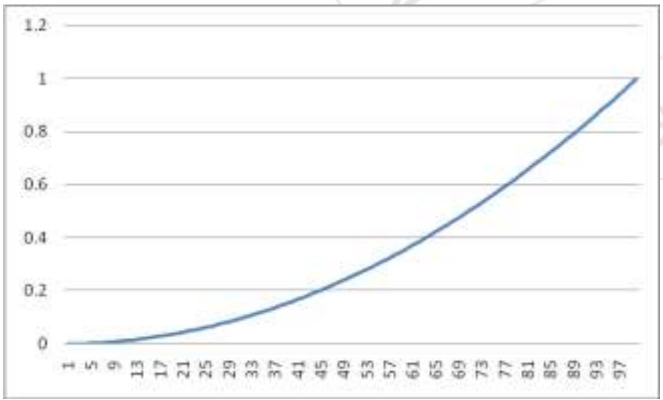

Figure 3: Membership function of Mathematics

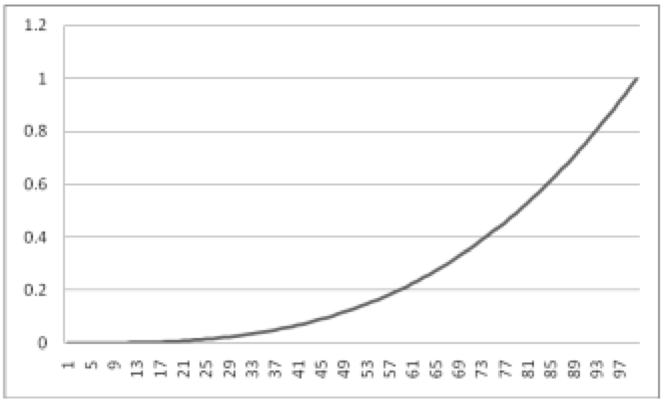

Figure 4: Membership function ofBiology

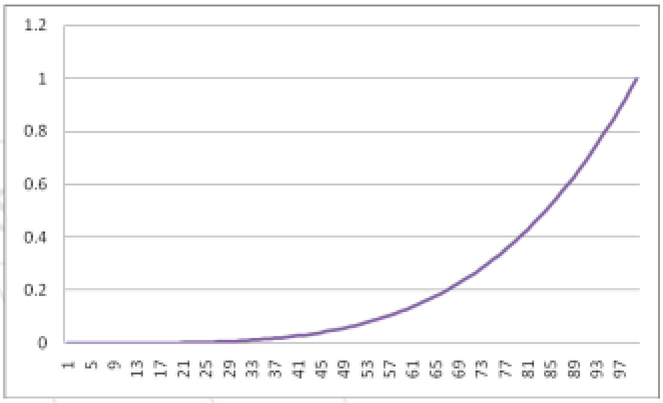

Figure 5: Membership function of History

We convert the marks obtained by the students into grades which are given below.

Table 2: Grades of the students

Subject /Student	S_1	S_2	S_3	S_4	S_5
English	0.70	0.78	0.74	0.66	0.68
Mathematics	0.46	0.53	0.23	0.55	0.36
Biology	0.39	0.26	0.27	0.47	0.47
History	0.41	0.39	0.27	0.24	0.41
λ	0.5	0.6	0.8	0.7	0.9

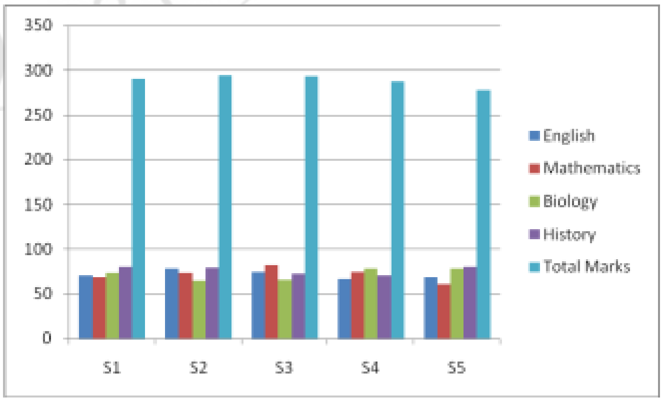

Figure 6: Grades of the students

We express the results in matrix form as

$$[a_{ij}, \lambda_j] = \begin{bmatrix} (0.7,0.5) & (0.78,0.6) & (0.74,0.8) & (0.66,0.7) & (0.68,0.9) \\ (0.46,0.5) & (0.53,0.6) & (0.23,0.8) & (0.55,0.7) & (0.36,0.9) \\ (0.39,0.5) & (0.26,0.6) & (0.27,0.8) & (0.47,0.7) & (0.47,0.9) \\ (0.41,0.5) & (0.39,0.6) & (0.27,0.8) & (0.24,0.7) & (0.47,0.9) \end{bmatrix}$$

We calculate generalized resultant matrix and generalized choice value matrix respectively as follows:

$$[r_{ij}] = \begin{bmatrix} 0.35 & 0.23 & 0.19 & 0.20 \\ 0.47 & 0.32 & 0.16 & 0.23 \\ 0.59 & 0.18 & 0.22 & 0.21 \\ 0.46 & 0.38 & 0.33 & 0.17 \\ 0.61 & 0.32 & 0.43 & 0.37 \end{bmatrix}$$

$$\text{And } C_i = \begin{bmatrix} 0.98 \\ 1.17 \\ 1.21 \\ 1.35 \\ 1.73 \end{bmatrix}$$

Clearly the maximum value is 1.73 of the s_5, so the student s_5 has to be given Student of the Year Award. Although he does not have maximum marks in academic tests but due to difficulties associated to his socio economic background he would be the best choice for the award.

6. Conclusion

This new ranking is not based on actual marks obtained by a student. It converts the marks obtained by the students in each subject into grades. Therefore the credit can be assigned according to the difficulties associated to each student due to different socio-economic backgrounds and not uniformly. In the present work we defined some new types of generalized fuzzy soft matrices and gave a new ranking method for the students of different socio economic backgrounds using the concept of generalized fuzzy soft matrices.

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