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Probing Students' Conceptual Understanding, Mathematical Fluency, and Mathematics Anxiety through Cognitive-Demand Mathematical Tasks

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Abstract: ***This paper deals with three big issues in mathematics education- conceptual understanding, mathematical fluency, and mathematics anxiety. In the experimental teaching, the relationship of the three factors and mental ability were investigated. To investigate and establish logical relationships, two intact groups from among freshman students in School Algebra were taken as participants and then decisive success factors were identified. This leads to the conclusion that infusing non-routine cognitive-demand mathematical tasks is effective in enhancing the participants' conceptual understanding and mathematical fluency. Mental ability influences conceptual understanding and mathematical fluency but it does not affect the participants' mathematics anxiety.***

Keywords: Conceptual Understanding, Mathematical Fluency, Mental Ability, Mathematics Anxiety

1. Introduction

The paradigm shift in approach to teaching from teacher-centered to student centered gives clear focus on student outcomes and constructive alignment on content, methods, and assessment tasks (Biggs,2003). To achieve the desired learning outcomes in Algebra for example, Keiser (2010) urged teachers to help and encourage students to develop mathematical fluency- the student's ability to perform basic computation and problem solving with speed, accuracy, flexibility, and efficiency; and conceptual understanding- the student's ability to write logical relationships and representations of concepts, explain, interpret and apply the concept in real life situations.

On one hand, many students demonstrate mathematical fluency in performing a routine problem, say, *Determine if the equation* $x^2 + y^2 = 4$ *describes a function.* On the other hand, a lot of them had a hard time in solving non-routine problems with similar concept like: *Find or define two equations* $y = f(x)$ *and* $y = g(x)$ *that describe a function and satisfy the equation* $x^2 + y^2 = 4$. As observed, mathematical procedure is often forgotten if students only build up mathematical fluency. This may be a sign that doing a mathematical task fluently without conceptual understanding is futile just as conceptual understanding without mathematical fluency may also deter the problem solving performance. It is advantageous for students to maintain and harmonize the acquisition of conceptual understanding and mathematical fluency for better performance in tests. What better way to achieve the desired student outcomes than providing students with opportunities for sustained engagement with cognitive-demand mathematical tasks (Strayer and Brown , 2012).

According to Stein, Smith, Henningsen, and Silver (2000), "not all tasks can cause equal result and different tasks will provoke different levels and kinds of student thinking", and "the level of thinking in which the students engage determines what they will learn" (Hiebert, 1997). In this regard, there is a need to test if the exposure of students to cognitive-demand mathematical tasks using non-routine problems and routine problems with worked-out examples works and makes sense. The questions really are: How do the students' mathematical fluency, conceptual understanding, and mathematics anxiety compare as influenced by cognitive-demand mathematical tasks? Is there interaction of students' mathematical fluency, conceptual understanding, and mathematics anxiety as influenced by their mental ability and the types of mathematical tasks they engage in? What sustains the attention of the students to complete the tasks? While the students are guided to develop conceptual understanding and mathematical fluency through the task design, the researcher also verified if the students' mathematics anxiety is lessened or if their confidence increases as they engage in more challenging mathematical tasks.

2. Conceptual Understanding, Mathematical Fluency, Mental Ability, and Math Anxiety

Conceptual understanding and mathematical fluency encompass many instructional tasks so that the mind of the learners will be able to assimilate all the skills needed for proficiency. Piaget (1936) and Bruner (1966) on the Theory of Cognitive Development believed that learning can be achieved by doing. In addition, Bensford (1990) and Dewey (1964) support that learning is constructed by the learner and learning can take place if there are others who can help on the social process, that is, the Zone of Proximal Development or ZPD (Vygotsky,1970). There are numerous theories on mental ability but among those that this study is anchored on are Thorndike's (1949) and Thurstone's (1955) Theory of Intelligence. Both of them do not believe that intelligence is a single or general ability. While Thorndike believes that intelligence can be measured by level, range, speed and area, Thurstone believes that mental ability can be measured through many factors: number, fluency, reasoning, verbal, space, and memory. These factors are covered in the school's aptitude test.

Conceptual understanding refers to the student's ability to write logical relationships and representations of concepts, explain, interpret and apply the concept in real life situations. It is measured through the problem solving test scores and written works of students and the ability to write about mathematics concept, procedures of the tasks assigned to them, including their difficulties on the topics and those being well understood. Ketterlin-Geller (2007) said that conceptual understanding requires students to decipher what it means to do fundamental operations in mathematics and why the algorithm works. Conceptual understanding requires one to explain and interpret what they are doing in a mathematical operation, why it works, where and when it can be applied. To achieve conceptual understanding teachers should start with concrete objects to model the process, show visual models, and finally use the model in terms of symbols of the abstract concept.

Mathematical Fluency refers to the students' achievement score using the skill in carrying out mathematical steps and computations with speed, accuracy, flexibility and efficiency in performing basic computation and problem solving and in pen and paper test. Building mathematical fluency however, like most issues in education, cannot be done overnight. There is no universally agreed best method for teaching mathematical fluency. In fact, considerable debate on this exists (Woodward, 2006). Isaacs and Carroll (1999) believed that students can develop mathematical fluency through a more natural progression, rather than through a rote memorization because the latter just encourages students to believe that mathematics is more on memorizing than on thinking (p. 509). Educators under this view believe that students must develop number sense to be able to effectively solve mathematics problems. They believe that to solve a problem like 8 + 5, the student may change the problem into 10 + 3. For a multiplication problem such as 16 x 4, the student may split the problem into 8 x 4 and then double the product 2(8x4), the process which trains flexibility of thought. Yet, research by Cumming and Elkins (1999), suggests that applying strategies such as applying the split then double method is not enough to develop automaticity; other means can be used.

Engelbrecht (1995) said in his general perception that high school teaching of mathematics tends to be fairly procedural in South Africa and that students who enter university are better equipped to deal with procedural problems rather than conceptual. In his study, he compared the conceptual and procedural skills of first year calculus students in life sciences. He also investigated students' confidence in handling conceptual and procedural problems. His study indicated that students do not perform better in procedural problems than in conceptual problems. They are also more confident of their ability to handle conceptual problems than to handle procedural problems. Furthermore the study also indicates that, agreeing with common opinion amongst university teachers, students do not have more misconceptions about procedural mathematics than about conceptual issues. This study has bearing with the present study since it deals with procedural skills which is a feature of mathematical fluency.

Moreover, Siadat (2011) found out that time-restricted test provides additional benefit to the educational process. It builds students' concentration skill and focus on the task, thus, honing their mathematical fluency. In this manner, the comfort that one experiences by achieving mathematical fluency far transcends stress and students gain the ability to learn and retain knowledge.

While the students are trained to develop mathematical fluency and conceptual understanding, the researcher has also verified if students' mathematics anxiety is lessened and if their confidence increases as they engage in more challenging mathematical tasks.

3. Methods

In this study, a quasi-experimental non-equivalent pretest-posttest control group design was employed. Two groups: the control group being exposed to worked-out examples on cognitive-demand routine mathematical tasks and the experimental group under cognitive-demand non-routine mathematical tasks, composed the participants. Mental ability of students in each group were categorized based on the result of the Aptitude Test conducted by the Admissions Office. Students' mathematical fluency which includes accuracy, speed and flexibility was measured in terms of their cognitive-demand achievement test scores in Algebra and their conceptual understanding was measured in terms of their cognitive-demand open-ended problem solving test scores. Mathematics anxiety was measured using the standardized mathematics anxiety self-test. To compare the effects of the treatments on students' conceptual understanding, mathematical fluency, and mathematics anxiety, the mean and standard deviation were used. Two-Way Analysis of Covariance was used to determine if the types of mathematical tasks and mental ability have significant influence on the students' conceptual understanding, mathematical fluency, and mathematics anxiety. The students' post opinion on the cognitive-demand mathematical tasks was qualitatively analyzed.

4. Results and Discussion

The result of the analysis of student achievement are shown in the following tables:

Table 1: Summary of Two-Factor ANCOVA with Unequal n of Students' Mathematical Fluency in Terms of Achievement Test Scores

Sources of Variation	SSA	df	MS	F-Ratio	Prob. Value
Factor A					
Mental Ability	56.1	1	56.1	133.57	0.001*
Factor B					
Types of Mathematical Tasks	16.32	1	16.32	38.86	0.003*
Interaction					
A & B	15.97	1	15.97	38.02	0.003*
Error within	14.58	35	0.42		

***Significant at .05 level**

The quantitative analyses of data yielded the result that cognitive-demand algebra tasks using non-routine problems can develop and improve students' mathematical fluency. Students have managed to transfer their previous knowledge to a new situation and problem, making use of more complex mental process. Training students to solve problems which are not yet familiar to them also developed conceptual understanding because students were challenged to think critically as they connect concepts previously studied to solve the present task. In this manner their conceptual understanding and mathematical fluency skills were honed, thus, it became easier for them to tackle the achievement test. When students were trained to do challenging tasks, they developed higher critical thinking skills which resulted to a higher problem-solving performance. Students with average and above-average mental ability have comparable fear towards mathematics. The non-routine mathematical tasks may have triggered mathematics anxiety in them but this also provided additional benefits in their educational process. The non-routine mathematical tasks gave them an impression that they need to struggle and work to be able to solve the difficult task. The difficulty of the task may have scared them but it also gave them the chance to struggle productively, thus honing them to perform better and pushing them to express their thinking and engage in meaningful discourse.

Table 2: Summary of Two-Factor ANCOVA with Unequal n of Students' Conceptual Understanding in Terms of Problem Solving Test Scores

Sources of Variation	SSA	Df	MS	F-Ratio	Prob. Value
Factor A					
Mental Ability	234.44	1	234.44	3907.33	0.001*
Factor B					
Types of Mathematical Tasks	34.21	1	34.21	570.67	0.001*
Interaction					
A & B	54.18	1	54.18	903	0.001*
Error within	2.12	35	0.06		

***Significant at 0.05 level**

With regard to the interaction of students' scores due to the type of mathematical tasks and mental ability, the analysis yielded that generally, non-routine algebra tasks have improved the conceptual understanding and mathematical fluency of students with average and above-average mental ability. Based on their test scores, the effect of the treatment seems to benefit the above-average students than those with average mental ability. However, students with average mental ability who got high achievement scores exemplify that one does not need a very high mental ability to be able to do a difficult task. The key to solving a task is the interest to solve and the desire to do it so especially on the topic that the student had prior knowledge.

Table 3: Summary of Two-Factor ANCOVA with Unequal n of Students' Mathematics Anxiety

Sources of Variation	SSA	Df	MS	F-Ratio	Prob. Value
Factor A					
Mental Ability	0.11	1	0.11	0.44	0.26
Factor B					
Types of Mathematical Tasks	1.13	1	1.13	4.52	0.003*
Interaction					
A & B	0.12	1	0.1	0.48	0.26
Error within	8.72	35	0.25		

***Significant at 0.05 level**

The students' mental ability and the tasks they undertook did not have mixed-effect on their anxiety towards mathematics. In fact, the mean anxiety of the control group remained constant in their pretest and posttest and only the students in the experimental group have built fear towards mathematics in the posttest due to the non-routine mathematical tasks.

A better analysis of the effects of the cognitive-demand mathematical task on students' performance was also based on the students' post opinions. Factors that should be considered and looked into for a successful implementation of cognitive-demand algebra tasks are found to be the appropriateness of time to do the tasks; the mental ability and academic background of the students; and the support of the faculty and their classmates in building and completing the non-routine tasks.

5. Conclusion and Recommendations

On the basis of the findings, mental ability influences the participants' conceptual understanding and mathematical fluency but it does not influence mathematics anxiety. Cognitive-demand algebra tasks using non-routine problems built the participants' anxiety towards mathematics but they have proven to be effective in enhancing their conceptual understanding and mathematical fluency. Participants with above average mental ability have positive perception on non-routine mathematical tasks. With the school-wide attention on this method, schools and universities are encouraged to implement or plan implementation and adaptation of the instructional design to improve students' performance in cognitive-demand mathematics assessments.

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