

A New Statistical Averaging Method to Solve Multi-Objective Linear Programming Problem

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Abstract: *In this paper, a new statistical averaging technique is suggested to solve MOLPP by using new arithmetic averaging method, new geometric averaging method and new harmonic averaging method. Statistical averaging technique (arithmetic averaging, geometric averaging and harmonic averaging) has also been used to solve the same problem. Among all the solution techniques, the proposed new harmonic averaging method gives better result than all these averaging methods.*

Keywords: Linear Programming Problem, new Statistical averaging method, Arithmetic average, Geometric average, Harmonic average

1. Introduction

Linear Programming is without doubt the most natural mechanism for formulating a vast array of problems with modest effort. A Linear Programming Problem is characterized, as the name implies, by linear functions of the unknowns; the objective is linear in the unknowns, and the constraints are linear equalities or linear inequalities in the unknowns. A learning of multi-objective linear programming problem (MOLPP) is introduced in [1] which suggests an method to set up multi-objective function (MOF) under the limitation so that the optimum value of individual problem was greater than zero. Using mean and median the MOF was studied by solving multi-objective programming problem [2].

A popular technique-named as Chandra Sen's technique-has been used to solve the multi-objective linear fractional programming problem (MOLFPP) [1]. To solve these problems there are several methods discussed in [3] where linear fractional programming is transformed to an equivalent linear program. The paper, [4], shows a useful study about the optimality condition in fractional programming. In [5], a study on MOLFPP has been conducted. Arithmetic mean was used to study MOLPP in [6]. The MOLPP was transformed to the single objective linear programming problem using harmonic mean for values of functions in [7]. In [11], a new geometric average technique was used to solve MOLFPP where multi-objective functions were converted to a single objective function.

In this paper, in order to extend my work on solution to optimization problem, I have defined a MOLPP and convert to a single objective function. Optimization results using arithmetic mean, geometric mean and harmonic mean have been analyzed and compared. I have suggested new arithmetic averaging method, new geometric averaging method and new harmonic averaging method. Optimization results from new harmonic averaging method have compared with the results from that of all other techniques to solve optimization problems. Optimization results using harmonic averaging and new harmonic averaging method are better than that of obtained from statistical methods. Also, the new statistical averaging method gives better

results than the statistical averaging method. From the analysis, I found that MOLPP is best optimized using new harmonic averaging method.

2. Formulation of Problem

The main objective of this study is to solve MOLPP. The mathematical form of MOLPP is given as follows:

$$\begin{aligned}
 \text{Max } z_1 &= C_1^t x + r_1 \\
 \text{Max } z_2 &= C_2^t x + r_2 \\
 \dots \dots \dots \dots \dots & \\
 \text{Max } z_r &= C_r^t x + r_r \\
 \text{Min } z_{r+1} &= C_{r+1}^t x + r_{r+1} \\
 \dots \dots \dots \dots \dots & \\
 \text{Min } z_s &= C_s^t x + r_s
 \end{aligned}
 \quad \text{s/t } \begin{aligned}
 A\bar{x} &= \bar{b} \\
 \bar{x} &\geq 0
 \end{aligned} \quad (1)$$

where, \mathbf{b} is m -dimensional vector of constants, \mathbf{x} is n -dimensional vector of decision variables and A is $m \times n$ matrix of constants. Both types of objective functions must be present.

3. Techniques of MOLPP Solution

At first, the Chandra Sen's technique has been used to solve MOLPP which gives comparatively poor result of the objective function. Then other existing and suggested techniques have been used solve MOLPP. These techniques are briefly described below.

3.1 Solving MOLPP by Chandra Sen's Technique:

Using simplex method to solve MOLPP in equation (1), a single value corresponding to each of the objective functions is obtained which are in equation (2).

$$\begin{aligned}
 \text{Max } z_1 &= \phi_1 \\
 \text{Max } z_2 &= \phi_2 \\
 \dots \dots \dots \dots \dots & \\
 \text{Max } z_r &= \phi_r \\
 \text{Min } z_{r+1} &= \phi_{r+1} \\
 \dots \dots \dots \dots \dots &
 \end{aligned} \quad (2)$$

Min $z_s = \varphi_s$;

where $\varphi_1, \varphi_2, \dots, \varphi_s$ are values of the objective functions.

These values are used in Chandra Sen's technique to obtain a single objective function as shown in equation (3).

$$\max z = \sum_{i=1}^r \frac{z_i}{|\varphi_i|} - \sum_{i=r+1}^s \frac{z_i}{|\varphi_i|} \quad (3)$$

where, $\varphi_i \neq 0, i = 1, 2, \dots, s$. Subject to the constraints of equation (1) and the optimum value of the objective functions φ_i may be positive or negative.

3.2 Applied Techniques

3.2.1 Statistical Averaging Method

$$\max z = \sum_{i=1}^r \frac{z_i}{A.M.(AA_i)} - \sum_{i=r+1}^s \frac{z_i}{A.M.(AL_i)} \quad (4)$$

$$\max z = \sum_{i=1}^r \frac{z_i}{G.M.(AA_i)} - \sum_{i=r+1}^s \frac{z_i}{G.M.(AL_i)} \quad (5)$$

$$\max z = \sum_{i=1}^r \frac{z_i}{H.M.(AA_i)} - \sum_{i=r+1}^s \frac{z_i}{H.M.(AL_i)} \quad (6)$$

Where, $AA_i = |\varphi_i|, i = 1 \dots r$ and $AL_i = |\varphi_i|,$

$i = 1 + r \dots s$

And A.M. is Arithmetic mean, G.M. is Geometric mean and H.M. is Harmonic mean.

3.2.2 Solving MOLPP by using the new arithmetic averaging technique:

Let $m_1 = \min \langle AA_i \rangle$, where $AL_i = |\varphi_i|,$

φ_i is maximum value of $z_i, i = 1 \dots r$

$m_2 = \min \langle AL_i \rangle$, where $AA_i = |\varphi_i|,$

φ_i is minimum value of $z_i, i = r + 1 \dots s$

$$A.Av = \frac{m_1 + m_2}{2} \text{ so}$$

$$\max z = \left(\sum_{i=1}^r z_i - \sum_{i=r+1}^s z_i \right) / A.Av \quad (7)$$

3.2.3 Solving MOLPP by using the new geometric averaging technique:

Using m_1 and m_2 , we can find the geometric average as follows:

$$G.Av = \sqrt{m_1 m_2}$$

$$\max z = \left(\sum_{i=1}^r z_i - \sum_{i=r+1}^s z_i \right) / G.Av \quad (8)$$

3.2.4 Solving MOLPP by using the new harmonic averaging technique:

Using m_1 and m_2 , we can find the harmonic average as follows:

$$H.Av = \frac{2}{\frac{1}{m_1} + \frac{1}{m_2}}$$

$$\max z = \left(\sum_{i=1}^r z_i - \sum_{i=r+1}^s z_i \right) / H.Av \quad (9)$$

3.3 Algorithm for new Arithmetic, Geometric and Harmonic averaging technique:

Step 1: Find the value of each of individual objective functions which is to be maximized or minimized.

Step 2: Solve the first objective problem by simplex method.

Step 3: Check the feasibility of the solution in step 2. If it is feasible then go to step 4. Otherwise, use dual simplex method to remove infeasibility.

Step 4: Assign a name to the optimum value of the first objective function z_1 say φ_1 .

Step 5: Repeat the step 2, $i=1, 2, \dots$

Step 6:

$$\text{Select } m_1 = \min \langle AA_i \rangle, m_2 = \min \langle AL_i \rangle$$

$$i = 1 \dots s$$

$$A.Av = \frac{m_1 + m_2}{2}, G.Av = \sqrt{m_1 m_2} \text{ and } H.Av = \frac{2}{\frac{1}{m_1} + \frac{1}{m_2}}$$

Step 7: Optimize the combined objective function with the same constraints

$$\max z = \left(\sum_{i=1}^r z_i - \sum_{i=r+1}^s z_i \right) / A.Av$$

$$\max z = \left(\sum_{i=1}^r z_i - \sum_{i=r+1}^s z_i \right) / G.Av \text{ and}$$

$$\max z = \left(\sum_{i=1}^r z_i - \sum_{i=r+1}^s z_i \right) / H.Av$$

3.4 Program solution for new Arithmetic, Geometric and Harmonic averaging technique:

To solve MOLPP by proposed method, the following program can be used. For this, let

φA_i = value of objective functions which is to be maximized

φL_i = value of objective functions which is to be minimized

So

$$AA_i = |\varphi A_i|; \forall i = 1 \dots r;$$

$$AL_i = |\varphi L_i|; \forall i = 1 + r \dots s$$

$$SM = \sum_{i=1}^r z_i; \quad SN = \sum_{i=r+1}^s z_i$$

$$m_1 = \min \langle AA_i \rangle; \quad m_2 = \min \langle AL_i \rangle$$

$$\max z = (SM - SN) / A.Av$$

$$\max z = (SM - SN) / G.Av$$

$$\max z = (SM - SN) / H.Av$$

4. Mathematical Illustration

Example: Here is an MOLPP with the constraints shown in equation (10). The values of these objective functions can be obtained using simplex method first. Then with these values, a single objective function is developed using Chandra Sen's technique.

Multi-objective functions:

$$\max z_1 = x_1 + 2x_2$$

$$\max z_2 = x_1$$

$$\min z_3 = -2x_1 - 3x_2$$

$$\min z_4 = -x_2$$

$$\begin{aligned} \text{s/t } & 6x_1 + 8x_2 \leq 48 \\ & x_1 + x_2 \geq 3 \\ & x_1 \leq 4 \\ & x_2 \leq 3 \\ & x_1, x_2 \geq 0 \end{aligned} \quad (10)$$

Introducing the slack variables s_1, s_2, s_3, s_4

Standard form is

$$\begin{aligned} 6x_1 + 8x_2 + s_1 &= 48 \\ -x_1 - x_2 + s_2 &= -3 \\ x_1 + s_3 &= 4 \\ x_2 + s_4 &= 3 \\ s_1, s_2, s_3, s_4, x_1, x_2 &\geq 0 \end{aligned}$$

For the first objective function, we get

$$\max z_1 = x_1 + 2x_2$$

Subject to $6x_1 + 8x_2 + s_1 = 48$

$$\begin{aligned} -x_1 - x_2 + s_2 &= -3 \\ x_1 + s_3 &= 4 \\ x_2 + s_4 &= 3 \end{aligned}$$

$$s_1, s_2, s_3, s_4, x_1, x_2 \geq 0$$

Setting the decision variables $x_1, x_2 = 0$, the feasible solution is

$$\begin{aligned} x_1 = 0 & \quad s_1 = 48, \quad s_2 = -3 \\ x_2 = 0 & \quad s_3 = 4, \quad s_4 = 3 \\ z_1 = 0 & \end{aligned}$$

Table 1

C _B	C _J Basis	1	2	0	0	0	0		
		x ₁	x ₂	s ₁	s ₂	s ₃	s ₄		
0	s ₁	6	8	1	0	0	0	48	6
0	s ₂	-1	-1	0	1	0	0	-3	3
0	s ₃	1	0	0	0	1	0	4	
0	s ₄	0	1	0	0	0	1	3	3

	C _J -E _j	1	2	0	0	0	0	0	0
0	s ₁	-2	0	1	8	0	0	24	3
2	x ₂	1	1	0	-1	0	0	3	3
0	s ₃	1	0	0	0	1	0	4	
0	s ₄	-1	0	0	1	0	1	0	0
	C _J -E _j	-1	0	0	2	0	0	6	
0	s ₁	6	0	1	0	0	-8	24	4
2	x ₂	0	1	0	0	0	1	3	
0	s ₃	1	0	0	0	1	0	4	4
0	s ₂	-1	0	0	1	0	1	0	
	C _J -E _j	1	0	0	0	0	-2	6	
1	x ₁	1	0	1/6	0	0	-4/3	4	
2	x ₂	0	1	0	0	0	1	3	
0	s ₃	0	0	-1/6	0	1	4/3	0	
0	s ₂	0	0	1/6	1	0	-1/3	4	
	C _J -E _j	0	0	-1/6	0	0	-2/3	10	

As C_J-E_j is not positive under any column in the Table.1 Thus we get optimal solution. From Table 1, $x_1=4, x_2=3, Z_{max}=10$

Table 2

C _B	C _J Basis	1	0	0	0	0	0		
		x ₁	x ₂	s ₁	s ₂	s ₃	s ₄		
1	x ₁	1	0	1/6	0	0	-4/3	4	
0	x ₂	0	1	0	0	0	1	3	
0	s ₃	0	0	-1/6	0	1	4/3	0	
0	s ₂	0	0	1/6	1	0	-1/3	4	
	C _J -E _j	0	0	-1/6	0	0	4/3	4	
1	x ₁	1	0	0	0	1	0	4	
0	x ₂	0	1	1/8	0	-3/4	0	3	
0	s ₄	0	0	-1/8	0	3/4	1	0	
0	s ₂	0	0	-5/24	1	1/4	0	4	
	C _J -E _j	0	0	0	0	-1	0	4	

For second objective function

$$\max z_2 = x_1$$

Subject to: $6x_1 + 8x_2 + s_1 = 48$

$$\begin{aligned} -x_1 - x_2 + s_2 &= -3 \\ x_1 + s_3 &= 4 \\ x_2 + s_4 &= 3 \end{aligned}$$

$$s_1, s_2, s_3, s_4, x_1, x_2 \geq 0$$

Thus the optimal solution from Table 2 is $x_1 = 4,$

$$x_2 = 3 \text{ and } z_{\max} = 4$$

For third objective function:

$$\min z_3 = -2x_1 - 3x_2$$

Subject to $6x_1 + 8x_2 \leq 48$

$$\begin{aligned} -x_1 - x_2 &\leq -3 \\ x_1 &\leq 4 \\ x_2 &\leq 3 \\ x_1, x_2 &\geq 0 \end{aligned}$$

Which implies $\max z_3 = 2x_1 + 3x_2$

Subject to

$$6x_1 + 8x_2 \leq 48$$

$$-x_1 - x_2 \leq -3$$

$$x_1 \leq 4$$

$$x_2 \leq 3$$

$$x_1, x_2 \geq 0$$

Table 3

C _B	C _j Basis	2	3	0	0	0	0	
		x ₁	x ₂	s ₁	s ₂	s ₃	s ₄	
2	x ₁	1	0	1/6	0	0	-4/3	4
3	x ₂	0	1	0	0	0	1	3
0	s ₃	0	0	-1/6	0	1	4/3	0
0	s ₂	0	0	1/6	1	0	-1/3	4
	C _j -E _j	0	0	-1/3	0	0	1/3	17
							↑	
2	x ₁	1	0	0	0	1	0	4
3	x ₂	0	1	1/8	0	-3/4	0	3
0	s ₄	0	0	-1/8	0	3/4	1	0
0	s ₂	0	0	1/8	1	1/4	0	4
	C _j -E _j	0	0	-3/8	0	1/4	0	17
							↑	
2	x ₁	1	0	1/6	0	0	-4/3	4
3	x ₂	0	1	0	0	0	1	3
0	s ₃	0	0	-1/6	0	1	4/3	0
0	s ₂	0	0	1/6	1	0	-1/3	4
	C _j -E _j	0	0	-1/3	0	0	-1/3	17

Thus the optimal solution from Table 3 is $x_1 = 4$, $x_2 = 3$ and $z_{\min} = -17$

For fourth objective function

$$\min z_4 = -x_2$$

$$6x_1 + 8x_2 \leq 48$$

$$-x_1 - x_2 \leq -3$$

$$x_1 \leq 4$$

$$x_2 \leq 3$$

$$x_1, x_2 \geq 0$$

Which implies

$$\max z_4 = x_2$$

$$6x_1 + 8x_2 \leq 48$$

$$-x_1 - x_2 \leq -3$$

$$x_1 \leq 4$$

$$x_2 \leq 3$$

$$x_1, x_2 \geq 0$$

Table 4

C _B	C _j Basis	0	1	0	0	0	0	
		x ₁	x ₂	s ₁	s ₂	s ₃	s ₄	
0	x ₁	1	0	1/6	0	0	-4/3	4
1	x ₂	0	1	0	0	0	1	3
0	s ₃	0	0	-1/6	0	1	4/3	0
0	s ₂	0	0	1/6	1	0	-1/3	4
	C _j -E _j	0	0	0	0	0	-1	3

Thus the optimal solution from Table 4 is $x_1 = 4$,

$$x_2 = 3 \text{ and } z_{\min} = -3$$

Thus the optimum values of the objective functions with same constraints are given in Table 5.

Table 5

I	φ_i	x _i	AA _i = φ_i	AL _i = φ_i
1	10	(4, 3)	10	
2	4	(4, 3)	4	
3	-17	(4, 3)		17
4	-3	(4, 3)		3

By Chandra Sen's approach,

$$\text{Max } Z = \sum_{k=1}^r \frac{z_k}{|\varphi_k|} - \sum_{k=r+1}^s \frac{z_k}{|\varphi_k|}$$

$$\text{Max } Z = \frac{x_1 + 2x_2}{10} + \frac{x_1}{4} - \left(\frac{-2x_1 - 3x_2}{17} - \frac{x_2}{3} \right)$$

Max Z = 0.4676x₁ + 0.7098x₂ with

$$6x_1 + 8x_2 \leq 48$$

$$-x_1 - x_2 \leq -3$$

$$x_1 \leq 4$$

$$x_2 \leq 3$$

$$x_1, x_2 \geq 0$$

Table 6

C _B	C _j Basis	0.4676	0.7098	0	0	0	0	
		x ₁	x ₂	s ₁	s ₂	s ₃	s ₄	
0.4676	x ₁	1	0	1/6	0	0	-4/3	4
0.7098	x ₂	0	1	0	0	0	1	3
0	s ₃	0	0	-1/6	0	1	4/3	0
0	s ₂	0	0	1/6	1	0	-1/3	4
	C _j -E _j	0	0	-0.0779	0	0	-0.0863	3.9998

Thus from Table 6 Z_{max} = 3.9998 with $x_1=4$, $x_2=3$

Using Arithmetic averaging approach,

From Table-5, we get

$$A.M. (10, 4) = 7, \quad A.M. (17, 3) = 10$$

So, by using equation (7) we have

$$\begin{aligned} \text{Max } Z &= \sum_{i=1}^r \frac{z_i}{A.M(AA_i)} - \sum_{i=r+1}^s \frac{z_i}{A.M(AL_i)} \\ &= \frac{1}{7} \sum z_i - \frac{1}{10} \sum z_i \end{aligned}$$

$$= \frac{1}{7}[x_1 + 2x_2 + x_1] - \frac{1}{10}[-2x_1 - 3x_2 - x_2]$$

$$= 0.4857x_1 + 0.6857x_2$$

s/t

$$6x_1 + 8x_2 \leq 48$$

$$-x_1 - x_2 \leq -3$$

$$x_1 \leq 4$$

$$x_2 \leq 3$$

$$x_1, x_2 \geq 0$$

Table 7

C _B	C _j Basis	0.4857	0.6857	0	0	0	0	
		x ₁	x ₂	s ₁	s ₂	s ₃	s ₄	
0.4857	x ₁	1	0	1/6	0	0	-4/3	4
0.6857	x ₂	0	1	0	0	0	1	3
0	s ₃	0	0	-1/6	0	1	4/3	0
0	s ₂	0	0	1/6	1	0	-1/3	4
	C _j -E _j	0	0	-0.08095	0	0	-0.0381	3.9999

Thus from Table 7 $Z_{max} = 3.9999$ with $x_1=4, x_2=3$
 Using Geometric averaging approach
 From Table-5, we get

$$G.M.(10, 4) = \sqrt{10 \times 4} = \sqrt{40} = 6.324,$$

$$G.M.(17, 3) = \sqrt{17 \times 3} = \sqrt{51} = 7.1414$$

So, by using equation (8) we have

$$\text{Max } Z = \sum_{i=1}^r \frac{z_i}{G.M.(AA_i)} - \sum_{i=r+1}^s \frac{z_i}{G.M.(AL_i)}$$

$$= \frac{1}{6.324}(2x_1 + 2x_2) + \frac{1}{7.1414}(2x_1 + 4x_2)$$

$$= 0.5963x_1 + 0.8763x_2$$

s/t

$$6x_1 + 8x_2 \leq 48$$

$$-x_1 - x_2 \leq -3$$

$$x_1 \leq 4$$

$$x_2 \leq 3$$

$$x_1, x_2 \geq 0$$

Table 8

C _B	C _j Basis	0.596	0.876	0	0	0	0	
		x ₁	x ₂	s ₁	s ₂	s ₃	s ₄	
0.5963	x ₁	1	0	1/6	0	0	-4/3	4
0.8763	x ₂	0	1	0	0	0	1	3
0	s ₃	0	0	-1/6	0	1	4/3	0
0	s ₂	0	0	1/6	1	0	-1/3	4
	C _j -E _j	0	0	-0.0994	0	0	-1.8338	5.0141

Thus from Table 8 $Z_{max} = 5.0141$ with $x_1=4, x_2=3$
 Using Harmonic averaging approach,
 From Table-5, we get

$$H.M.(10, 4) = 5.7143, \quad H.M.(17, 3) = 5.1$$

So, by using equation (9) we have

$$\text{Max } Z = \sum_{i=1}^r \frac{z_i}{H.M.(AA_i)} - \sum_{i=r+1}^s \frac{z_i}{H.M.(AL_i)}$$

$$= \frac{1}{5.7143}(2x_1 + 2x_2) + \frac{1}{5.1}(2x_1 + 4x_2) = 0.7421x_1 + 1.1342x_2$$

s/t

$$6x_1 + 8x_2 \leq 48$$

$$-x_1 - x_2 \leq -3$$

$$x_1 \leq 4$$

$$x_2 \leq 3$$

$$x_1, x_2 \geq 0$$

Table 9

C _B	C _j Basis	0.7421	1.13421	0	0	0	0	
		x ₁	x ₂	s ₁	s ₂	s ₃	s ₄	
0.7421	x ₁	1	0	1/6	0	0	-4/3	4
1.1342	x ₂	0	1	0	0	0	1	3
0	s ₃	0	0	-1/6	0	1	4/3	0
0	s ₂	0	0	1/6	1	0	-1/3	4
	C _j -E _j	0	0	-0.1237	0	0	-0.1447	6.37103

Thus from Table 9 $Z_{max} = 6.37103$ with $x_1=4, x_2=3$

4.1 New Arithmetic Averaging technique:

Let $m_1=4, m_2=3; \frac{m_1 + m_2}{2} = 3.5$

$$\text{max } z = \left(\sum_{i=1}^r z_i - \sum_{i=r+1}^s z_i \right) / A.Av$$

$$= \frac{1}{3.5}[4x_1 + 6x_2] = 1.1428x_1 + 1.7143x_2$$

s/t

$$6x_1 + 8x_2 \leq 48$$

$$-x_1 - x_2 \leq -3$$

$$x_1 \leq 4$$

$$x_2 \leq 3$$

$$x_1, x_2 \geq 0$$

Table 10

C _B	C _j Basis	1.1428	1.7143	0	0	0	0	
		x ₁	x ₂	s ₁	s ₂	s ₃	s ₄	
1.1428	x ₁	1	0	1/6	0	0	-4/3	4
1.7143	x ₂	0	1	0	0	0	1	3
0	s ₃	0	0	-1/6	0	1	4/3	0
0	s ₂	0	0	1/6	1	0	-1/3	4
	C _j -E _j	0	0	-0.1904	0	0	-0.19056	9.7141

Thus from Table 10 $Z_{max} = 9.7141$ with $x_1=4, x_2=3$

4.2 New Geometric Averaging technique:

$$\max z = \left(\sum_{i=1}^r z_i - \sum_{i=r+1}^s z_i \right) / G.Av$$

$$\sqrt{4 \times 3} = 3.4641$$

$$= \frac{1}{3.4641} [2x_1 + 2x_2 + 2x_1 + 4x_2] = 1.1547x_1 + 1.73205x_2$$

s/t

$$6x_1 + 8x_2 \leq 48$$

$$-x_1 - x_2 \leq -3$$

$$x_1 \leq 4$$

$$x_2 \leq 3$$

$$x_1, x_2 \geq 0$$

Table 11

C _B	C _j Basis	1.1547	1.73205	0	0	0	0	
		x ₁	x ₂	s ₁	s ₂	s ₃	s ₄	
1.1547	x ₁	1	0	1/6	0	0	-4/3	4
1.73205	x ₂	0	1	0	0	0	1	3
0	s ₃	0	0	-1/6	0	1	4/3	0
0	s ₂	0	0	1/6	1	0	-1/3	4
	C _j -E _j	0	0	-0.1924	0	0	-0.1924	9.81415

Thus from Table 11 $Z_{max} = 9.81415$ with $x_1=4, x_2=3$

4.3 New Harmonic Averaging Technique

$$\max z = \left(\sum_{i=1}^r z_i - \sum_{i=r+1}^s z_i \right) / H.Av$$

$$\frac{2}{\frac{1}{4} + \frac{1}{3}} = 3.4483$$

Table 13

Chandra Sen's Approach	Statistical Averaging Method			New Statistical Averaging Method		
	Using A.M.	Using G.M.	Using H.M	New A. Av method	New G. Av method	New H. Av method
Max Z=3.9998 with $x_1=4, x_2=3$	Max Z=3.9999 with $x_1=4, x_2=3$	Max Z=5.0141 with $x_1=4, x_2=3$	Max Z=6.37103 with $x_1=4, x_2=3$	Max Z=9.7141 with $x_1=4, x_2=3$	Max Z=9.81415 with $x_1=4, x_2=3$	Max Z=9.8593 with $x_1=4, x_2=3$

5. Conclusion

In this paper, different methods such as Chandra Sen's approach and proposed statistical averaging methods are used to solve a MOLPP, and the results are compared in Table-13. In statistical average method, harmonic, geometric and arithmetic average approaches are proposed. We also proposed a new statistical average approach to solve the problem. It is observed that statistical average method results better optimization than Chandra Sen's approach of the MOLPP. The proposed new statistical averaging methods optimize the problem better than that of statistical average method. We also found that harmonic average technique is

$$\max Z = \frac{1}{3.4483} [2x_1 + 2x_2 + 2x_1 + 4x_2] = 1.1599x_1 + 1.7399x_2$$

s/t

$$6x_1 + 8x_2 \leq 48$$

$$-x_1 - x_2 \leq -3$$

$$x_1 \leq 4$$

$$x_2 \leq 3$$

$$x_1, x_2 \geq 0$$

Table 12

C _B	C _j Basis	1.1599	1.7399	0	0	0	0	
		x ₁	x ₂	s ₁	s ₂	s ₃	s ₄	
1.1599	x ₁	1	0	1/6	0	0	-4/3	4
1.7399	x ₂	0	1	0	0	0	1	3
0	s ₃	0	0	-1/6	0	1	4/3	0
0	s ₂	0	0	1/6	1	0	-1/3	4
	C _j -E _j	0	0	-0.1933	0	0	0.1934	9.8593
1.1599	x ₁	1	0	0	0	1	0	4
1.7399	x ₂	0	1	1/8	0	-3/4	0	3
0	s ₄	0	0	-1/8	0	3/4	1	0
0	s ₂	0	0	1/8	1	1/4	0	4
		0	0	-0.2175	0	-0.1450	0	9.8593

Thus from Table 12 $Z_{max} = 9.8593$ with $x_1=4, x_2=3$

Table 13 summarizes the solutions of the MOLPP using approaches. It shows that the solution of the objective function is improved when we used the proposed new statistical averaging method in this paper. In the new approach, new harmonic average technique gives better optimization of the MOLPP.

suited for optimizing MOLPP better than that of arithmetic average and geometric average techniques.

6. Conflict of Interest

Statement: The authors declare here that there is no conflict of interest regarding the publication of this paper.

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