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On Reliability Estimation for the Rayleigh Distribution Based on Monte Carlo Simulation

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Abstract: ***This Research deals with estimation the reliability function of two-parameters Rayleigh distribution, using different estimation methods like, Maximum likelihood, Median-First Order Statistics, Ridge Regression, Modified Thompson-Type Shrinkage and Single Stage Shrinkage methods. Comparisons among the suggestion estimators were made using Monte Carlo Simulation based on statistical indicter mean squared error (MSE).***

Keywords: The Rayleigh distribution, Maximum likelihood method, Median-First Order Statistics, Ridge regression, Modified Thompson, Shrinkage methods estimators, Mean squared error

1. Introduction

"The study of the industrial equipment reliability is a challenge for the most industries, in order to develop the efficiency and improve the quality and the performance of these equipment's. Recently, research on the reliability increased in all areas of science and has experienced the use of several mathematical methods and techniques that include the use of Rayleigh distribution for modeling and estimation of reliability and parameters. Indeed, the Rayleigh distribution model is important widely used in large scale trials in life and reliability of the industrial equipment. This distribution also used for other applications for modeling and analysis of data in the different fields of science"; [1].

The Rayleigh distribution is commonly used to model the behavior of units that have an increasing failure rate. The two-parameter Rayleigh distribution provides a simple but nevertheless useful model for the analysis of lifetimes, especially when investigating reliability of technical equipment.

"Lord Rayleigh (1880) introduced the Rayleigh distribution in connection with a problem in the field of acoustics";[2]. Since then, extensive work has taken place related to this distribution in different areas of science and technology. It has some nice relations with some of the well-known distributions like Weibull, chi-square or extreme value distributions. An important characteristic of the Rayleigh distribution is that its hazard function is an increasing function of time. It means that when the failure times are distributed according to the Rayleigh law, an intense aging of the equipment/ item takes place. The Rayleigh distribution was originally derived in connection with a problem in acoustics, and has been used in modeling certain features of electronic waves and as the distance distribution between individuals in a spatial Poisson process. Most frequently however it appears as a suitable model in life testing and reliability theory.

Rayleigh distribution, which is a special case of Weibull distribution has wide application, such as, in life testing, "Palovko (1978), in clinical studies deals with cancer patients";[3], "Gross and Clark (1975) ";[4] and" Lee, E. T (1980) ";[5]. "Cohen and Whitten (1982) used the moment and modified moment estimators for the Weibull distribution";[6]. "Ariyawansa and Templeton (1984) have also discussed some of its applications";[7]. " Lalitha and Anand (1996) used the modified maximum likelihood to estimate the scale parameter of the Rayleigh distribution";[8] . "Meintan is and Iliopoulos (2003) proposed a class of goodness of fit tests for the Rayleigh distribution";[9]. The tests are based on a weighted integral involving the empirical Laplace transform.

"Al– Naqeep and Hamed in(2009) suggested, as well, a conventional method for estimating the two parameters of generalized Rayleigh distribution for different sample (small, medium and large) and compare the estimators by using mean square error for simulation data";[10]. " Dhwyia et al in (2012) developed the moment generating function which was derived to help in finding the moment , also cumulative distribution function , then obtaining the least squares estimators for the unknown parameters and moment estimator method ";[11]. " Parvin , Ali and Hossein in (2013) considered the estimation of $R=P(y<x)$ where x and y have two parameters of generalized Rayleigh distribution , then obtained the maximum likelihood estimations of parameters with simple iterative procedure for several values of parameters and calculate the MLE of $R=P(y<x)$ with Simpson integration using maple codes";[12]. "Iden Kanani and Shaima Abbas studies compare Non-Bayesian and Bayesian Estimation for Generalized Rayleigh Distribution and get moment estimator method is the best";[13].

The aim of this research is to estimate the reliability function of two-parameters Rayleigh distribution, using different estimation methods like, Maximum likelihood, Median-First Order Statistics, Ridge Regression, Modified Thompson-Type Shrinkage and Single Stage Shrinkage methods. Comparisons among the suggestion estimators were made using Monte Carlo simulation based on mean squared error criteria.

The probability density function of two parameters Rayleigh distribution is given by

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$$f(t_i,\alpha,\beta) = \begin{cases} \frac{(t_i-\alpha)}{\beta^2} exp - \frac{(t_i-\alpha)^2}{2\beta^2} & \alpha < t_i < \infty \\ 0 & \frac{o}{w} \end{cases}$$

$$\Omega = \{(\alpha,\beta); \alpha > 0, \beta > 0\}$$

Where α is a location parameter and β is a scale parameter.

The parameters β and α are interpreted as measure of guarantee and failure rate respectively.
The mean and the variance of Rayleigh distribution are:

$$M_t = E(t) = \sqrt{\frac{\pi}{2}}\,\beta + \alpha \qquad (2)$$

$$\sigma^2{}_t = var(t) = \left(2 - \frac{\pi}{2}\right)\beta^2 \qquad (3)$$

$$E(t^2) = 2\beta^2 + \sqrt{2\pi}\,\alpha\beta + \alpha^2 \qquad (4)$$

The reliability function and hazard function of generalized Rayleigh distribution are:

$$F(t) = 1 - exp - \frac{(t_i-\alpha)^2}{2\beta^2} \qquad (5)$$

$$R(t) = exp - \frac{(t_i-\alpha)^2}{2\beta^2} \qquad (6)$$

$$h(t) = \frac{(t_i-\alpha)}{\beta^2} \qquad (7)$$

2. Theoretical Part (Method of Estimation)

In this section, we discuss five estimation methods for the parameters and the reliability function of two-parameter Rayleigh distribution

2.1 Maximum likelihood method

This is the way one of the most important methods of appreciation aims to make possible a function at the end of maximizes . "All that is done to write down the likelihood function L(t_i , α , β), and then find the valueˆof which maximizes L(t_i , α , β;). The log-likelihood function based on the random sample of t_1 ,t_2 ,,,,,,t_n is given by" ;[2]:

$$L = \prod_{i=1}^{n} f(t_i\,,\alpha\,,\beta) \qquad (8)$$

$$L = \frac{\prod_{i=1}^{n}(t_i-\alpha)}{\beta^2} exp - \frac{(t_i-\alpha)^2}{2\beta^2} \qquad (9)$$

Taking the Natural for equation (9)logarithm for the above likelihood function, so we get the following:

$$\text{Ln L} = -2n\,\text{Ln}\,\beta + \sum_{i=1}^{n}\text{Ln}(t_i-\alpha) - \sum_{i=1}^{n}\frac{(t_i-\alpha)^2}{2\beta^2} \quad (10)$$

The partial derivative for equation (10) with respect to unknown parameter β, is:

$$\frac{\partial \text{Ln L}}{\partial\beta} = \frac{-2n}{\beta} + \sum_{i=1}^{n}\frac{(t_i-\alpha)^2}{\beta^3} \qquad (11)$$

Equating equation (11) to zero to solve this equation:

$$\frac{-2n}{\beta} + \sum_{i=1}^{n}\frac{(t_i-\alpha)^2}{\beta^3} = 0$$

$$\frac{-2n\,\hat\beta^2 + \sum_{i=1}^{n}(t_i-\hat\alpha)^2}{\hat\beta^3} = 0$$

$$-2n\,\hat\beta^2 + \sum_{i=1}^{n}(t_i-\hat\alpha)^2 = 0$$

(2)

$$2n\,\hat\beta^2 = \sum_{i=1}^{n}(t_i-\hat\alpha)^2 =$$

$$\hat\beta^2 = \frac{\sum_{i=1}^{n}(t_i-\hat\alpha)^2}{2n}$$

$$\therefore\ \hat\beta_{ml} = \sqrt{\frac{\sum_{i=1}^{n}(t_i-\hat\alpha)^2}{2n}} \qquad (14)$$

Then the estimation of Reliability function for the two-parameters Rayleigh distribution using ML technique will be

$$\hat R_{ML} = exp - \frac{(t_i-\alpha)^2}{2\hat\beta_{ml}{}^2} \qquad (15)$$

2.2 Median-First Order Statistics Method

"In this modification the second moment is replaced by Me_t = t_{me}, where Me_t is the population median and t_{me} is the sample median";[11]. The median of two parameters Rayleigh distribution is given by

$$0.5 = \int_0^A f(x)\,dt \qquad (16)$$

$$\frac{(t_i-\alpha)}{\beta^2}\int_0^A \exp[- \frac{(t_i-\alpha)^2}{2\beta^2}]\,dt = \frac{1}{2}$$

$$A = \alpha + \beta\sqrt{(2ln2)}$$

The cumulative distribution function of the random variable

$$F_{t_{(1)}}t = pr[t_{(1)} < t] = 1 - pr[t_{(1)} > t)]$$
$$= 1 - pr[t_{(1)} > t\,,t_{(2)} > t\,,,,,,,,t_{(n)} > t]$$
$$= 1 - [pr(T > t)]^n = 1 - [R(t)]^n$$

So, the cumulative distribution function of the random variable $t_{(1)}$ is

$$;\alpha < t_1 < \infty \quad ,\beta > 0$$

$$f_{t_{(1)}}(t) = \frac{dF_{t_{(1)}}}{dt} = \frac{n(t-\alpha)}{\beta^2}\exp\left[-\frac{(t_i-\alpha)^2}{2\beta^2}\right]$$

Then, the mathematical expectation of a random variable $t_{(1)}$ is

$$E[t_{(1)}] = \int_\alpha^\infty \frac{tn(t-\alpha)}{\beta^2}\exp\left[-\frac{(t_i-\alpha)^2}{2\beta^2}\right]dt$$
$$= \alpha + \beta\sqrt{\pi/2n}$$

$t_{(1)}$: The first statistic represents the values of t
If $(\hat\alpha\,,\ \hat\beta)$ *is unbiased estimator to* $(\alpha\,,\beta)$ respectively then the equationrealized

$$\hat\alpha = t_{(1)} - \hat\beta\sqrt{\pi/2n} \qquad (17)$$

Thus, we have $\alpha + \beta\sqrt{(2ln2)} = t_{me}$ (18)

$$\alpha + \beta\sqrt{(2ln2)} = t_{me} \quad \rightarrow \quad \alpha = t_{me} - \beta\sqrt{(2ln2)}$$
$$\alpha + \beta\sqrt{\pi/2n} = t_{(1)} \quad \rightarrow \quad \alpha = t_{(1)} - \sqrt{\pi/2n}\,\beta$$
$$t_{me} - \beta\sqrt{(2ln2)} = t_{(1)} - \beta\sqrt{\pi/2n} \quad \rightarrow \quad t_{me} - t_{(1)} = \beta\sqrt{(2ln2)} - \beta\sqrt{\pi/2n}$$

$$\hat\beta_{Md} = \frac{t_{me} - t_{(1)}}{\sqrt{(2ln2)} - \sqrt{\pi/2n}} \qquad (19)$$

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Then the estimation of Reliability function for the two-parameters Rayleigh distribution using Median-First Order Statistics Method(MD) will be

$$\hat{R}_{Md} = exp - \frac{(t_i - \alpha)^2}{2\hat{\beta}_{Md}{}^2} \qquad (20)$$

2.3 Ridge regression method (RR)

"The ridge regression (RR) estimates of A and B can be obtained by minimizing the error sum of squares for the model Y_i = a+bX_i Subject to the single constraint that a^2+b^2= ρ where ρ is a finite positive constant";[15].

The method of Lagrange's multiplier requires the differentiation of

$$L = \sum_{i=1}^{n}(Y_i - a - bX_i)^2 + \lambda(a^2 + b^2 - \rho)$$

w.r.t a and b. when these derivatives are equated to zero, we obtain the following two equations

$$\sum_{i=1}^{n} Y_i = (n+\lambda)a + b\sum_{i=1}^{n} X_i$$

$$\sum_{i=1}^{n} X_i Y_i = a\sum_{i=1}^{n} X_i + b(\lambda + \sum_{i=1}^{n} X_i^2)$$

Solving above two equations for a and b we get

$$\hat{a} = \frac{(\sum_{i=1}^{n} X_i)(\sum_{i=1}^{n} X_i Y_i) - \sum_{i=1}^{n} Y_i\ (\lambda + \sum_{i=1}^{n} X_i^2)}{(\sum_{i=1}^{n} X_i)^2 - (n+\lambda)(\lambda + \sum_{i=1}^{n} X_i^2)}$$

$$\hat{b} = \frac{(\sum_{i=1}^{n} X_i)(\sum_{i=1}^{n} Y_i) - (n+\lambda)\sum_{i=1}^{n} X_i Y_i}{(\sum_{i=1}^{n} X_i)^2 - (n+\lambda)(\lambda + \sum_{i=1}^{n} X_i^2)}$$

For two parameter Rayleigh distribution when α is known we know that $Y_i = t_i$, , b = β

X_i= [-ln (1-F (t_i))] $\qquad i = 1, 2 \dots n$

$$\hat{\beta} = \frac{\sum t_i \sum(-\ln(1-F(t_i))) - (n+\lambda)\sum t_i(-\ln(1-F(t_i)))}{[\sum(-\ln(1-F(t_i)))]^2 - (n+\lambda)(\lambda + \sum[-\ln(1-F(t_i))]^2)} \qquad (21)$$

Then the estimation of Reliability function for the two-parameters Rayleigh distribution using ridge regression technique (RR) will be

$$\hat{R}_{RR} = exp - \frac{(t_i - \alpha)^2}{2\hat{\beta}_{RR}{}^2} \qquad (22)$$

Where 0<λ<1 is the ridge coefficient the readers may see Ronald and Raymond (1978) if λ=0, we obtain the least square estimates. I suggested λ= exp((-n+1) /(n^2)) and 0< exp((-n+1) /(n^2)) <1 .

2.4 Modified Thompson-Type Shrinkage Estimator

"The shrinkage estimation method is the Bayesian approach depending on prior information regarding the value of the specific parameter θ from past experiences or previous studies.However, in certain situations, prior information is available only from of an initial guess value (natural origin) $\theta_\circ$ of θ";[17]. In such a situation, it is naturalto start with an estimator $\hat{\theta}$ (e.g., MLE) of $\theta$and modify it by moving it closer to $\theta_\circ$.Thompson has suggested the problem of shrink an unbiased estimator $\hat{\theta}$ of the parameter θ toward prior information (a natural origin) $\theta_\circ$by shrinkage estimator $\psi(\hat{\theta})\hat{\theta} + (1-\psi(\hat{\theta}))\theta_\circ$, 0≤ $\psi(\hat{\theta}) \le 1$, which is more efficient than $\hat{\theta}$ if $\theta_\circ$ is close to θ and less efficient than $\hat{\theta}$ otherwise.

According to Thompson[17] $\theta_\circ$ is a natural origin and, as such, may arise for any one of a number of reasons—e.g., we are estimating θ and (a) we believe $\theta_\circ$ is closed to the true value of θ, or (b) we fear that $\theta_\circ$ may be near the true value of θ, that is, something bad happens if $\theta_\circ = \theta$, and we do not know about it (that is, something bad happens if $\theta_\circ \approx \theta$ and we do not use $\theta_\circ$).

Where, $\psi(\hat{\theta})$ is so called shrinkage weight factor; 0≤ $\psi(\hat{\theta}) \le 1$ which represent the belief of $\hat{\theta}$,and (1- $\psi(\hat{\theta})$) represent the belief of $\theta_\circ$.Thompson noting that the shrinkage weight factor may be a function of $\hat{\theta}$ or may be constant and the chosen of the shrinkage weight factor is(ad hoc basis).

The shrinkage weight function $\psi(\hat{\theta})$ can be found by minimizing the mean square error of $\tilde{\theta}$:

$$MSE(\tilde{\theta}_{TH}) = E(\tilde{\theta}_{TH} - \theta)^2 \qquad (23)$$

$$= E(\psi(\hat{\theta})\,\hat{\theta}_{ML} + (1-\psi(\hat{\theta}))\,\theta_\circ - \theta)^2$$

The partial derivative for equation $(2-60)$ w .r. t. to $\psi(\hat{\theta})$ is

$$\frac{\partial MSE(\tilde{\theta})}{\partial \psi(\hat{\theta})} = 2\psi(\hat{\theta})E(\hat{\theta}_{ML} - \theta_\circ)^2 - 2\left(1-\psi(\hat{\theta})\right)(\hat{\theta}_{ML} - \theta_\circ)^2 \quad (24)$$

Equating equation (24) to zero to solve this equation:

$$\frac{\partial MSE(\tilde{\theta})}{\partial \psi(\hat{\theta})} = 0$$

$$2\psi(\hat{\theta})MSE(\hat{\theta}_{ML}) - 2\left(1-\psi(\hat{\theta})\right)(\hat{\theta}_{ML} - \theta_\circ)^2 = 0$$

$$\psi(\hat{\theta}) = \frac{(\hat{\theta}_{ML} - \theta_\circ)^2}{\left[MSE(\hat{\theta}_{ML}) + (\hat{\theta}_{ML} - \theta_\circ)^2\right]} * (0.01) \qquad (25)$$

Therefore, the shrinkage estimators of α and β are respectively becomes as below:

$$\hat{\beta}_{MT} = \beta_\circ + \frac{(\hat{\beta}_{ML} - \beta_\circ)^3}{\left[MSE(\hat{\beta}_{ML}) + (\hat{\beta}_{ML} - \beta_\circ)^2\right]} * (0.001) \quad (26)$$

Then the estimation of Reliability function for the two-parameters Rayleigh distribution using modified Thompson-type shrinkage estimator (MT) will be

$$\hat{R}_{MT} = exp - \frac{(t_i - \alpha)^2}{2\hat{\beta}_{RR}{}^2} \qquad (27)$$

2.5 Single Stage Shrinkage Estimator

"Single stage shrinkage estimation method is the same as the method of Thompson-Type shrinkage estimator $\psi(\hat{\theta})\hat{\theta} + (1-\psi(\hat{\theta}))\theta_\circ$, 0≤ $\psi(\hat{\theta}) \le 1$, which is define in section 2.4 above but we consider the shrinkage weight factor $\psi(\hat{\theta})$ as a function of n ; I put $\psi(\hat{\theta}) = e^{-n}$ ";[16].

$$\hat{\beta}_{ST} = \psi(\hat{\theta})\hat{\beta}_{ST} + \left(1-\psi(\hat{\theta})\right)\beta_\circ \qquad (28)$$

using single stage shrinkage estimator (ST)will be

$$\hat{R}_{ST} = exp - \frac{(t_i - \alpha)^2}{2\hat{\beta}_{ST}{}^2} \qquad (29)$$

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3. Practical Part

3.1 Simulation Study

We carried out Monte Carlo simulation in order to compare the performance of all the estimators proposed in the preceding section. The programs were written in Matlab (2013b). The results are based on 1000 simulation runs. We generated random samples of different sizes by observing that if U is uniform (0, 1), then $t = \alpha + \beta\,[-2\log(1-U)]^{\frac{1}{2}}$ is Rayleigh of (α , β). The sample sizes considered were n=25, 50, 75, 100 and the shape parameter was taken as α = 2, 2.5. In all cases, we set the scale parameter β =3, 3.5, 4. We used 1000 replications to estimate by using the ML, MD,RR ,MT and ST methods. The process of simulation strategy is explained the numerical results in the Table (1) and Table (2). And comparison among all propose estimators where made on MSE which is defined as follow

$$MSE\left(\hat{R}(t)\right) = \frac{\sum_{i=1}^{L}\left(\hat{R}_i(t) - R_i(t)\right)^2}{L}$$

$\hat{R}_i(t)$ is the specific estimated survival
$R_i(t)$ is the specific real survival

Where L=1000 is the number of replications, the results of the simulation study
are reported in the following tables(1) to (2):

Table 1: Estimate Reliability of Rayleigh distribution Based on simulations

n	β	α	R_{RL}	$\hat{R}_{ML1}$	$\hat{R}_{MD}$	$\hat{R}_{RR}$	$\hat{R}_{MT}$	$\hat{R}_{ST}$
25	3	2	0.0838	0.6733	0.6325	0.2378	0.0844	0.0837
		2.5	0.1150	0.5382	0.7560	0.2854	0.1154	0.1148
	3.5	2	0.1550	0.5945	0.7334	0.2297	0.1555	0.1549
		2.5	0.1799	0.6815	0.6654	0.2581	0.1805	0.1797
	4	2	0.1605	0.6387	0.6192	0.1510	0.1611	0.1603
		2.5	0.1869	0.5890	0.6974	0.1765	0.1874	0.1868
50	3	2	0.0077	0.0114	0.0066	0.3500	0.0077	0.0077
		2.5	0.0063	0.0233	0.0045	0.3353	0.0063	0.0063
	3.5	2	0.0019	0.0140	0.0009	0.1580	0.0019	0.0019
		2.5	0.0186	0.0355	0.0319	0.3100	0.0186	0.0186
		2	0.0058	0.0048	0.0209	0.1381	0.0058	0.0058
	4	2.5	0.0018	0.0108	0.0016	0.0880	0.0018	0.0018
75	3	2	0.0204	0.0257	0.0336	0.6333	0.0204	0.0203
		2.5	0.0467	0.1237	0.1227	0.6982	0.0468	0.0466
	3.5	2	0.0010	0.0039	0.0065	0.3328	0.0010	0.0010
		2.5	0.0507	0.0494	0.0027	0.6212	0.0507	0.0506
	4	2	0.0361	0.0191	0.0480	0.5002	0.0361	0.0361
		2.5	0.0195	0.0163	0.0287	0.4397	0.0195	0.0195
100	3	2	0.0004	0.0009	0.0005	0.5496	0.0004	0.0004
		2.5	0.0096	0.0136	0.0057	0.7045	0.0096	0.0095
	3.5	2	0.0056	0.0140	0.0173	0.5873	0.0056	0.0056
		2.5	0.0181	0.0080	0.0058	0.6628	0.0181	0.0181
		2	0.0047	0.0011	0.0031	0.4879	0.0047	0.0047
	4	2.5	0.0017	0.0030	0.0021	0.4256	0.0017	0.0017

Table 2: Mean squared error (MSE) of Reliability estimates based on simulations

n	β	α	$\hat{R}_{ML}$	$\hat{R}_{MD}$	$\hat{R}_{RR}$	$\hat{R}_{MT}$	$\hat{R}_{ST}$
25	3	2	3.4743e-04	3.0108e-04	0.0012	2.0498e-05	2.0129e-05
		2.5	1.7914e-04	4.1091e-04	0.0012	3.8708e-05	3.8314e-05
	3.5	2	1.9319e-04	3.3452e-04	0.0012	7.0891e-05	7.0284e-05
		2.5	2.5160e-04	2.3579e-04	0.0012	9.5849e-05	9.4949e-05
	4	2	2.2865e-04	2.1040e-04	0.0011	7.6097e-05	7.5407e-05
		2.5	1.6167e-04	2.6057e-04	0.0011	1.0333e-04	1.0263e-04
50	3	2	1.3581e-08	1.2143e-09	1.1716e-04	6.5130e-16	6.2325e-13
		2.5	2.8854e-07	3.3224e-09	1.0822e-04	6.6647e-14	4.5580e-13
	3.5	2	1.4616e-07	9.1223e-10	2.4354e-05	1.8694e-14	4.5922e-14
		2.5	2.8482e-07	1.7706e-07	8.4925e-05	6.5076e-14	1.7907e-12
	4	2	1.0362e-09	2.2632e-07	1.7494e-05	4.8897e-18	2.2348e-13
		2.5	8.1367e-08	4.0395e-11	7.4339e-06	1.1743e-14	3.2932e-14
75	3	2	2.8093e-08	1.7638e-07	3.7574e-04	1.1184e-15	2.7887e-12
		2.5	5.9334e-06	5.7761e-06	4.2440e-04	2.9589e-12	9.0913e-12
	3.5	2	8.2761e-09	3.0345e-08	1.1009e-04	1.4920e-15	1.6285e-14
		2.5	1.7472e-09	2.3019e-06	3.2544e-04	6.0127e-20	7.4547e-12
	4	2	2.8865e-07	1.3970e-07	2.1532e-04	2.0676e-13	3.5958e-12
		2.5	1.0206e-08	8.5086e-08	1.7652e-04	1.9641e-16	1.4741e-12

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100	3	2	2.8823e-10	3.3102e-11	3.0164e-04	4.4510e-17	3.5208e-15
		2.5	1.6346e-08	1.5079e-08	4.8297e-04	1.7906e-15	8.7752e-13
	3.5	2	7.1728e-08	1.3809e-07	3.3836e-04	2.1464e-14	2.7250e-13
		2.5	1.0233e-07	1.5270e-07	4.1562e-04	1.0346e-13	1.7222e-12
	4	2	1.3369e-08	2.5295e-09	2.3349e-04	2.5829e-14	1.5977e-13
		2.5	1.6305e-09	1.4597e-10	1.7969e-04	2.0684e-16	2.9547e-14

3.2 Numerical Analysis

1) When n=25, the estimator $\hat{R}_{ST}$ preformed good and will best than the others estimators in the sense of MSE, then follow by $\hat{R}_{MT}$, $\hat{R}_{MD}$, $\hat{R}_{ML}$ and $\hat{R}_{RR}$ respectively for all α and β.
2) When n=50,75 and 100 , we can see from the Table (2) , the MSE of estimator $\hat{R}_{MT}$ less than of the MSE of the other estimators , thus it will be the best in the sense of MSE and follow by the estimators by $\hat{R}_{ST}$, $\hat{R}_{MD}$, $\hat{R}_{ML}$ and $\hat{R}_{RR}$ respectively.
3) For all n and for all β, the MSE of $\hat{R}_{ML}$ are decreases function with respect to α.
4) For all n and for all β, the MSE of $\hat{R}_{MD}$, $\hat{R}_{MT}$ and $\hat{R}_{ST}$ increase function with regard to α.
5) For all *β, the* MSE of the estimator $\hat{R}_{RR}$ are fixed with respect to α when n=25 but when n=50, 75 and 100 , the MSE of the estimator $\hat{R}_{RR}$ will be vibration with regard to α.

4. Conclusions

From the Table (2), one can be noticed that, the shrinkage method perform better than the other method in the sense of MSE, and we recommend to use this type of estimation which is depend on prior information from the past experiences or previous studies.

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References

[1] Mouloud Guemana, Ahmed Hafaifa and Ben Rahmoune Mohamed, Reliability modeling using Rayleigh distribution: Industrial pump application. Proceedings of the 19th European Conference on Mathematics for Industry ECMI2016, 13-17th June 2016, Santiago de Compostela, Spain.

[2] Rayleigh, J.W.S. (1880). On the resultant of a large number of vibrations of the some pitch and of arbitrary phase. Philosophical Magazine, 5-th Series, 10, 73 - 78.

[3] Palovko, A.M (1968). Fundamentals of reliability theory. Academic Press. New-**12**-York.

[4] Gross, A.J. and Clark, V.A.(1975).Survival distributions, Reliability Applications in Biomedical Sciences. Wiley, New York.

[5] Lee, E. T.(1980).Statistical Methods for survival data analysis. Lifetime, learning Publications. Inc., Belmount.

[6] Cohen,A.C. and Whitten,B.J.(1982) Modified maximum likelihood and moment estimation of three parameters Weibull distribution.Commun. Statist.,theory and Method., 11(23)2631-2656

[7] Ariyawansa,K.A. and Templeton,J.G.C.(1984) Structural inference on the parameter of the Rayleigh distribution from doubly censored samples.Statist.Hefte,25,181-199.

[8] Lalitha, S. and Anand, M. (1996). Modified maximum likelihood estimation for Rayleigh distribution. Commun. Statist., Theory Meth., 25 (2) , 389-401 .

[9] Meintanis,S.and Iliopoulos,G.(2003) Tests of fit for the Rayleigh distribution based on the empirical Laplace transform. Annals of the Institute of Statististical Mathematics, Vol.55, No.1, pp. 137-151.

[10] Al – Nachawati H. and Abu – Youssef S. E. (2009) A Bayesian Analysis of Order Statistics from the Generalized Rayleigh Distribution, *Applied Mathematics Sciences* , 3 (27) : 1315-1325.

[11] Dhwyia S. H. , Faten F. A. , Nathier A. I. and Hani A. A. (2012) Proposed Methods for Estimating Parameters of the Generalized Rayleigh Distribution in the Presence of One Outlier, *American Journal of Mathematics and Statistics*, 2 (6): 178-183.

[12] Parvin F., Ali A. and Hossein J. K. (2013). Estimating *R*=(*Y*<*X*) in the Generalized Rayleigh Distribution with Different Scale Parameters, *Applied Mathematics Sciences*, 7(2)(2013): 87-92.

[13] Iden Kanani and Shaima Abbas(2014)..studies compare Non-Bayesian and Bayesian Estimation for Generalized Rayleigh Distribution *Int. J. Modern Math. Sci.* 10(2): 103-115

[14] Wagner, H.(1959). Linear programming techniques for regression analysis. J. Am. Stat. Assoc.,54, 206-212.

[15] Pfaffenberger, R.C. and Dielman,T.E.(1984).A modified ridge regressionestimator using the least absolute value criterion in multiple linear regression model. Proceeding of the American Institute for Decision Sciences. Tronto, PP.791-793.

[16] Al-Joboori, A.N., el al (2014), "Single and Double Stage Shrinkage Estimators for the Normal Mean with the Variance Cases", International Journal of statistic, Vol. 38, No. 2, pp.1127-1134.

[17] Thompson, J.R. (1968). Some Shrinkage Techniques for Estimating the Mean. J. Amer. Statist. Assoc.vol.63. pp.113-122.

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