The Weakly Coupled Channels Contribution Effects in Neutron Induced Complex Particles Emission in $^{98}$Mo Nuclei

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Abstract: The non-equidistant space model (non-ESM) for two-component state densities has been modified to include different correction parameters. The calculated results have been used with neutron induced reaction with $^{56}$Fe target nuclei. The comparisons with other theoretical and experimental results show a very good agreement at excitation energy less than 15 MeV. The modified state density has been applied to calculate the cross-sections of emitting a complex particles in $^{98}$Mo(n,n+p)$^{97}$Nb$^*$ and $^{98}$Mo(n,4He)$^{96}$Zr reactions for different incident neutron energies. Furthermore, the results are compared with the available experimental results of EXFOR. It was found the weaker probability of emitting complex particles comes from weaker primary pre-equilibrium emission due to high binding energies of complex particles. Different stages of probabilities for neutron induced complex particles emissions have been studied to evaluate the depletion factor DF that weakly contributed from the direct or PE processes in (neutron +$^{98}$Mo) reaction by consume the flux in complex emission particles, (n+p) and ($^4$He).

Keywords: Depletion factor, Pre-Equilibrium fraction, $^{98}$Mo(n,n+p)$^{97}$Nb$^*$ and $^{98}$Mo(n,$^4$He)$^{96}$Zr reactions

1. Introduction

Griffin proposed a semi-classical approach to explain and predicted the probability of the pre-equilibrium (PE) region toward the tracking and understanding the scattering process at equilibrium stage [1]. This approach is called exxon model (ExM), which is assumed the incident projectile on a certain target nuclei gradually induced more excited states in the frame of the compound system. These excited states are characterized by excitation energy and the exciton number [n=p (particle)+h(hole)] below the Fermi energy ($\varepsilon_F$). Base on experimental results of light nuclei-target nuclei at different excitation energies, this model has been reformulated and included system properties and features [2-5]. Furthermore, Griffin’s model is integrated with quantum mechanic’s approaches to describe the state density of the possible PE channels of reactions [6]. This development of ExM gathered the idea of equidistant space model (ESM) and non-equidistant space model (NESM) which gave promising and satisfactory results [7-9].The addition of the Feshbach, Kerman, Koonin theory (FKK-theory) with ExM gave more improvements to the probabilities’ predictions of the reaction channels. The idea of multistep direct (MSD) and multistep compound (MSC) stages of this model is succeeded in justification the appearance of particles involved in the reaction at short time and high energy. In MSD, most of the particles involved in the reactions are unbound with at least one particle is bound, and a significant forward peak may be achieved. While in MSC stage, all particles are bound and the emission is rather slow with continuous distribution or symmetric about 90°.

The ExM has been developed with different physical corrections, such as, Pauli’s correction [10], implication of the angular momentum dependence[11], developed a method to calculate by master equation[12], back shift [13], surface correction [14], bound state correction and finite depth [15], the extension to the two-component formalism [16], the distinction of neutron and proton through the reaction chained the implication of the extension to gamma-ray emission [17], pairing correction [18] and isospin distribution and parity corrections [18-20].

2. Theory

The depletion factor, DF, is applied to the weakly coupled channels that deplete the flux, such as contributions from the direct or PE processes. For the deformed nuclides, the effect of direct transitions to discrete levels is included directly in the coupled-channels scheme and the $\sigma_{(dis+dir)}$ is omitted in this equation. Then, the DF can formulate as:

$$DF = 1 - \left( \frac{\sigma_{dis+dir}}{\sigma_{rxn}} + \frac{\sigma_{PE}}{\sigma_{rxn}} \right)$$ (1)

where the $\sigma_{dis+dir}$ is the total discrete and direct reaction cross-section, $\sigma_{PE}$ is the PE cross-section and $\sigma_{rxn}$ is the reaction cross-section.

In order to calculate equation (1), the following formulae are considered.

a) The PE cross-section

The quantum mechanical theories have been developed to describe pre-equilibrium processes. The most used theory is that of FKK [19]. The theory partitions the reaction process into two types of scattering: multistep compound (important at the lowest incident energies), and multistep direct (important at higher incident energies). The above theory of reactions will be applied to the spectrum calculations. The PE cross-section is formulated in terms of The MSD and MSc differential cross-section can be formulated as [21]:

$$\left( \frac{d\sigma}{d\varepsilon} \right)_{MSD} = \sum_{i=1}^{3} \left( \frac{d\sigma}{d\varepsilon} \right)_{i}$$

and

$$\left( \frac{d\sigma}{d\varepsilon} \right)_{MSC} = \sum_{i=1}^{3} \left( \frac{d\sigma}{d\varepsilon} \right)_{i}$$ (2)

Where $i=1,2,3$ for primary and secondary PE, and knockout or direct differential cross-sections, respectively.
When the cross-sections in eq.(2) are combined in the forward peak for the energy channel particle b \((E_b)\) the fraction yields:

\[
\frac{d\sigma_{PE}}{dE_b} = \frac{d\sigma_{PE}}{dE_b} \frac{d\sigma_{SE}}{dE_b} + \frac{d\sigma_{SE}}{dE_b}
\]  

(3)

where the factor \(P\) represents the part of the PE population that has survived emission, the compound formation cross section \(\sigma_{CF} \) yields:

\[
\frac{d\sigma_{PE}}{dE_b} = \sigma_{CF}(E_b) \sum_{\gamma_{PE}}^{} \frac{dE_b}{dE_b} \sum_{\gamma_{SE}}^{} \frac{dE_b}{dE_b} \times \rho(p_x, h_x, p_y, h_y, E_x, E_y, T_x, T_y) \times P(p_x, h_x, p_y, h_y) \times P(p_y, h_y, p_x, h_x)
\]  

(4)

The primary PE differential cross section, for the emission of a cluster b particle \((E_b)\) and can then be expressed in terms of lifetimes \(\tau\) for various classes of stages, with the composite nucleus formation cross section \(\sigma_{CF}\) and an emission rate \(\zeta_b\) in two-component, can be defined as:

\[
\frac{d\sigma_{PE}}{dE_b} = \sigma_{PE}(E_b) = \sigma_{PE}(E_b) \sum_{\gamma_{CF}}^{} \frac{dE_b}{dE_b} + \frac{dE_b}{dE_b} \times \rho(p_x, h_x, p_y, h_y, E_x, E_y, T_x, T_y) \times P(p_x, h_x, p_y, h_y) \times P(p_y, h_y, p_x, h_x)
\]  

The emission rate, \(W_b\), (Cline and Blann [16], from a state in the isospin mixed case that has the form of the particle b emission rate at the equilibrium stage with relative reduced mass \(\mu_b\), spin \(s_b\), isospin quantum number T and the final state isospin \(T_B\) is:

\[
W_b = \frac{\sum_{\gamma_{PE}}^{} \frac{dE_b}{dE_b} + \frac{dE_b}{dE_b}}{\gamma_{CF}} \sum_{\gamma_{SE}}^{} \frac{dE_b}{dE_b} + \frac{dE_b}{dE_b}
\]  

(5)

The Total Reaction Cross Section

The total reaction cross section for an incident particle of type \(a\), used in calculating the pre-equilibrium and equilibrium spectrum emission, was estimated using the relation [21]:

\[
\sigma_a(E) = \frac{\sum_{\gamma_{PE}}^{} \frac{dE_b}{dE_b} + \frac{dE_b}{dE_b}}{\gamma_{CF}} \sum_{\gamma_{SE}}^{} \frac{dE_b}{dE_b} + \frac{dE_b}{dE_b}
\]  

(6)

The relationships between the p-h single level density, \(g_{p-h}(E)\) and the total density, \(g_{n}(E)\) formula has been established depending on the different modifications and ideas, [22-25].

\[
\sigma_b(E_b) = \begin{cases} 
0 & \text{for } E_b \lesssim E_{b_{min}} \\
\chi E_b^2 + \alpha E_b + \beta & \text{for } E_{b_{min}} \leq E_b < E_{b\text{tot,a}} \\
\lambda E_b + \mu + \frac{\delta}{E_b} & \text{for } E_{b\text{tot,a}} \leq E_b \leq E_{b\text{tot,e}} \\
\max(\lambda E_b + \mu + \frac{\delta}{E_b}, \sigma_g) & \text{for } E_{b\text{tot,e}} < E_b
\end{cases}
\]

(7)

The variables \(\alpha\) and \(\beta\) are given by:

\[
\beta = \chi B_{\text{tot,a}}^2 + \mu + \frac{2B_{\text{tot,a}}^2}{E_{\text{tot,a}}} \quad \text{and} \quad E_{b_{min}} = \begin{cases} 
\alpha + \sqrt{\alpha^2 - 4\chi \beta} / 2\chi & \alpha^2 - 4\chi \beta > 0 \\
-\alpha / 2\chi & \alpha^2 - 4\chi \beta \leq 0
\end{cases}
\]

(8)

The parameters \(\chi, \lambda, \mu, \nu, B_{\text{tot,a}}\) and \(E_{b_{min}}\) are listed in table (1). The quantities \(\chi_0, \chi_1, \chi_2, p_0, h_1, v_1, v_2\) and \(v_2\) can be find in [21].

The geometrical cross section is given by [21]:

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Table 1: The parameters used in calculating the reaction cross section [21]

<table>
<thead>
<tr>
<th>variable</th>
<th>Neutron</th>
<th>Proton</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_{\text{coul}}$</td>
<td>0.5 MeV</td>
<td>$1.44 Z_a Z_{\text{tar}} / \left( 1.5 A_{\text{tar}}^{1/2} + R_{\text{tar}} \right)$</td>
</tr>
<tr>
<td>$E_{\text{test}}$</td>
<td>32 MeV</td>
<td>$\sqrt{\nu / \lambda + 7}$ (proton)</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>$\kappa_0$</td>
<td>$\kappa_x + \kappa_0 / B_{\text{coul}}$</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>$\lambda_0 + A_{\text{tar}} + \lambda_1$</td>
<td>$\lambda_0 + A_{\text{tar}} + \lambda_1$</td>
</tr>
<tr>
<td>$\mu$</td>
<td>$\mu_0 A_{\text{tar}}^{1/2}$</td>
<td>$\mu_0 A_{\text{tar}}^{1/2}$</td>
</tr>
<tr>
<td>$\nu$</td>
<td>$\nu_0 A_{\text{tar}}^{1/2} + \nu_1 A_{\text{tar}}^{2/2}$</td>
<td>$\nu_0 A_{\text{tar}}^{1/2} + \nu_1 A_{\text{tar}}^{2/2} + \nu_2$</td>
</tr>
</tbody>
</table>

where $\sigma_b(\epsilon)$ is the inverse cross section, $K_{\alpha,p}$ is the enhancement factor for $(\alpha, N)$ and $(N, \alpha)$ reactions, $E_a$ is incident energy in the laboratory system, $V_I$ is the average potential drop seen by projectile between infinity and Fermi's level, $C_a$ and $N_a$ are the normalization constants can be determined by references [29] while in the second process, knock out process, the projectile will excite a proton, neutron or clusters in the target and resulting particles in the composite nucleus can be emitted. The energy spectrum for this process has the form [30]:

$$E_{\alpha,p} = \left( \frac{Z_a}{A_a} \right)^2 (Z_a+2) h_a + 2 p_a \omega_{NT} (n, U)$$

3. Results and Discussions

In the present work the two-component state density, using the non-ESM excitation model, eq.(6), for neutron induced reactions with $^{56}$Fe target nuclei has been compared with other theoretical and experimental results, see fig.1. The comparisons look at very good agreement at excitation energy less than 15 MeV. After adding all the corrections to the state density equation and compare the results with other practical data and theoretical results, eq.(4), it was found that the most important corrections need to be added to the state density and later enhanced the spectrum are: The Pauli, pairing, surface, back shift, isospin, and finite depth corrections.

After added the corrections to the state density equation, the calculated energy spectrums were also corrected, eqs.(5,7,8,9,10,11) and table (1). As shown in fig.2, the results are compared with a large variety of experimental data and theoretical results, eq.(4), it was found that the most important corrections need to be added to the state density and later enhanced the spectrum are: The Pauli, pairing, surface, back shift, isospin, and finite depth corrections.

In fig.3, the inter-comparison has been made from the data calculated in the present work for $^{58}$Mo(n,n)$^{58}$Mo energy
spectrum, with the experimental measured data from EXFOR [40].

The DF for weakly coupled channel's energy spectrum that depletes the flux, eq.(1), which contributed from the direct or pre-equilibrium processes and PE factor, eq.(3), is shown in fig.4, where the spectrum of the depleted flux decreased as the energy shares the nucleons in the discrete excited states is increased. From this figure, one can indicate the dominant weakly depleted flux in incident neutron energy below 20 MeV. Furthermore, as shown in fig.5 the distribution of \( \sigma_{\text{rxn}} \), \( \sigma_{\text{dir}} \), \( \sigma_{\text{PE}} \), \( \sigma_{\text{disc}} \) and \( \sigma_{\text{compound}} \) cross-sections as a function of incident neutrons. It shows the weakly depleted probability increase and the PE probability decrease as the neutron energy less than 20 MeV.

4. Conclusion

Different corrections have been implemented in predicting the state density, for the configuration of emitting complex particles, to compute and evaluate the cross-section in neutron interacted with 98Mo target nuclei. From the behavior of the emitting \( ^4\text{He} \) and \( (n+p) \) spectrums, it was concluded the weaker probability of these particles came from the weaker primary PE emission due to high binding energy of complex particles. Also, due to consume the flux in complex emission particles, \( (n+p) \) and \( ^4\text{He} \), the depletion factors DF, that weakly contributed from the direct or PE processes in \( (\text{neutron} + ^{98}\text{Mo}) \) reaction, is increased at incident neutron energy less than 20 MeV, where the PE fractions in forward direction are decreased, and conversely at neutron energy greater than 20 MeV.

![Graph 1: The two component state density as a function of excitation energy using eq.(6)](image1)

Solid curves: \( \Omega(p,h,E) \) without any correction; dashed curves: with energy single particle level density (EDSPLD), pairing and finite depth corrections; dotted curves: represent the state density with EDSPLD, pairing and finite depth and parity corrections only; dashed dotted curves: with EDSPLD, pairing, finite depth and spin distribution corrections. These results are compared with experimental data in (a) [34,35] and (b) [36,39], also with other theoretical results (c) [37] and (d) [38].

![Graph 2: The \(^{98}\text{Mo} \ (n, n + p) ^{97}\text{Nb}^{m} \) and \(^{98}\text{Mo} \ (n, ^4\text{He}) ^{95}\text{Zr} \) cross-sections at different incident neutrons compared with data for EXFOR [40].](image2)
**Figure 3:** The $^{98}$Mo(n,n)$^{98}$Mo energy spectrum compared with data for EXFOR [40].

**Figure 4:** The DF and PE-fraction via incident neutron energy on $^{98}$Mo.

**Figure 5:** The possible stage’s cross-sections in n+$^{98}$Mo reaction.

**References**