On Mathematical Modeling in Optimizing Telephone Communication Network Using Fuzzy Weighted Graph

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Abstract: ***In this work, the shortest path algorithm is proposed in order to minimize the cost of telephone cable based on the weighted fuzzy graph and spanning tree. The problem of telephone line cable connection in some districts of Tamil Nadu is illustrated in showing the effectiveness of matrix algorithm which is based on the weighted Fuzzy graph.***

Keyword: Fuzzy Weighted Graph, Spanning Tree, Weighted Matrix, Shortest path algorithm.

1. Introduction

We introduce an algorithm for finding the shortest path from source node to destination node. This chapter is aimed at providing algorithms to locate fuzzy trees with in a given fuzzy graph. The fuzzy tree represents a generalized concept in fuzzy graph; it is the spanning fuzzy tree that is of a greater interest to the Mathematician. The result deals with locating the spanning fuzzy trees. A fuzzy digraph is said to be a fuzzy tree if its underlying fuzzy graph is a fuzzy tree.

Optimizing the length of telephone communication network is one of the major problems in the digital world. It is also placing an important role in internet connection. As minimizing the length of telephone communication network, the cost and the structure of the network will be minimized. Moreover, every pair of nodes will have the sufficient connection if the one of the connection is disconnected between the roads.

At present communication and transportation has an important place in every person life in their all business and non-business task. In daily life everybody is facing a problem of choosing a shortest path from one location to another location. Shortest path means the path which has minimum mileage or distance covered.

2. Definitions

Definition 2.1:
A fuzzy graph with S as the underlying set is a pair $G:(\sigma,\mu)$ where $\sigma: S \rightarrow [0,1]$ is a fuzzy subset, $\mu: S\times S \rightarrow [0,1]$ is a fuzzy relation on the fuzzy subset σ such that, $\mu(x,y) \le \sigma(x)\wedge\sigma(y)$ for all $x,y \in S$, where $\wedge$ stands for minimum value.

Example 2.1:

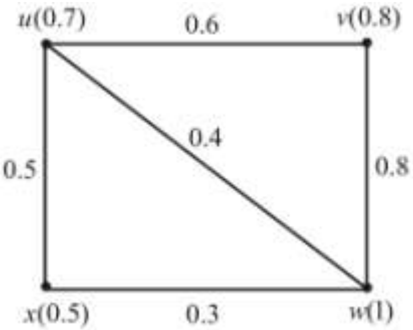

Figure 1: Fuzzy Graph

Let $G:(\sigma,\mu)$ and $G':(\sigma',\mu')$ be the fuzzy graphs with underlying sets $\sigma^* = \{u,v,w,x\}$ where $\sigma: V \rightarrow [0,1]$, $V\times V \rightarrow [0,1]$ are defined as $\sigma(u)=0.7, \sigma(v)=0.8, \sigma(w)=1, \sigma(x)=0.5$ and $\mu(u,v)=0.6, \mu(v,w)=0.8,\ \mu(w,x)=0.3, \mu(x,u)=0.5$ and $\mu(u,w)=0.4$. Then G is a fuzzy graph since, $\mu(u,v)\le\sigma(u)\wedge\sigma(v)$ for all $u,v\in\sigma^*$.

Definition 2.2:
A fuzzy spanning tree is a fuzzy tree which covers all the vertices of a fuzzy graph. If there are *n* vertices in the fuzzy graph, then each fuzzy spanning tree has $n-1$ edges.

Example 2.2:

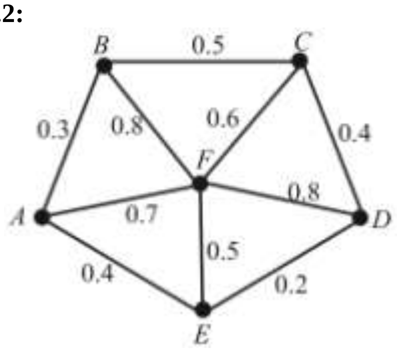

Figure 2(a): Fuzzy Graph *G*.

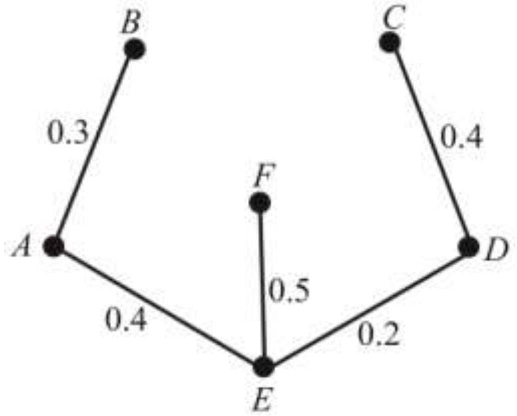

Figure 2(b): Fuzzy spanning tree.

Definition 2.3:
A Fuzzy minimum spanning tree or minimum weight spanning tree is a subset of the edges of a connected, edge - weighted undirected fuzzy graph that connects all the vertices together, without any cycles and with the minimum possible total edge weight.

Example 2.3:

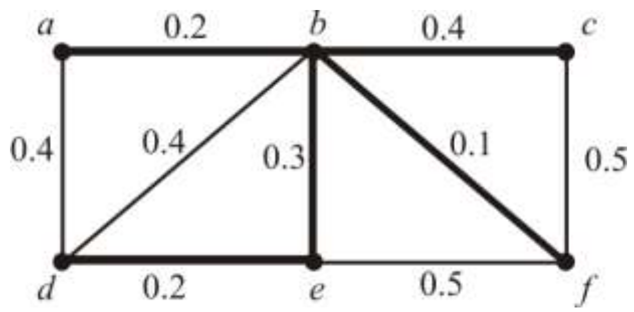

Figure 3: Fuzzy Minimum Weight Spanning Tree

Definition 2.4:
A fuzzy Hamiltonian Circuit is a circuit that visits every vertex in a fuzzy graph once with no repeats; being fuzzy Hamiltonian Circuit must start and end at the same vertex.

Example 2.4:

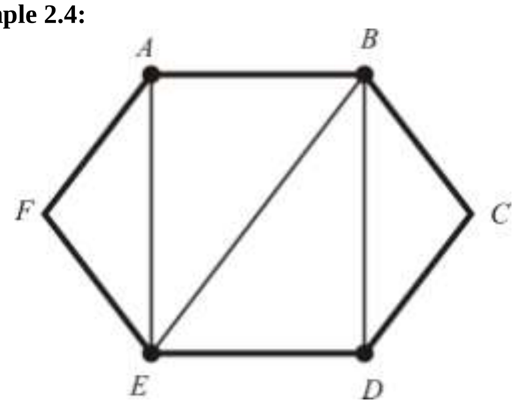

Figure 4: Fuzzy Hamiltonian Circuit

Definition 2.5:
For two vertices u and v in a Fuzzy graph G, the distance from u to v is denoted by $d(u,v)$ and defined as the length of the shortest path in Fuzzy graph G.

Example 2.5:

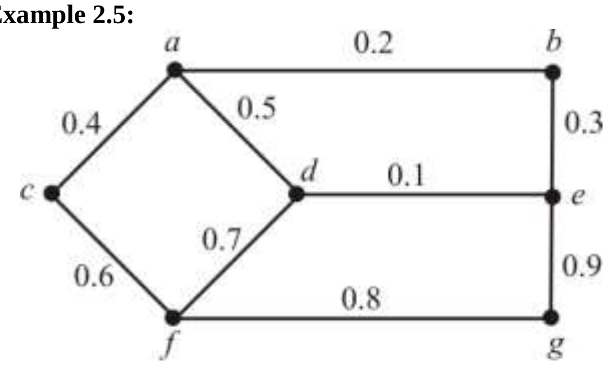

Figure 5: Distance between two vertices

To find the distance from d to e.

There are many paths from vertex $'d'$ to vertex $'e'$.

$da-ab-be \Rightarrow 0.5, 0.2, 0.3$

$df-fg-ge \Rightarrow 0.7, 0.8, 0.9$

de (It is considered for distance $\Rightarrow 0.1$ between the vertices).

$df-fc-ca-ab-be \Rightarrow 0.7,\ 0.6,\ 0.4,\ 0.2,\ 0.3$

$da-ac-cf-fg-ge \Rightarrow 0.5, 0.4, 0.6, 0.8, 0.9$

Therefore, $'de'$ is the shortest path is the distance between the two vertices.

Definition 2.6:
The Fuzzy Eccentricity $E(v)$ of a vertex v of a connected Fuzzy Graph G is the maximum distance between a vertex to all other vertices.

$$E(v) = \max_{u \varepsilon v(G)} \quad d(u,v)$$

Example 2.6:
In the above (Fig.5) graph, the eccentricity of $'a'$ is 3.

The distance from a to b is 1 (i.e., ab)

The distance from a to c is 1 (i.e., ac)

The distance from a to d is 1 (i.e., ad)

The distance from a to e is 2 (i.e., $ab-be$ (or) $ad-de$)

The distance from a to f is 2 (i.e., $ac-cf$ (or) $ad-df$)

The distance from a to g is 3 (i.e., $ac-cf-fg$ (or) $ad-df-fg$)

So, the eccentricity is 3, which is maximum from vertex $'a'$ from the distance between $'ag'$ which is maximum.

3. Computation of Minimum Spanning Tree

Let $G(V,E,W)$ be an undirected connected weighted fuzzy graph with n vertices, where

V – the set of vertices of the fuzzy graph.

E – the set of edges of the fuzzy graph

W – the set of weights associated to corresponding edges of the fuzzy graph

e_{ij} – the edges adjacent to vertices to v_i and v_j.

W_{ij} -the weight associated to the edge e_{ij} the weight matrix *M* of the fuzzy graph *G* is constructed as follows:
If there is an edge between the vertices v_i to v_j in G.
Then set,

$$M_{[i,j]} = W_{ij}$$

else

$$M_{[i,j]} = 0$$

4. Shortest Route Algorithm

The path of minimum cost between the point *s* (source) to *t* (sink) of fuzzy graph *G* can be obtained using the following steps.

4.1. Proposed weighted matrix algorithm

Input: The weight matrix $M = [W_{ij}]\, n \times n$ for undirected Weighted Fuzzy Graph G.

Step 1: Start by assigning the notation $x_i \quad (i = 1,2,\ldots,n)$ to the variables connected with each of the cities.

Step 2: Repeat step 3 to step 4 until all $(n-1)$ element of upper triangular matrix or lower triangular matrix of M are marked or set to zero or in other words all the non-zero elements are marked.

Step 3: Search the upper triangular matrix or lower triangular matrix M by row-wise to find unmarked non-zero minimum element $M_{[i,j]}$, which is the weight of the corresponding edge $e_{i,j}$ in M .

Step 4: If the corresponding edge $e_{i,j}$ of selected $M_{[i,j]}$, forms cycle with the already marked element in the element of $M_{[i,j]}$ then set $M_{[i,j]} = 0$ else mark $M_{[i,j]}$.

Step 5: Construct the fuzzy graph T including only the marked elements from the weight matrix M which shall be desired minimum cost spanning tree of G.

Step 6: We get the fuzzy shortest path of fuzzy network.

Output: Minimum Spanning Tree T of undirected weighted fuzzy graph G.

4.2 Problem definition

We shall illustrate the technique with example and provide the mathematical verification.

Telephone Company providing the telephone cable connection to connect the district of Tamilnadu, laying the cable along the roads shown on the map. They want to connect all of these districts to their telephone cable network using the minimum total length of the cable.

This telephone connection problem is converted as a fuzzy graph theoretical problem. In this example we applied the matrix algorithm to connect the shortest route telephone cable connection for the problem. Consider the Tamilnadu map and point some cities and consider them as vertices.

All the Districts in Tamil Nadu are shown on the map is given below.

Figure 6: Tamil Nadu Map

The following network diagram represents the telephone line connection between ten districts of Tamil Nadu, laying the cable along the road is shown on the map.

Figure 7: Network diagram representing the connection between districts.

Consider fig.7, each District as vertex such as

V_1 - Madurai
V_2 - Theni
V_3 - Dindigul
V_4 - Thirunelveli
V_5 - Tuticorin
V_6 - Trichy
V_7 - Sivagangai
V_8 - Perambalur
V_9 - Thanjavur
V_{10} - Pudukottai

In order to develop the above problem in terms of fuzzy graph, the distances between any two districts are to be considered in the following manner.

Hence we consider each city as vertex and the distance between any two cities is represented as edge or arc. In order to develop the above problem in terms of fuzzy graph, the distances between any two cities are to be considered in the following manner.

The length of the roads joining adjacent districts is given in Kms in the table below.

Table 1

Distance in Km	*Membership grades*
0 – 20	0.12
20 – 40	0.17
40 – 60	0.24
60 – 80	0.28
80 – 100	0.19
100 – 120	0.16
120 – 140	0.32
140 – 160	0.9
160 – 180	0.13
180 – 200	0.25
200 – 220	0.7

Network input information is given below table

Table 2: Network input information

Edge	Starting District	Destination District	Distance in Km	Membership grade (length of the cable)
1	Madurai	Theni	77	0.28
2	Madurai	Dindigul	66	0.28
3	Theni	Tuticorin	210	0.7
4	Madurai	Sivagangai	45	0.24
5	Thirunelveli	Tuticorin	50	0.24
6	Madurai	Thirunelveli	162	0.13
7	Theni	Dindigul	74	0.28
8	Sivagangai	Pudukottai	83	0.19
9	Sivagangai	Thanjavur	144	0.9
10	Dindigul	Trichy	98	0.19
11	Trichy	Thanjavur	60	0.24
12	Thirunelveli	Sivagangai	154	0.9
13	Thanjavur	Pudukottai	71	0.28
14	Perambalur	Thanjavur	110	0.16
15	Trichy	Perambalur	61	0.28

The length of the cable represents the membership grade between the Districts is given in the table below.

Table 3: Distance matrix of the fuzzy weighted graph

-	V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8	V_9	V_{10}
V_1	-	0.28	0.28	0.13	0	0	0.24	0	0	0
V_2	0.28	-	0.28	0	0.7	0	0	0	0	0
V_3	0.28	0.28	-	0	0	0.19	0	0	0	0
V_4	0.13	0	0	-	0.24	0	0.9	0	0	0
V_5	0	0.7	0	0.24	-	0	0	0	0	0
V_6	0	0	0.19	0	0	-	0	0.28	0.24	0
V_7	0.24	0	0	0.9	0	0	-	0	0.9	0.19
V_8	0	0	0	0	0	0.28	0	-	0.16	0
V_9	0	0	0	0	0	0.24	0.9	0.16	-	0.28
V_{10}	0	0	0	0	0	0	0.19	0	0.28	-

In order to give the telephone connection between the districts with minimum, the company has to identify the shortest path between every pair of the districts. The proposed weighted matrix algorithm is employed in the following way to find the shortest path between the districts giving the telephone connection with minimum cost.

$$\begin{pmatrix} 0 & 0.28 & 0.28 & 0.13 & 0 & 0 & 0.24 & 0 & 0 & 0 \\ 0.28 & 0 & 0.28 & 0 & 0.7 & 0 & 0 & 0 & 0 & 0 \\ 0.28 & 0.28 & 0 & 0 & 0 & 0.19 & 0 & 0 & 0 & 0 \\ 0.13 & 0 & 0 & 0 & 0.24 & 0 & 0.9 & 0 & 0 & 0 \\ 0 & 0.7 & 0 & 0.24 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.19 & 0 & 0 & 0 & 0 & 0.28 & 0.24 & 0 \\ 0.24 & 0 & 0 & 0.9 & 0 & 0 & 0 & 0 & 0.9 & 0.19 \\ 0 & 0 & 0 & 0 & 0 & 0.28 & 0 & 0 & 0.16 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.24 & 0.9 & 0.16 & 0 & 0.28 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.19 & 0 & 0.28 & 0 \end{pmatrix}$$

Modify weight matrix of Fuzzy Graph

$$\begin{pmatrix} 0 & 0.28 & 0.28 & \boxed{0.13} & 0 & 0 & 0.24 & 0 & 0 & 0 \\ 0.28 & 0 & 0.28 & 0 & \boxed{0.7} & 0 & 0 & 0 & 0 & 0 \\ 0.28 & 0.28 & 0 & 0 & 0 & \boxed{0.19} & 0 & 0 & 0 & 0 \\ 0.13 & 0 & 0 & 0 & \boxed{0.24} & 0 & \boxed{0.9} & 0 & 0 & 0 \\ 0 & 0.7 & 0 & 0.24 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.19 & 0 & 0 & 0 & 0 & 0.28 & \boxed{0.24} & 0 \\ 0.24 & 0 & 0 & 0.9 & 0 & 0 & 0 & 0 & \boxed{0.9} & \boxed{0.19} \\ 0 & 0 & 0 & 0 & 0 & 0.28 & 0 & 0 & \boxed{0.16} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.24 & 0.9 & 0.16 & 0 & 0.28 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.19 & 0 & 0.28 & 0 \end{pmatrix}$$

Non-zero minimum (n-1) elements are marked

From the above weighted matrix the minimum weight is selected and colored, their corresponding edges were drawn and repeat the process until the algorithm terminates.

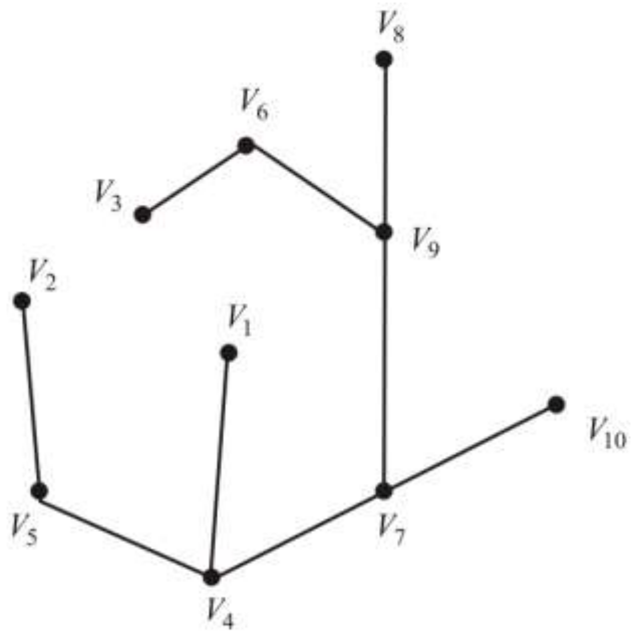

Figure 8: Updated fuzzy shortest route telephone line connection

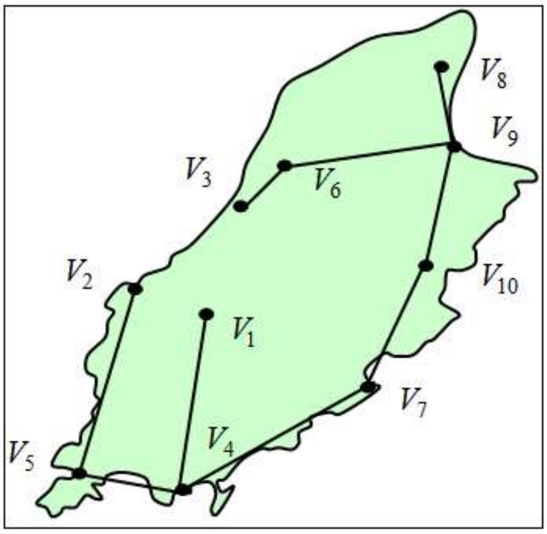

Figure 9: Minimum length of the cable between districts

Using the matrix algorithm the minimum total length of the telephone cable

$= (V_1,V_4)+(V_2,V_5)+(V_4,V_8)+(V_7,V_{10})+(V_4,V_5)+(V_7,V_9)+(V_3,V_6)+(V_6,V_9)+(V_8,V_9)$

= 0.13 + 0.7 + 0.9 + 0.19 + 0.24 + 0.9 + 0.19 + 0.24 + 0.16

= 3.65

5. Conclusion

Finally, we present a simple and efficient technique to compute minimum spanning tree of the weighted fuzzy graph using matrix algorithm. This proposed algorithm will be handy to the companies whose providing communication network through cables.

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