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A New Statistical Averaging Method to Solve Multi-Objective Linear Programming Problem

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Abstract: ***In this paper, a new statistical averaging technique is suggested to solve MOLPP by using new arithmetic averaging method, new geometric averaging method and new harmonic averaging method. Statistical averaging technique (arithmetic averaging, geometric averaging and harmonic averaging) has also been used to solve the same problem. Among all the solution techniques, the proposed new harmonic averaging method gives better result than all these averaging methods.***

Keywords: Linear Programming Problem, new Statistical averaging method, Arithmetic average, Geometric average, Harmonic average

1. Introduction

Linear Programming is without doubt the most natural mechanism for formulating a vast array of problems with modest effort. A Linear Programming Problem is characterized, as the name implies, by linear functions of the unknowns; the objective is linear in the unknowns, and the constraints are linear equalities or linear inequalities in the unknowns. A learning of multi-objective linear programming problem (MOLPP) is introduced in [1] which suggests an method to set up multi-objective function (MOF) under the limitation so that the optimum value of individual problem was greater than zero. Using mean and median the MOF was studied by solving multi-objective programming problem [2].

A popular technique-named as Chandra Sen's technique-has been used to solve the multi-objective linear fractional programming problem (MOLFPP) [1]. To solve these problems there are several methods discussed in [3] where linear fractional programming is transformed to an equivalent linear program. The paper, [4], shows a useful study about the optimality condition in fractional programming. In [5], a study on MOLFPP has been conducted. Arithmetic mean was used to study MOLPP in [6]. The MOLPP was transformed to the single objective linear programming problem using harmonic mean for values of functions in [7]. In [11], a new geometric average technique was used to solve MOLFPP where multi-objective functions were conversed to a single objective function.

In this paper, in order to extend my work on solution to optimization problem, I have defined a MOLPP and convert to a single objective function. Optimization results using arithmetic mean, geometric mean and harmonic mean have been analyzed and compared. I have suggested new arithmetic averaging method, new geometric averaging method and new harmonic averaging method. Optimization results from new harmonic averaging method have compared with the results from that of all other techniques to solve optimization problems. Optimization results using harmonic averaging and new harmonic averaging method are better than that of obtained from statistical methods. Also, the new statistical averaging method gives better results than the statistical averaging method. From the analysis, I found that MOLPP is best optimized using new harmonic averaging method.

2. Formulation of Problem

The main objective of this study is to solve MOLPP. The mathematical form of MOLPP is given as follows:

$$
\begin{aligned}
&\text{Max } z_1 = C_1^t x + r_1 \\
&\text{Max } z_2 = C_2^t x + r_2 \\
&\dots\dots\dots\dots\dots \\
&\text{Max } z_r = C_r^t x + r_r \\
&\text{Min } z_{r+1} = C_{r+1}^t x + r_{r+1} \\
&\dots\dots\dots\dots\dots \\
&\text{Min } z_s = C_s^t x + r_s
\end{aligned}
\qquad \text{s/t} \quad
\begin{aligned}
A\vec{x} &= \vec{b} \\
\vec{x} &\geq 0
\end{aligned}
\qquad (1)
$$

where, ***b*** is *m*-dimensional vector of constants, ***x*** is *n*-dimensional vector of decision variables and ***A*** is $m \times n$ matrix of constants. Both types of objective functions must be present.

3. Techniques of MOLPP Solution

At first, the Chandra Sen's technique has been used to solve MOLPP which gives comparatively poor result of the objective function. Then other existing and suggested techniques have been used solve MOLPP. These techniques are briefly described below.

3.1 Solving MOLPP by Chandra Sen's Technique:

Using simplex method to solve MOLPP in equation (1), a single value corresponding to each of the objective functions is obtained which are in equation (2).

$$
\begin{aligned}
&\text{Max } z_1 = \varphi_1 \\
&\text{Max } z_2 = \varphi_2 \\
&\dots\dots\dots\dots\dots\dots \\
&\text{Max } z_r = \varphi_r \\
&\text{Min } z_{r+1} = \varphi_{r+1} \\
&\dots\dots\dots\dots\dots\dots
\end{aligned}
\qquad (2)
$$

Min $z_s = \varphi_s$;

where $\varphi_1, \varphi_2, \cdots\cdots\varphi_s$ are values of the objective functions.

These values are used in Chandra Sen's technique to obtain a single objective function as shown in equation (3).

$$\max \quad z = \sum_{i=1}^{r} \frac{z_i}{|\varphi_i|} - \sum_{i=r+1}^{s} \frac{z_i}{|\varphi_i|} \tag{3}$$

where, $\varphi_i \neq 0, \quad i = 1, 2, \quad \cdots s$. Subject to the constraints of equation (1) and the optimum value of the objective functions φ_i may be positive or negative.

3.2 Applied Techniques

3.2.1 Statistical Averaging Method

$$\max \quad z = \sum_{i=1}^{r} \frac{z_i}{A.M.(AA_i)} - \sum_{i=r+1}^{s} \frac{z_i}{A.M.(AL_i)} \tag{4}$$

$$\max \quad z = \sum_{i=1}^{r} \frac{z_i}{G.M.(AA_i)} - \sum_{i=r+1}^{s} \frac{z_i}{G.M.(AL_i)} \tag{5}$$

$$\max \quad z = \sum_{i=1}^{r} \frac{z_i}{H.M.(AA_i)} - \sum_{i=r+1}^{s} \frac{z_i}{H.M.(AL_i)} \tag{6}$$

Where, $AA_i = |\varphi_i|$, $i = 1 \cdots r$ and $AL_i = |\varphi_i|$, $i = 1 + r \cdots s$

And A.M. is Arithmetic mean, G.M. is Geometric mean and H.M. is Harmonic mean.

3.2.2 Solving MOLPP by using the new arithmetic averaging technique:

Let $m_1 = \min\langle AA_i \rangle$, where $AL_i = |\varphi_i|$,

φ_i is maximum value of z_i, $i = 1 \cdots r$

$m_2 = \min\langle AL_i \rangle$, where $AA_i = |\varphi_i|$,

φ_i is minimum value of z_i, $i = r+1 \cdots s$

$A.Av = \dfrac{m_1 + m_2}{2}$ so

$$\max \quad z = \left(\sum_{i=1}^{r} z_i - \sum_{i=r+1}^{s} z_i \right) / A.Av \tag{7}$$

3.2.3 Solving MOLPP by using the new geometric averaging technique:

Using m_1 and m_2, we can find the geometric average as follows:

$G.Av = \sqrt{m_1 m_2}$

$$\max \quad z = \left(\sum_{i=1}^{r} z_i - \sum_{i=r+1}^{s} z_i \right) / G.Av \tag{8}$$

3.2.4 Solving MOLPP by using the new harmonic averaging technique:

Using m_1 and m_2, we can find the harmonic average as follows:

$$H.Av = \frac{2}{\frac{1}{m_1} + \frac{1}{m_2}}$$

$$\max \quad z = \left(\sum_{i=1}^{r} z_i - \sum_{i=r+1}^{s} z_i \right) / H.Av \tag{9}$$

3.3 Algorithm for new Arithmetic, Geometric and Harmonic averaging technique:

Step 1: Find the value of each of individual objective functions which is to be maximized or minimized.

Step 2: Solve the first objective problem by simplex method.

Step 3: Check the feasibility of the solution in step 2. If it is feasible then go to step 4. Otherwise, use dual simplex method to remove infeasibility.

Step 4: Assign a name to the optimum value of the first objective function z_1 say φ_1.

Step5: Repeat the step 2, i=1, 2, …s

Step 6:

Select $m_1 = \min\langle AA_i \rangle, m_2 = \min\langle AL_i \rangle$

$i = 1 \cdots s$

$A.Av = \dfrac{m_1 + m_2}{2}$, $G.Av = \sqrt{m_1 m_2}$ and $H.Av = \dfrac{2}{\frac{1}{m_1} + \frac{1}{m_2}}$

Step 7: Optimize the combined objective function with the same constraints

$$\max \quad z = \left(\sum_{i=1}^{r} z_i - \sum_{i=r+1}^{s} z_i \right) / A.Av$$

$$\max \quad z = \left(\sum_{i=1}^{r} z_i - \sum_{i=r+1}^{s} z_i \right) / G.Av \text{ and}$$

$$\max \quad z = \left(\sum_{i=1}^{r} z_i - \sum_{i=r+1}^{s} z_i \right) / H.Av$$

3.4 Program solution for new Arithmetic, Geometric and Harmonic averaging technique:

To solve MOLPP by proposed method, the following program can be used. For this, let

φA_i = value of objective functions which is to be maximized

φL_i = value of objective functions which is to be minimized

So

$AA_i = |\varphi A_i|; \ \forall \ i = 1 \cdots r;$

$AL_i = |\varphi L_i|; \forall \ i = 1 + r \cdots s$

$SM = \sum_{i=1}^{r} z_i$; $\quad SN = \sum_{i=r+1}^{s} z_i$

$m_1 = \min\langle AA_i \rangle$; $\qquad m_2 = \min\langle AL_i \rangle$

$\max \quad z = (SM - SN) / A.Av$

$\max \quad z = (SM - SN) / G.Av$

$\max \quad z = (SM - SN) / H.Av$

4. Mathematical Illustration

Example: Here is an MOLPP with the constraints shown in equation (10). The values of these objective functions can ob obtained using simplex method first. Then with these values, a single objective function is developed using Chandra Sen's technique.

Multi-objective functions:

$$\max z_1 = x_1 + 2x_2$$

$$\max z_2 = x_1$$

$$\min z_3 = -2x_1 - 3x_2$$

$$\min z_4 = -x_2$$

$$\text{s/t} \quad \begin{aligned} 6x_1 + 8x_2 &\le 48 \\ x_1 + x_2 &\ge 3 \\ x_1 &\le 4 \\ x_2 &\le 3 \\ x_1, x_2 &\ge 0 \end{aligned} \qquad (10)$$

Introducing the slack variables s_1, s_2, s_3, s_4

Standard form is

$$\begin{aligned} 6x_1 + 8x_2 + s_1 &= 48 \\ -x_1 - x_2 + s_2 &= -3 \\ x_1 \qquad + s_3 &= 4 \\ x_2 \quad + s_4 &= 3 \\ s_1, s_2, s_3, s_4, x_1, x_2 &\ge 0 \end{aligned}$$

For the first objective function, we get

$$\max z_1 = x_1 + 2x_2$$

Subject to

$$\begin{aligned} 6x_1 + 8x_2 + s_1 &= 48 \\ -x_1 - x_2 + s_2 &= -3 \\ x_1 \qquad + s_3 &= 4 \\ x_2 \quad + s_4 &= 3 \\ s_1, s_2, s_3, s_4, x_1, x_2 &\ge 0 \end{aligned}$$

Setting the decision variables $x_1, x_2 = 0$, the feasible solution is

$$x_1 = 0 \qquad s_1 = 48,\ s_2 = -3$$

$$x_2 = 0 \qquad s_3 = 4,\ s_4 = 3$$

$$z_1 = 0$$

Table 1

C_B	Cj / Basis	1 / x_1	2 / x_2	0 / s_1	0 / s_2	0 / s_3	0 / s_4		
0	s_1	6	8	1	0	0	0	48	6
0	s_2	-1	-1	0	1	0	0	-3	3
0	s_3	1	0	0	0	1	0	4	
0	s_4	0	1	0	0	0	1	3	3
	Cj-Ej	1	2 ↑	0	0	0	0	0	
0	s_1	-2	0	1	8	0	0	24	3
2	x_2	1	1	0	-1	0	0	3	3
0	s_3	1	0	0	0	1	0	4	
0	s_4	-1	0	0	1	0	1	0	0
	Cj-Ej	-1	0	0	2 ↑	0	0	6	
0	s_1	6	0	1	0	0	-8	24	4
2	x_2	0	1	0	0	0	1	3	
0	s_3	1	0	0	0	1	0	4	4
0	s_2	-1	0	0	1	0	1	0	
	Cj-Ej	1 ↑	0	0	0	0	-2	6	
1	x_1	1	0	1/6	0	0	-4/3	4	
2	x_2	0	1	0	0	0	1	3	
0	s_3	0	0	-1/6	0	1	4/3	0	
0	s_2	0	0	1/6	1	0	-1/3	4	
	C_j-E_j	0	0	-1/6	0	0	-2/3	10	

As C_j-E_j is not positive under any column in the Table.1 Thus we get optimal solution. From Table 1, x_1=4, x_2=3, Z_{max}=10

Table 2

C_B	Cj / Basis	1 / x_1	0 / x_2	0 / s_1	0 / s_2	0 / s_3	0 / s_4	
1	x_1	1	0	1/6	0	0	-4/3	4
0	x_2	0	1	0	0	0	1	3
0	s_3	0	0	-1/6	0	1	4/3	0
0	s_2	0	0	1/6	1	0	-1/3	4
	Cj-Ej	0	0	-1/6	0	0	4/3 ↑	4
1	x_1	1	0	0	0	1	0	4
0	x_2	0	1	1/8	0	-3/4	0	3
0	s_4	0	0	-1/8	0	3/4	1	0
0	s_2	0	0	-5/24	1	1/4	0	4
	Cj-Ej	0	0	0	0	-1	0	4

For second objective function

$$\max z_2 = x_1$$

Subject to:

$$\begin{aligned} 6x_1 + 8x_2 + s_1 &= 48 \\ -x_1 - x_2 + s_2 &= -3 \\ x_1 \qquad + s_3 &= 4 \\ x_2 \quad + s_4 &= 3 \\ s_1, s_2, s_3, s_4, x_1, x_2 &\ge 0 \end{aligned}$$

Thus the optimal solution from Table 2 is $x_1 = 4$, $x_2 = 3$ and $z_{\max} = 4$

For third objective function:

$$\min z_3 = -2x_1 - 3x_2$$

Subject to

$$\begin{aligned} 6x_1 + 8x_2 &\le 48 \\ -x_1 - x_2 &\le -3 \\ x_1 &\le 4 \\ x_2 &\le 3 \\ x_1, x_2 &\ge 0 \end{aligned}$$

Which implies $\max z_3 = 2x_1 + 3x_2$

Subject to

$$6x_1 + 8x_2 \le 48$$
$$-x_1 - x_2 \le -3$$
$$x_1 \le 4$$
$$x_2 \le 3$$
$$x_1, x_2 \ge 0$$

Table 3

C_B	C_j Basis	2	3	0	0	0	0	
		x_1	x_2	s_1	s_2	s_3	s_4	
2	x_1	1	0	1/6	0	0	-4/3	4
3	x_2	0	1	0	0	0	1	3
0	s_3	0	0	-1/6	0	1	4/3	0
0	s_2	0	0	1/6	1	0	-1/3	4
	Cj-Ej	0	0	-1/3	0	0	1/3 ↑	17
2	x_1	1	0	0	0	1	0	4
3	x_2	0	1	1/8	0	-3/4	0	3
0	s_4	0	0	-1/8	0	3/4	1	0
0	s_2	0	0	1/8	1	1/4	0	4
	Cj-Ej	0	0	-3/8	0	1/4 ↑	0	17
2	x_1	1	0	1/6	0	0	-4/3	4
3	x_2	0	1	0	0	0	1	3
0	s_3	0	0	-1/6	0	1	4/3	0
0	s_2	0	0	1/6	1	0	-1/3	4
	Cj-Ej	0	0	-1/3	0	0	-1/3	17

Thus the optimal solution from Table 3 is $x_1 = 4$, $x_2 = 3$ and. $z_{\min} = -17$

For fourth objective function

$$\min z_4 = -x_2$$
$$6x_1 + 8x_2 \le 48$$
$$-x_1 - x_2 \le -3$$
$$x_1 \le 4$$
$$x_2 \le 3$$
$$x_1, x_2 \ge 0$$

Which implies

$$\max z_4 = x_2$$
$$6x_1 + 8x_2 \le 48$$
$$-x_1 - x_2 \le -3$$
$$x_1 \le 4$$
$$x_2 \le 3$$
$$x_1, x_2 \ge 0$$

Table 4

C_B	C_j Basis	0	1	0	0	0	0	
		x_1	x_2	s_1	s_2	s_3	s_4	
0	x_1	1	0	1/6	0	0	-4/3	4
1	x_2	0	1	0	0	0	1	3
0	s_3	0	0	-1/6	0	1	4/3	0
0	s_2	0	0	1/6	1	0	-1/3	4
	Cj-Ej	0	0	0	0	0	-1	3

Thus the optimal solution from Table 4 is $x_1 = 4$, $x_2 = 3$ and $z_{\min} = -3$

Thus the optimum values of the objective functions with same constraints are given in Table 5.

Table 5

I	φ_i	x_i	$AA_i = \lvert\varphi_i\rvert$	$AL_i = \lvert\varphi_i\rvert$
1	10	(4, 3)	10	
2	4	(4, 3)	4	
3	-17	(4, 3)		17
4	-3	(4, 3)		3

By Chandra Sen's approach,

$$\text{Max } Z = \sum_{k=1}^{r} \frac{z_k}{|\varphi_k|} - \sum_{k=r+1}^{s} \frac{z_k}{|\varphi_k|}$$

$$\text{Max } Z = \frac{x_1 + 2x_2}{10} + \frac{x_1}{4} - \left(\frac{-2x_1 - 3x_2}{17} - \frac{x_2}{3}\right)$$

Max Z= $0.4676x_1 + 0.7098x_2$ with

$$6x_1 + 8x_2 \le 48$$
$$-x_1 - x_2 \le -3$$
$$x_1 \le 4$$
$$x_2 \le 3$$
$$x_1, x_2 \ge 0$$

Table 6

C_B	C_j Basis	0.4676	0.7098	0	0	0	0	
		x_1	x_2	s_1	s_2	s_3	s_4	
0.4676	x_1	1	0	1/6	0	0	-4/3	4
0.7098	x_2	0	1	0	0	0	1	3
0	s_3	0	0	-1/6	0	1	4/3	0
0	s_2	0	0	1/6	1	0	-1/3	4
	Cj-Ej	0	0	-0.0779	0	0	-0.0863	3.9998

Thus from Table 6 Z_{max}= 3.9998 with x_1=4, x_2=3

Using Arithmetic averaging approach,
From Table-5, we get
A.M. (10, 4) = 7, *A.M.* (17, 3) = 10

So, by using equation (7) we have

$$Max\,Z = \sum_{i=1}^{r} \frac{z_i}{A.M(AA_i)} - \sum_{i=r+1}^{s} \frac{z_i}{A.M(AL_i)}$$
$$= \frac{1}{7}\sum z_i - \frac{1}{10}\sum z_i$$

$$=\frac{1}{7}[x_1+2x_2+x_1]-\frac{1}{10}[-2x_1-3x_2-x_2]$$

$$=0.4857x_1+0.6857x_2$$

s/t
$$6x_1+8x_2\le 48$$
$$-x_1-x_2\le -3$$
$$x_1\le 4$$
$$x_2\le 3$$
$$x_1,x_2\ge 0$$

Table 7

C_B	C_j Basis	0.4857 x_1	0.6857 x_2	0 s_1	0 s_2	0 s_3	0 s_4	
0.4857	x_1	1	0	1/6	0	0	-4/3	4
0.6857	x_2	0	1	0	0	0	1	3
0	s_3	0	0	-1/6	0	1	4/3	0
0	s_2	0	0	1/6	1	0	-1/3	4
	Cj-Ej	0	0	-0.08095	0	0	-0.0381	3.9999

Thus from Table 7 Z_{max}= 3.9999 with x_1=4, x_2=3

Using Geometric averaging approach

From Table-5, we get

$$G.M.(10,4)=\sqrt{10\times 4}=\sqrt{40}=6.324,$$

$$G.M.(17,3)=\sqrt{17\times 3}=\sqrt{51}=7.1414$$

So, by using equation (8) we have

Max Z= $\sum_{i=1}^{r}\frac{z_i}{G.M(AA_i)}-\sum_{i=r+1}^{s}\frac{z_i}{G.M(AL_i)}$

$$=\frac{1}{6.324}(2x_1+2x_2)+\frac{1}{7.1414}(2x_1+4x_2)$$

$$=0.5963x_1+0.8763x_2$$

s/t
$$6x_1+8x_2\le 48$$
$$-x_1-x_2\le -3$$
$$x_1\le 4$$
$$x_2\le 3$$
$$x_1,x_2\ge 0$$

Table 8

C_B	C_j Basis	0.5963 x_1	0.8763 x_2	0 s_1	0 s_2	0 s_3	0 s_4	
0.5963	x_1	1	0	1/6	0	0	-4/3	4
0.8763	x_2	0	1	0	0	0	1	3
0	s_3	0	0	-1/6	0	1	4/3	0
0	s_2	0	0	1/6	1	0	-1/3	4
	Cj-Ej	0	0	-0.0994	0	0	-1.8338	5.0141

Thus from Table 8 Z_{max}= 5.0141 with x_1=4, x_2=3

Using Harmonic averaging approach,

From Table-5, we get

$$H.M.(10,4)=5.7143,\quad H.M.(17,3)=5.1$$

So, by using equation (9) we have

Max Z= $\sum_{i=1}^{r}\frac{z_i}{H.M(AA_i)}-\sum_{i=r+1}^{s}\frac{z_i}{H.M(AL_i)}$

$$=\frac{1}{5.7143}(2x_1+2x_2)+\frac{1}{5.1}(2x_1+4x_2)=0.7421x_1+1.13421x_2$$

s/t
$$6x_1+8x_2\le 48$$
$$-x_1-x_2\le -3$$
$$x_1\le 4$$
$$x_2\le 3$$
$$x_1,x_2\ge 0$$

Table 9

C_B	C_j Basis	0.7421 x_1	1.13421 x_2	0 s_1	0 s_2	0 s_3	0 s_4	
0.7421	x_1	1	0	1/6	0	0	-4/3	4
1.1342	x_2	0	1	0	0	0	1	3
0	s_3	0	0	-1/6	0	1	4/3	0
0	s_2	0	0	1/6	1	0	-1/3	4
	Cj-Ej	0	0	-0.1237	0	0	-0.1447	6.37103

Thus from Table 9 Z_{max}= 6.37103 with x_1=4, x_2=3

4.1 New Arithmetic Averaging technique:

Let m_1=4, m_2=3; $\frac{m_1+m_2}{2}=3.5$

$$\max\quad z=\left(\sum_{i=1}^{r}z_i-\sum_{i=r+1}^{s}z_i\right)/\,A.Av$$

$$=\frac{1}{3.5}[4x_1+6x_2]=1.1428x_1+1.7143x_2$$

s/t
$$6x_1+8x_2\le 48$$
$$-x_1-x_2\le -3$$
$$x_1\le 4$$
$$x_2\le 3$$
$$x_1,x_2\ge 0$$

Table 10

C_B	C_j Basis	1.1428 x_1	1.7143 x_2	0 s_1	0 s_2	0 s_3	0 s_4	
1.1428	x_1	1	0	1/6	0	0	-4/3	4
1.7143	x_2	0	1	0	0	0	1	3
0	s_3	0	0	-1/6	0	1	4/3	0
0	s_2	0	0	1/6	1	0	-1/3	4
	Cj-Ej	0	0	-0.1904	0	0	-0.19056	9.7141

Thus from Table 10 Z_{max}= 9.7141with x_1=4, x_2=3

4.2 New Geometric Averaging technique:

$$\max \quad z=\left(\sum_{i=1}^{r} z_i-\sum_{i=r+1}^{s} z_i\right)/G.Av$$

$$\sqrt{4\times 3}=3.4641$$

$$=\frac{1}{3.4641}[2x_1+2x_2+2x_1+4x_2]=1.1547x_1+1.73205x_2$$

s/t
$$6x_1+8x_2\le 48$$
$$-x_1-x_2\le -3$$
$$x_1\le 4$$
$$x_2\le 3$$
$$x_1,x_2\ge 0$$

Table 11

C_B	C_j / Basis	1.1547 x_1	1.73205 x_2	0 s_1	0 s_2	0 s_3	0 s_4	
1.1547	x_1	1	0	1/6	0	0	-4/3	4
1.73205	x_2	0	1	0	0	0	1	3
0	s_3	0	0	-1/6	0	1	4/3	0
0	s_2	0	0	1/6	1	0	-1/3	4
	Cj-Ej	0	0	-0.1924	0	0	-0.1924	9.81415

Thus from Table 11 Z_{max}= 9.81415 with x_1=4, x_2=3

4.3 New Harmonic Averaging Technique

$$\max \quad z=\left(\sum_{i=1}^{r} z_i-\sum_{i=r+1}^{s} z_i\right)/H.Av$$

$$\frac{2}{\frac{1}{4}+\frac{1}{3}}=3.4483$$

$$Max\,Z=\frac{1}{3.4483}[2x_1+2x_2+2x_1+4x_2]=1.1599x_1+1.7399x_2$$

s/t
$$6x_1+8x_2\le 48$$
$$-x_1-x_2\le -3$$
$$x_1\le 4$$
$$x_2\le 3$$
$$x_1,x_2\ge 0$$

Table 12

C_B	C_j / Basis	1.1599 x_1	1.7399 x_2	0 s_1	0 s_2	0 s_3	0 s_4	
1.1599	x_1	1	0	1/6	0	0	-4/3	4
1.7399	x_2	0	1	0	0	0	1	3
0	s_3	0	0	-1/6	0	1	4/3	0
0	s_2	0	0	1/6	1	0	-1/3	4
	Cj-Ej	0	0	-0.1933	0	0	0.1934	9.8593
1.1599	x_1	1	0	0	0	1	0	4
1.7399	x_2	0	1	1/8	0	-3/4	0	3
0	s_4	0	0	-1/8	0	3/4	1	0
0	s_2	0	0	1/8	1	1/4	0	4
		0	0	-0.2175	0	-0.1450	0	9.8593

Thus from Table 12 Z_{max}= 9.8593 with x_1=4, x_2=3

Table 13 summarizes the solutions of the MOLPP using approaches. It shows that the solution of the objective function is improved when we used the proposed new statistical averaging method in this paper. In the new approach, new harmonic average technique gives better optimization of the MOLPP.

Table 13

Chandra Sen's Approach	Statistical Averaging Method			New Statistical Averaging Method		
	Using *A.M.*	Using *G.M.*	Using *H.M*	New *A. Av* method	New *G. Av* method	New *H. Av* method
Max Z=3.9998 with x_1=4, x_2=3	Max Z=3.9999 with x_1=4, x_2=3	Max Z=5.0141 with x_1=4, x_2=3	Max Z=6.37103 with x_1=4, x_2=3	Max Z=9.7141 with x_1=4, x_2=3	Max Z=9.81415 with x_1=4, x_2=3	Max Z=9.8593 with x_1=4, x_2=3

5. Conclusion

In this paper, different methods such as Chandra Sen's approach and proposed statistical averaging methods are used to solve a MOLPP, and the results are compared in Table-13. In statistical average method, harmonic, geometric and arithmetic average approaches are proposed. We also proposed a new statistical average approach to solve the problem. It is observed that statistical average method results better optimization than Chandra Sen's approach of the MOLPP. The proposed new statistical averaging methods optimize the problem better than that of statistical average method. We also found that harmonic average technique is suited for optimizing MOLPP better that that of arithmetic average and geometric average techniques.

6. Conflict of Interest

Statement: The authors declare here that there is no conflict of interest regarding the publication of this paper.

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