A New Statistical Averaging Method to Solve Multi-Objective Linear Programming Problem

Samsun Nahar¹, Md. Abdul Alim²

¹Department of Basic Sciences and Humanities, University of Asia Pacific
²Department of Mathematics, Bangladesh University of Engineering and Technology, Dhaka-1000, Bangladesh

Abstract: In this paper, a new statistical averaging technique is suggested to solve MOLPP by using new arithmetic averaging method, new geometric averaging method and new harmonic averaging method. Statistical averaging technique (arithmetic averaging, geometric averaging and harmonic averaging) has also been used to solve the same problem. Among all the solution techniques, the proposed new harmonic averaging method gives better result than all these averaging methods.

Keywords: Linear Programming Problem, new Statistical averaging method, Arithmetic average, Geometric average, Harmonic average

1. Introduction

Linear Programming is without doubt the most natural mechanism for formulating a vast array of problems with modest effort. A Linear Programming Problem is characterized, as the name implies, by linear functions of the unknowns; the objective is linear in the unknowns, and the constraints are linear equalities or linear inequalities in the unknowns. A learning of multi-objective linear programming problem (MOLPP) is introduced in [1] which suggests an method to set up multi-objective function (MOF) under the limitation so that the optimum value of individual problem was greater than zero. Using mean and median the MOF was studied by solving multi-objective programming problem [2].

A popular technique—named as Chandra Sen’s technique—has been used to solve the multi-objective linear fractional programming problem (MOLFPP) [1]. To solve these problems there are several methods discussed in [3] where linear fractional programming is transformed to an equivalent linear program. The paper, [4], shows a useful study about the optimality condition in fractional programming. In [5], a study on MOLFPP has been conducted. Arithmetic mean was used to study MOLPP in [6]. The MOLPP was transformed to the single objective linear programming problem using harmonic mean for values of functions in [7]. In [11], a new geometric average technique was used to solve MOLFPP where multi-objective functions were conversed to a single objective function.

In this paper, in order to extend my work on solution to optimization problem, I have defined a MOLPP and convert to a single objective function. Optimization results using arithmetic mean, geometric mean and harmonic mean have been analyzed and compared. I have suggested new arithmetic averaging method, new geometric averaging method and new harmonic averaging method. Optimization results from new harmonic averaging method have compared with the results from that of all other techniques to solve optimization problems. Optimization results using harmonic averaging and new harmonic averaging method are better than that of obtained from statistical methods. Also, the new statistical averaging method gives better results than the statistical averaging method. From the analysis, I found that MOLPP is best optimized using new harmonic averaging method.

2. Formulation of Problem

The main objective of this study is to solve MOLPP. The mathematical form of MOLPP is given as follows:

Max \( z_1 = C'_1 x + r_1 \)
Max \( z_2 = C'_2 x + r_2 \)
... ... ...
Max \( z_r = C'_r x + r_r \)

s.t. \( A\bar{x} = \bar{b} \) \( \bar{x} \geq 0 \) (1)

Min \( z_{r+1} = C'_{r+1} x + r_{r+1} \)
... ... ...

Min \( z_r = C'_r x + r_r \)

where, \( b \) is \( m \)-dimensional vector of constants, \( x \) is \( n \)-dimensional vector of decision variables and \( A \) is \( m \times n \) matrix of constants. Both types of objective functions must be present.

3. Techniques of MOLPP Solution

At first, the Chandra Sen’s technique has been used to solve MOLPP which gives comparatively poor result of the objective function. Then other existing and suggested techniques have been used solve MOLPP. These techniques are briefly described below.

3.1 Solving MOLPP by Chandra Sen’s Technique:

Using simplex method to solve MOLPP in equation (1), a single value corresponding to each of the objective functions is obtained which are in equation (2).

Max \( z_1 = \phi_1 \)
Max \( z_2 = \phi_2 \)
... ... ...
Max \( z_r = \phi_r \)

Min \( z_{r+1} = \phi_{r+1} \)
... ... ...

Volume 6 Issue 8, August 2017

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Paper ID: ART20175911
DOI: 10.21275/ART20175911
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Min \( z_i = \varphi_i \)

where \( \varphi_1, \varphi_2, \cdots, \varphi_i \) are values of the objective functions.

These values are used in Chandra Sen’s technique to obtain a single objective function as shown in equation (3).

\[
\max \quad z = \sum_{i=1}^{s} z_i \frac{1}{|\varphi_i|} - \sum_{i=r+1}^{s} z_i \frac{1}{|\varphi_i|}
\]

(3)

where, \( \varphi_i \neq 0, \quad i = 1, 2, \cdots, s \). Subject to the constraints of equation (1) and the optimum value of the objective functions \( \varphi_i \) may be positive or negative.

3.2 Applied Techniques

3.2.1 Statistical Averaging Method

\[
\max \quad z = \sum_{i=1}^{r} A.M. (A_i) - \sum_{i=r+1}^{s} A.M. (A_i)
\]

(4)

\[
\max \quad z = \sum_{i=1}^{r} G.M. (A_i) - \sum_{i=r+1}^{s} G.M. (A_i)
\]

(5)

\[
\max \quad z = \sum_{i=1}^{r} H.M. (A_i) - \sum_{i=r+1}^{s} H.M. (A_i)
\]

(6)

Where, \( A_i = |\varphi_i|, \quad i = 1 \cdots r \) and \( A_i = |\varphi_i|, \quad i = r+1 \cdots s \),

And A.M. is Arithmetic mean, G.M. is Geometric mean and H.M. is Harmonic mean.

3.2.2 Solving MOLPP by using the new arithmetic averaging technique:

Let \( m_1 = \min\{A_i\} \), where \( A_i = |\varphi_i| \),

\( \varphi_i \) is maximum value of \( z_i \), \( i = 1 \cdots r \)

\( m_2 = \min\{A_i\} \), where \( A_i = |\varphi_i| \),

\( \varphi_i \) is minimum value of \( z_i \), \( i = r+1 \cdots s \)

\[
A.Av = \frac{m_1 + m_2}{2}
\]

so

\[
\max \quad z = \left( \sum_{i=1}^{r} z_i - \sum_{i=r+1}^{s} z_i \right) / A.Av
\]

(7)

3.2.3 Solving MOLPP by using the new geometric averaging technique:

Using \( m_1 \) and \( m_2 \), we can find the geometric average as follows:

\[
G.Av = \sqrt{m_1 m_2}
\]

\[
\max \quad z = \left( \sum_{i=1}^{r} z_i - \sum_{i=r+1}^{s} z_i \right) / G.Av
\]

(8)

3.2.4 Solving MOLPP by using the new harmonic averaging technique:

Using \( m_1 \) and \( m_2 \), we can find the harmonic average as follows:

\[
H.Av = \frac{2}{m_1 + \frac{m_2}{1}}
\]

\[
\max \quad z = \left( \sum_{i=1}^{r} z_i - \sum_{i=r+1}^{s} z_i \right) / H.Av
\]

(9)

3.3 Algorithm for new Arithmetic, Geometric and Harmonic averaging technique:

Step 1: Find the value of each of individual objective functions which is to be maximized or minimized.

Step 2: Solve the first objective problem by simplex method.

Step 3: Check the feasibility of the solution in step 2. If it is feasible then go to step 4. Otherwise, use dual simplex method to remove infeasibility.

Step 4: Assign a name to the optimum value of the first objective function \( z_i \) say \( \varphi_1 \).

Step 5: Repeat the step 2, \( i = 1, 2, \cdots s \).

Step 6: Select \( m_1 = \min\{A_i\} \), \( m_2 = \min\{A_i\} \),

\[ A.Av = \frac{m_1 + m_2}{2}, \; G.Av = \sqrt{m_1 m_2} \text{ and } H.Av = \frac{2}{m_1 + \frac{m_2}{1}} \]

Step 7: Optimize the combined objective function with the same constraints

\[
\max \quad z = \left( \sum_{i=1}^{r} z_i - \sum_{i=r+1}^{s} z_i \right) / A.Av
\]

\[
\max \quad z = \left( \sum_{i=1}^{r} z_i - \sum_{i=r+1}^{s} z_i \right) / G.Av \text{ and }
\]

\[
\max \quad z = \left( \sum_{i=1}^{r} z_i - \sum_{i=r+1}^{s} z_i \right) / H.Av
\]

3.4 Program solution for new Arithmetic, Geometric and Harmonic averaging technique:

To solve MOLPP by proposed method, the following program can be used. For this, let \( \varphi_1 \) = value of objective functions which is to be maximized

\( \varphi_2 \) = value of objective functions which is to be minimized

So

\[ A_i = |\varphi_i|, \quad \forall \; i = 1 \cdots r; \]

\[ A_i = |\varphi_i|, \quad \forall \; i = r+1 \cdots s \]

\[ SM = \sum_{i=1}^{r} z_i; \quad SN = \sum_{i=r+1}^{s} z_i \]

\[ m_1 = \min\{A_i\}; \quad m_2 = \min\{A_i\} \]

\[
\max \quad z = (SM - SN) / A.Av
\]

\[
\max \quad z = (SM - SN) / G.Av
\]

\[
\max \quad z = (SM - SN) / H.Av
\]
4. Mathematical Illustration

Example: Here is an MOLPP with the constraints shown in equation (10). The values of these objective functions can be obtained using simplex method first. Then with these values, a single objective function is developed using Chandra Sen’s technique.

Multi-objective functions:

\[
\begin{align*}
\text{max } z_1 &= x_1 + 2x_2 \\
\text{max } z_2 &= x_1 \\
\text{min } z_3 &= -2x_1 - 3x_2 \\
\text{min } z_4 &= -x_2 \\
\end{align*}
\]

s.t. \[6x_1 + 8x_2 \leq 48\]

\[x_1 + x_2 \geq 3\]

\[x_1 \leq 4\]

\[x_2 \leq 3\]

\[x_1, x_2 \geq 0\]

Introducing the slack variables \(s_1, s_2, s_3, s_4\).

Standard form is

\[
\begin{align*}
6x_1 + 8x_2 + s_1 &= 48 \\
-x_1 - x_2 + s_2 &= -3 \\
x_1 + s_3 &= 4 \\
x_2 + s_4 &= 3 \\
\end{align*}
\]

\[s_1, s_2, s_3, s_4, x_1, x_2 \geq 0\]

For the first objective function, we get

\[
\text{max } z_1 = x_1 + 2x_2
\]

Subject to

\[
\begin{align*}
6x_1 + 8x_2 + s_1 &= 48 \\
-x_1 - x_2 + s_2 &= -3 \\
x_1 + s_3 &= 4 \\
x_2 + s_4 &= 3 \\
\end{align*}
\]

\[s_1, s_2, s_3, s_4, x_1, x_2 \geq 0\]

Setting the decision variables \(x_1, x_2 = 0\), the feasible solution is

\[
\begin{align*}
x_1 &= 0 \\
x_2 &= 0 \\
z_1 &= 0
\end{align*}
\]

Thus the optimal solution from Table 2 is \(x_1 = 4\).

For second objective function

\[
\text{max } z_2 = x_1
\]

Subject to

\[
\begin{align*}
6x_1 + 8x_2 + s_1 &= 48 \\
-x_1 - x_2 + s_2 &= -3 \\
x_1 + s_3 &= 4 \\
x_2 + s_4 &= 3 \\
\end{align*}
\]

\[s_1, s_2, s_3, s_4, x_1, x_2 \geq 0\]

Thus the optimal solution from Table 2 is \(x_2 = 3\) and \(z_{\text{max}} = 4\).

For third objective function:

\[
\text{min } z_3 = -2x_1 - 3x_2
\]

Subject to

\[
\begin{align*}
6x_1 + 8x_2 &\leq 48 \\
-x_1 - x_2 &\leq -3 \\
x_1 + s_3 &= 4 \\
x_2 + s_4 &= 3 \\
\end{align*}
\]

\[s_1, s_2, s_3, s_4, x_1, x_2 \geq 0\]

\[
\begin{align*}
x_1 &\leq 4 \\
x_2 &\leq 3 \\
x_1, x_2 &\geq 0
\end{align*}
\]
Which implies $\max z_3 = 2x_1 + 3x_2$

Subject to

$6x_1 + 8x_2 \leq 48$

$-x_1 - x_2 \leq -3$

$x_1 \leq 4$

$x_2 \leq 3$

$x_1, x_2 \geq 0$

Thus the optimal solution from Table 3 is $x_1 = 4, x_2 = 3$ and $z_{\text{min}} = -3$

Thus the optimum values of the objective functions with same constraints are given in Table 5.

| I | $\phi_i$ | $x_i$ | $A_{\text{AA}} = |\phi_i|$ | $A_{\text{AL}} = |\phi_i|$ |
|---|---|---|---|---|
| 1 | 10 | (4, 3) | 10 |
| 2 | 4 | (4, 3) | 4 |
| 3 | -17 | (4, 3) | 17 |
| 4 | -3 | (4, 3) | 3 |

By Chandra Sen’s approach,

Max $Z = \max \sum z_i - \sum \frac{z_k}{r_k}$

Max $Z = x_1 + 2x_2 + \frac{x_1}{4} = \frac{-2x_1 - 3x_2}{17} - \frac{x_2}{3}$

Max $Z = 0.4676x_1 + 0.7098x_2$ with $6x_1 + 8x_2 \leq 48$

$-x_1 - x_2 \leq -3$

$x_1 \leq 4$

$x_2 \leq 3$

$x_1, x_2 \geq 0$

Thus from Table 6 $Z_{\text{max}} = 3.9998$ with $x_1 = 4, x_2 = 3$

Using Arithmetic averaging approach,

From Table-5, we get

$A.M. (10, 4) = 7, \quad A.M. (17, 3) = 10$

So, by using equation (7) we have

Max $Z = \sum \frac{z_i}{A.M.(A_i)} - \sum \frac{z_i}{A.M.(AL_i)}$

$= \frac{1}{7} \sum z_i - \frac{1}{10} \sum z_i$

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<th>Cj</th>
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<th>$x_2$</th>
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<th>Cj</th>
<th>Basis</th>
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<th>$x_2$</th>
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Thus the optimal solution from Table 3 is $x_1 = 4, x_2 = 3$ and $z_{\text{min}} = -17$

For fourth objective function

$\min z_4 = -x_2$

$6x_1 + 8x_2 \leq 48$

$-x_1 - x_2 \leq -3$

$x_1 \leq 4$

$x_2 \leq 3$

$x_1, x_2 \geq 0$

Which implies

$\max z_4 = x_2$

$6x_1 + 8x_2 \leq 48$

$-x_1 - x_2 \leq -3$

$x_1 \leq 4$

$x_2 \leq 3$

$x_1, x_2 \geq 0$

Thus the optimal solution from Table 6 is $x_1 = 4, x_2 = 3$.
\[= \frac{1}{7}[x_1 + 2x_2 + x_3] - \frac{1}{10}[-2x_1 - 3x_2 - x_3] = 0.4857x_1 + 0.6857x_2 \]

\[s/t \quad 6x_1 + 8x_2 \leq 48 \]
\[- x_1 - x_2 \leq -3 \]
\[x_1 \leq 4 \]
\[x_2 \leq 3 \]
\[x_1, x_2 \geq 0 \]

**Table 7**

<table>
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<th>(C_B)</th>
<th>(C_I)</th>
<th>Benefit</th>
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<td>0</td>
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Thus from Table 7, \(Z_{max} = 3.99999\) with \(x_1 = 4, x_2 = 3\)

Using Geometric averaging approach

From Table-5, we get

\[G.M.(10,4) = \sqrt[10]{4} = 6.324, \quad G.M.(17,3) = \sqrt[17]{3} = 7.1414 \]

So, by using equation (8) we have

\[\text{Max } Z = \frac{\sum_{i=1}^{r} G.M.(A_i)}{r} - \frac{\sum_{i=1}^{r} G.M.(A_L)}{r} \]

\[= \frac{1}{6.324} (2x_1 + 2x_2) + \frac{1}{7.1414} (2x_1 + 4x_2) = 0.5963x_1 + 0.8763x_2 \]

\[s/t \quad 6x_1 + 8x_2 \leq 48 \]
\[- x_1 - x_2 \leq -3 \]
\[x_1 \leq 4 \]
\[x_2 \leq 3 \]
\[x_1, x_2 \geq 0 \]

**Table 8**

<table>
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<th>(C_B)</th>
<th>(C_I)</th>
<th>Benefit</th>
<th>0.596</th>
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Thus from Table 8, \(Z_{max} = 5.0141\) with \(x_1 = 4, x_2 = 3\)

Using Harmonic averaging approach,

From Table-5, we get

\[H.M.(10,4) = 5.7143, \quad H.M.(17,3) = 5.1 \]

So, by using equation (9) we have

\[\text{Max } Z = \frac{\sum_{i=1}^{r} H.M.(A_i)}{r} - \frac{\sum_{i=1}^{r} H.M.(A_L)}{r} \]

\[= \frac{1}{5.7143} (2x_1 + 2x_2) + \frac{1}{5.1} (2x_1 + 4x_2) = 0.7421x_1 + 1.21342x_2 \]

\[s/t \quad 6x_1 + 8x_2 \leq 48 \]
\[- x_1 - x_2 \leq -3 \]
\[x_1 \leq 4 \]
\[x_2 \leq 3 \]
\[x_1, x_2 \geq 0 \]

**Table 9**

<table>
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<td>(s_2)</td>
<td>0</td>
<td>0</td>
<td>1/6</td>
<td>1</td>
<td>0</td>
<td>-1/3</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>(C_I + s)</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>6.3710</td>
</tr>
</tbody>
</table>

Thus from Table 9, \(Z_{max} = 6.3710\) with \(x_1 = 4, x_2 = 3\)

**4.1 New Arithmetic averaging technique:**

Let \(m_1 = 4\), \(m_2 = 3\); \(\frac{m_1 + m_2}{2} = 3.5\)

\[\text{max } Z = \left( \frac{\sum_{i=1}^{r} z_i - \sum_{i=1}^{r} z_i}{r} \right) / \text{AAv} \]

\[= \frac{1}{3.5} [4x_1 + 6x_2] = 1.1428x_1 + 1.7143x_2 \]

\[s/t \quad 6x_1 + 8x_2 \leq 48 \]
\[- x_1 - x_2 \leq -3 \]
\[x_1 \leq 4 \]
\[x_2 \leq 3 \]
\[x_1, x_2 \geq 0 \]

**Table 10**

<table>
<thead>
<tr>
<th>(C_B)</th>
<th>(C_I)</th>
<th>Benefit</th>
<th>1.1428</th>
<th>1.7143</th>
<th>0</th>
<th>0</th>
<th>0</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1428</td>
<td>(x_1)</td>
<td>1</td>
<td>0</td>
<td>1/6</td>
<td>0</td>
<td>0</td>
<td>-4/3</td>
<td>4</td>
</tr>
<tr>
<td>1.7143</td>
<td>(x_2)</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>0</td>
<td>(s_3)</td>
<td>0</td>
<td>0</td>
<td>-1/6</td>
<td>0</td>
<td>1</td>
<td>4/3</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>(s_2)</td>
<td>0</td>
<td>0</td>
<td>1/6</td>
<td>1</td>
<td>0</td>
<td>-1/3</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>(C_I + s)</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>7.1414</td>
</tr>
</tbody>
</table>

Thus from Table 10, \(Z_{max} = 9.7141\) with \(x_1 = 4, x_2 = 3\)
4.2 New Geometric Averaging Technique:

$$\max z = \left( \sum_{i=1}^{n} z_i - \sum_{i=1}^{n} z_i \right) / G.Av$$

$$\sqrt{4 \times 3} = 3.4641$$

$$= \frac{1}{3.4641} \left[ 2x_1 + 2x_2 + 2x_3 + 4x_4 \right] = 1.1547x_1 + 1.73205x_2$$

$$s/t \quad 6x_1 + 8x_2 \leq 48$$

$$-x_1 - x_2 \leq -3$$

$$x_1 \leq 4$$

$$x_2 \leq 3$$

$$x_1, x_2 \geq 0$$

<table>
<thead>
<tr>
<th>$C_B$</th>
<th>$C_j$</th>
<th>Basis</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_3$</th>
<th>$x_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1547</td>
<td>$x_1$</td>
<td>1</td>
<td>0</td>
<td>1/6</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1.73205</td>
<td>$x_2$</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>$s_3$</td>
<td>0</td>
<td>0</td>
<td>-1/6</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>$s_2$</td>
<td>0</td>
<td>0</td>
<td>1/6</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

Thus from Table 11 $Z_{max} = 9.81415$ with $x_1 = 4, x_2 = 3$

4.3 New Harmonic Averaging Technique

$$\max z = \left( \sum_{i=1}^{n} z_i - \sum_{i=1}^{n} z_i \right) / H.Av$$

$$\frac{2}{\frac{1}{4} + \frac{3}{4}} = 3.4483$$

<table>
<thead>
<tr>
<th>Chandra Sen’s Approach</th>
<th>Statistical Averaging Method</th>
<th>New Statistical Averaging Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max</td>
<td>Z=3.9999 with $x_1=4, x_2=3$</td>
<td>Max</td>
</tr>
<tr>
<td>Max</td>
<td>Z=9.81415 with $x_1=4, x_2=3$</td>
<td>Max</td>
</tr>
</tbody>
</table>

5. Conclusion

In this paper, different methods such as Chandra Sen’s approach and proposed statistical averaging methods are used to solve a MOLPP, and the results are compared in Table-13. In statistical average method, harmonic, geometric and arithmetic average approaches are proposed. We also proposed a new statistical average approach to solve the problem. It is observed that statistical average method results better optimization than Chandra Sen’s approach of the MOLPP. The proposed new statistical averaging methods optimize the problem better than that of statistical average method. We also found that harmonic average technique is suited for optimizing MOLPP better that that of arithmetic average and geometric average techniques.

6. Conflict of Interest

Statement: The authors declare here that there is no conflict of interest regarding the publication of this paper.

References


