

# Integer Linear Programming Applied to Nurses Rostering Problem

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**Abstract:** *The paper extended the work of (Satheeshkumar et al., 2014) by employing a different approach to solve a nurse scheduling problem in order to ascertain whether the new approach produce better result than the previous method. The problem is formulated into an integer linear programming model. A computer software known as QM for windows version 2.0 is used to solve the integer linear program on a I7 Intel core processor personal computer. The optimal result found has minimize the number of workforce to 300 as against 303.33 that was produce when linear programming model is applied and a balance schedule across the wards in the hospital. The significance of this findings cannot be over emphasizes since, the difference of 3.33 in terms of people is meaninglessness. The paper applies a novel approach to find solution to a practical problem of paramount importance to the development of health care system.*

**Keywords:** Nurse scheduling, Integer linear programming, optimization algorithm

## 1. Introduction

The nurse-scheduling problem (NSP), also known as nurse rostering problem (NRP), is an optimization problem for determining an optimal way to assign nurses to shifts duties. Nurse scheduling problem has been studied since before 1969<sup>[1], [2]</sup>. The problem requires assignment of shifts and holidays to nurses. Each nurse has his or her own desires and restrictions, as does the hospital. The problem is depicted as determining a good timetable that both respects the constraints of the nurses and meets the goals and aspirations of the hospital. Operations research is an application of mathematical, statistical and computing techniques to solve management decision-making problems. It comprises some sets of optimization, analytical and simulation techniques and models, such as linear programming, integer programming, combinatorial optimization artificial intelligence, computer science etc.

conclusion is drawn in section 5. Policy implication is presented in section 6

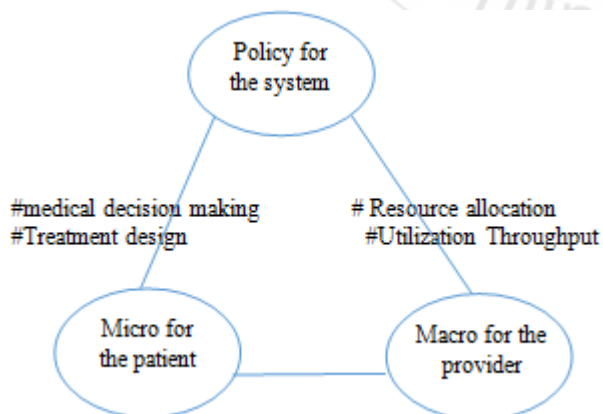
## 2. Integer linear Programming

Integer linear programming techniques is considered as mathematics based decision-making tool. Such technique requires two fundamental types of functions, objectives and constraints, that is developed to generate closed form solution. In a typical Operations research problem, the objective function, often expressed as Z, is formulated to determine the maximum profit while minimizing cost with given set of rules or constraints such as business policies, resource availability, preventive maintenance schedule, transportation distance or time and capacity.

## 3. Literature Review

(Kumar et al., 2014a) demonstrated how linear program was used to schedule nurses to duty taking into account the number of available hours, the number of days off and some other nurses' preference. Most hospital runs shift for eight hours in a day, with three shifts within twenty-four hours. The problems were model and solved using Microsoft excel solver. Results show a tremendous improvement over the manual method.

(Satheeshkumar et al., 2014) employed linear programming approach to assign nurse to shift duty. The problem was model as a linear program and solved on a computer. Results show that out of 1,250 nurses 304 were optimally assigned to duty. The paper illustrates how linear program has been effectively use in nurse scheduling at a multi-specialty hospital in coimbatore India. The work demonstrates a balanced schedule in terms of the distribution of shift duty, fairness and consecutive night duty. Nurse preferences were considered. This served as an improvement over the traditional manual approach, which is costly in terms of labor as well as inefficient in producing a good schedule.



**Figure 1:** Operations Research Applications in Healthcare- Three Categories

The paper is organized as follows: The algorithm is described in section 2, literature is reviewed in section 3. Section four, shows how the problem is formulated. Finally,

(Lin et al., 2015) employed integer programming to optimally schedule staff nurse on rotating shift over a schedule period. The plan schedule would meet the work force requirement of each shift and the number of days off of each staff member.

(Kumar et al., 2014b) demonstrated the application of linear programming (LP) on a practical nurse scheduling problem with the aim to improve the quality of the schedule. The significance of this study is to optimize the fairness of the current schedule. It was solve using a computer software, results show an optimal balance schedule using eight hours shift in a modern hospital.

#### 4. Problem Definition

Consider a hospital that is open seven days a week. Based on past experience, the number of nurse needed on a particular day is given as follows.

**Table 1:** Number of nurses needed per day

|                 | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | $x_7$ |
|-----------------|-------|-------|-------|-------|-------|-------|-------|
| Days            | Mon   | Tue   | Wed   | Thur  | Fri   | Sat   | Sun   |
| Number of nurse | 200   | 150   | 250   | 90    | 160   | 300   | 100   |

Every nurse works five consecutive days, and then takes two days off, repeating this pattern indefinitely. How can we minimize the number of nurses that staff the hospital?

$$\text{Minimize } Z = \sum_{i=1}^{m=7} x_i \quad (1)$$

subject to:

$$\sum_{i=1}^{m=7} x_i \geq b_i \quad (2)$$

$x_{ij} \geq 0 \quad \forall_{ij}$  all are integer and arbitrary constants

##### 4.1 Model

Primarily, the paper adopts (Satheeshkumar et al., 2014) assumption and procedure by letting:

$x_1$  = number of nurse starting duty on Sunday (Sunday–Thursday)

$x_2$  = number of nurse starting duty on Monday (Monday–Friday)

$x_3$  = number of nurse starting duty on Tuesday (Tuesday–Saturday)

$x_4$  = number of nurse starting duty on Wednesday (Wednesday–Sunday)

$x_5$  = number of nurse starting duty on Thursday (Thursday–Monday)

$x_6$  = number of nurse starting duty on Friday (Friday–Tuesday)

$x_7$  = number of nurse starting duty on Saturday (Saturday–Wednesday)

Demand for the number of nurses in the hospital on Sunday: 100, Monday: 200, Tuesday: 150, Wednesday: 250, Thursday: 90, Friday: 160 and Saturday: 300 respectively. Let  $x_i$  be the number of workers who begin their five-day shift on day  $i$ . The objective function is to:

$$\text{Minimize } Z = \sum_{i=1}^{m=7} x_i \quad (3)$$

Consider the constraint for Sunday staffing level of 100. Nurse that start its shift on Sunday will work for consecutive 5 days, that is Sunday to Thursday. Those, who start their shift on Monday, they will work for consecutively 5 days, that is, Monday to Friday. Similarly, those that start their work shift on Tuesday will work from Tuesday to Saturday and so on. Then the following table is produce as follows:

**Table 2:** Shift pattern

|                | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | $x_7$ |
|----------------|-------|-------|-------|-------|-------|-------|-------|
| Sunday shift   | *     | *     | *     | *     | *     | -     | -     |
| Monday shift   | -     | *     | *     | *     | *     | *     | -     |
| Tuesday shift  | -     | -     | *     | *     | *     | *     | *     |
| Wednesday      | *     | -     | -     | *     | *     | *     | *     |
| Thursday shift | *     | *     | -     | -     | *     | *     | *     |
| Friday shift   | *     | *     | *     | -     | -     | *     | *     |
| Saturday shift | *     | *     | *     | *     | -     | -     | *     |

Similar arguments give the model formulation.

$$\text{Minimize: } Z = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7$$

$$\text{subject to: } x_1 + x_4 + x_5 + x_6 + x_7 \geq 100$$

$$x_1 + x_2 + x_5 + x_6 + x_7 \geq 200$$

$$x_1 + x_2 + x_3 + x_6 + x_7 \geq 150$$

$$x_1 + x_2 + x_3 + x_4 + x_7 \geq 250$$

$$x_2 + x_3 + x_4 + x_5 + x_6 \geq 160$$

$$x_1 + x_2 + x_3 + x_4 + x_5 \geq 90$$

$$x_3 + x_4 + x_5 + x_6 + x_7 \geq 300$$

$$x_{ij} \geq 0 \quad \forall_{ij} \text{ all are integer}$$

##### 4.2 Solving the Model

Solving this model manually requires 8 additional surplus variables plus 7 variables that make up 15 non-basic variables. QM for windows version 2.0 software was used to solve the model. Optimum solution was obtained with  $Z = 300$ , in contrast with  $Z = 303.33$  (Satheeshkumar et al., 2014). This means that 300 nurses are required to optimally assign in a particular shift. The difference of 3.33 nurses that was produced by linear programming approach is meaningless.

##### 5. Conclusion

Nurse changing allotment is well-developed area. Note that our model has only one type of shift as was done in (Satheeshkumar et al., 2014), but with different methodology. The resulting schedule includes balanced schedules in terms of the distribution of shift duties, fairness in terms of the number of consecutive night duties and the preferences of the nurses. This is an improvement over analytical linear programming approach that was employed by (Satheeshkumar et al., 2014) which produce a better result unambiguous in producing good schedule. Nurse scheduling problem is a complex timetabling problem that affects hospital personnel on a daily basis all over the world. The need for robust solution is sharp for a number of reasons. In particular, it is very important to efficiently utilize time and effort, to evenly balance the workload among people and to

attempt to satisfy personnel preferences. A high quality schedule can result to a more contented and thus more efficient work force.

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## 6. Policy Implication of the Study

The ratio of nursing staff serving both in-patient and out patients' population is minimal in most hospital throughout the world today. The need to allocate nurse to duty post is of paramount importance. The paper has solved nurse scheduling problems using integer linear programming in contrast with the earlier paper that employed linear programming which has improved the result tremendously. It has shed more light on the existence of the shifting problem in nursing profession as a whole and help in choosing the right decision about the requirement of the nursing staff at different hour of the day. Finding of the research work if implemented would guide hospital management to efficiently allocate the available scarce resources optimally.

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