International Journal of Science and Research (IJSR)
ISSN (Online): 2319-7064
Index Copernicus Value (2015): 78.96 | Impact Factor (2015): 6.391

Purely Fully Cancellation Fuzzy Modules

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Abstract: ***In this paper we introduce the concept of purely fully cancellation fuzzy modules and give some characterizations and properties of this concept.***

Keywords: Fully cancellation Fuzzy modules, Invertible fuzzy ideal, Purely Fully-Cancellation fuzzy module

1. Introduction

Gilmer in [1] was introduced the concept of cancellation ideal, and Anderson in [2], studied the concept of cancellation ideals. In [3] A, S, Mijbass, gave some generalization of this concept namely cancellation module (weakly cancellation module). In [4], Buthyna Nijad Shihab, introduce and studied restricted (and weakly restricted) cancellation module.

Next, Dr. L.M. Salman and Buthyna Nigad Shihab introduced and studied Relatively cancellation module in[5]. In [6]. Hatam Yahya Khalaf and Hadi.G. Rashed, introduced the concept of Fully cancellation fuzzy modules, where a fuzzy module X of an R-module M is called fully cancellation fuzzy module if for each fuzzy ideal I of R and for each fuzzy submodules A and B of X such that IA=IB, implies A=B.

In this paper we will introduce the concept of Puerly-fully cancellation fuzzy module and gives some properties, examples of this concept.

2.1 Definition

Let X be a fuzzy module of an R-module M. X is called puerlly fully cancellation fuzzy module if for all non-empty fuzzy pure ideal I of R and for all non-empty fuzzy submodules A_1, A_2 of X such that $IA_1=IA_2$ then $A_1=A_2$.And follow up to this same idea will offer the defition of puerlly fully cancellation ideal. If for all non-empty pure fuzzy ideal J of R and for all non-empty fuzzy ideals A and B of R such that JA=JB, then A=B.

2.2 Proposition

Let X be a fuzzy module of an R-module M.X is purely-fully cancellation fuzzy module if and only if X_t is purely –fully cancellation module

Proof: (⇒) Let K, N be two submodules of an R-module M and let J be a pure ideal of R.

Let I: R→ [0, 1] such that $I(x)=\begin{cases} t & \text{if } x\in J \\ 0 & \text{otherwise}\end{cases}$ ∀t∈(01]

It is clear that I is afuzzy ideal of R

Let A: M → [0, 1] , B: M → [0, 1] such that : $A(x)=\begin{cases} t & \text{if } x\in K \\ 0 & \text{otherwise}\end{cases}$ ∀t∈(0, 1]

$B(x)=\begin{cases} t & \text{if } x\in N \\ 0 & \text{otherwise}\end{cases}$ ∀t∈(0, 1]

It is clear that A and B are two fuzzy submodules of X and A_t=K, B_t=N and I_t=J

Suppose that JA=JB, to prove A=B

$(IA)_t=(IB)_t$, so IA=IB by[7]

Thus A=B, since X is purely –fully cancellation fuzzy module

Therefore $A_t=B_t$.

Conversely, It is clear that X_t=M and M is purely –fully cancellation module.

Let A and B be two fuzzy submodules of a fuzzy module X and let I be a fuzzy ideal of R such that IA=IB, then $(IA)_t=(IB)_t$∀t∈(0, 1], which imples that A_t, B_t are submodules of X_t, but X_t is purely fully cancellation module, so $I_tA_t=I_tB_t$ implies $A_t=B_t$, hence A=B.

Thus X is purely fully cancellation fuzzy module

Examples (2.3)

(1) Let M=Z_{30}, R= Z_{30} and let X: M → [0, 1] such that X(x) $=\begin{cases} 1 & \text{if } x\in \overline{(6)} \\ 0 & \text{otherwise}\end{cases}$ ∀t∈(0, 1]

X is a fuzzy module of Z_{30} –module.

Let I: Z_{30}→ [0, 1] such that $I(x)=\begin{cases} t & \text{if } x\in \overline{(5)} \\ 0 & \text{otherwise}\end{cases}$ ∀ t∈(0.1]

Let A: M→ [0, 1] such that $A(x)=\begin{cases} t & \text{if } x\in \overline{(18)} \\ 0 & \text{otherwise}\end{cases}$ ∀ t∈(0, 1]

Let B: M→[0, 1] such that $B(x)=\begin{cases} t & \text{if } x\in \overline{(12)} \\ 0 & \text{otherwise}\end{cases}$ ∀ t∈(0, 1]

It is clear that A and B are fuzzy submodules of X.

X_t=M and $A_t=\overline{(18)}$, $B_t=\overline{(12)}$, $I_t=\overline{(5)}$ it is pure ideal by[8]

Now, $I_tA_t=I_tB_t$ (since $\overline{(5)}\ \overline{(18)}=\overline{(5)}\ \overline{(12)}$)

Thus $A_t=B_t$.

(2) Let M= Z_{12} and R=Z_{12}. Let X: M →[0, 1] such that $X(x)=\begin{cases} 1 & \text{if } x\in Z_{12} \\ 0 & \text{otherwise}\end{cases}$

Define I: R→[0, 1] such that :

$I(x)=\begin{cases} 1 & \text{if } x\in \overline{(3)} \\ 0 & \text{otherwise}\end{cases}$ ∀t∈(0, 1].

Let A :M→ [0, 1] such that : $A(x)=\begin{cases} t & \text{if } t\in \overline{(2)} \\ 0 & \text{otherwise}\end{cases}$

and B: M → [0, 1] such that : $B(x)=\begin{cases} t & \text{if } t\in \overline{(6)} \\ 0 & \text{otherwise}\end{cases}$.

It is clear that $I_t=\overline{(3)}$, $A_t=\overline{(2)}$ and B $=\overline{(6)}$

Then $\overline{(3)}$ is pure ideal of Z_{12} and $I_tA=\overline{(3)}.\overline{(2)}=\overline{(3)}.\overline{(6)}=I_t.B_t$. But $\overline{(2)}\neq\overline{(6)}$

Therefore $X_t=Z_{12}$ is not purely –fully cancellation module.

Then by proposition (2.2).X is not purely –fully cancellation fuzzy module.

Remark: (2.4)
Every fully cancellation fuzzy module is purely –fully cancellation fuzzy module
The converse of this remark is not true in general for example:
For example (1) we get $I_t=\overline{(5)}$ and $X_t=\overline{(6)}$ is a Z_{30}-module, X_t is purely –fully cancellation module and by proposition (2.2).We get X is purely –fully cancellation fuzzy module.
Now, define A: M→ [0, 1] where M $=Z_{30}$ by A(x) $=\begin{cases} t & if\ x\in\overline{(6)} \\ 0 & otherwise \end{cases}$ ∀ t∈ (0, 1]
Define B: M→ [0, 1] by B(x) $=\begin{cases} t & if x\in\overline{(0)} \\ 0 & otherwise \end{cases}$. Since $\overline{(5)}.\overline{(6)}=\overline{(5)}.\overline{(0)}=\overline{(0)}$, but $\overline{(6)}\neq\overline{(0)}$. Thus X_t is not fully cancellation module and by proposition (2.2).We get X is not fully cancellation fuzzy module.

Proposition (2.5)
Every fuzzy submodule of purely –fully cancellation fuzzy module is also purely –fully cancellation.

Proposition: (2.6)
Let X_1 and X_2 be two fuzzy submodules of an R-m0dule M_1, M_2 respectively such that $M_1\cong M_2$.Then X is purely –fully cancellation fuzzy module if and only if X_2 is purely –fully cancellation fuzzy module.

Proof:
Let X_1: M_1→ [0, 1] define by $X_1(x)=\begin{cases} 1 & if\ x\in M_1 \\ 0 & otherwise \end{cases}$
Let X_2: M_2→ [0, 1] define by $X_2(x)=\begin{cases} 1\ if\ x\in M_2 \\ 0\ otherwise \end{cases}$
It is clear that X_1 and X_2 are fuzzy modules of M_1 and M_2 respectively.

Since $(X_1)_t=M_1$, $(X_2)_t=M_2$ ∀t∈(0, 1], and $M_1\cong M_2$, then M_2 is purely –fully cancellation module by [8, (2.1.2)(5)].
Then X_2 is purlly –fully cancellation fuzzy module by proposition (2.2).

Definition (2.7): Let (x^{-1}) be the invertible element of x in R then $(x^{-1})_t$ is an invertible of a fuzzy singleton in A and $x_t(x^{-1})_t=(xx^{-1})_t=1_t=(x^{-1})_t.x_t$ where 1_t:R→ [0, 1] such that $1_t=\begin{cases} t\ if\ x=1 \\ 0\ if\ x\neq 1 \end{cases}\leq\begin{cases} 1\ if\ x\in R \\ 0\ otherwise \end{cases}=\lambda_R(x)=1$, [8].

Proposition: (2.8)
Let X be a fuzzy module of an R-module M and every non-empty fuzzy ideal I of R is fuzzy invertible. Then X over an R-module M is purely –fully cancellation module.
Proof:
Let A and B be a non- empty fuzzy submodules of a fuzzy module X and let I be a non-empty pure fuzzy ideal of R such that IA=IB.
Since I is an invertible fuzzy ideal, then $A=I^{-1}.I\ A=I^{-1}I\ B=B$.
Therefore X is purely –fully cancellation fuzzy module which is end the proof.

Definition: (2. 9)
Let I be a fuzzy invertible ideal of a ring R, I is called fuzzy invertible if there exsist I^{-1} in R such that $I.I^{-1}=\lambda_R(x)$ where $\lambda_R(x)$ =1 if x=1.

Theorem: (2.10)
Let X be a fuzzy module of an R-module M. If A, B are two non-empty fuzzy submodules of X and I be a puire fuzzy ideal of R. Then the following statements are equivalents:
(1) X is purely- fully cancellation fuzzy module.
(2) If IA ⊆ IB, then A⊆B.
(3) If $I(a_t)\subseteq IB$, then $a_t\subseteq A$, where $a_t\subseteq X$ ∀t∈(0, 1]
(4) If $(IA:_R IB)=(A:_R B)$.

Proof:
(1)⇨(2) Let IA ⊆ IB, then we have IB= IA +IB. Thus IB =I(A+B).
Since X is purely –fully by(1), then we get B= A+B
Therefore A⊆B which is end the proof.
(2)⇨(3) If $I(a_t)\subseteq IB$, then by (2) we get $a_t\subseteq B$.∀t∈(0, 1]
(1)⇨(4) Let $r_t\subseteq(IA:_R IB)$.Then $r_tIB\subseteq IA$ ∀t∈(0, 1].
Therefore, $Ir_tB\subseteq IA$, then we have $r_tB\subseteq A$, since (1) implies (2)
Thus $r_t\subseteq(A:_R B)$, and hence, we get $(IA:_R IB)$.
The other hand, let $r_t\subseteq(A:_R B)$ ∀t∈(0, 1].Then $r_tB\subseteq A$ which implies that
$Ir_tB\subseteq IA$, and hence $r_tIB\subseteq IA$. ∀t∈(0, 1].
Therefore $r_t\subseteq(IA:_R IB)$, then we get $(A:_R B)\subseteq(IA:_R IB)$.
Thus $(A:_B)B=(IA:_R IB)$.
(4) ⇒ (1) Let IA = IB and $(IA:_R IB)=(A:_R B)$
Since IA =IB, then $(IA:_R IB)=\lambda_R(x)$ where $\lambda_R(x)$ =1 if x =1.
Hence $(A:_R B)=\lambda_R(x)$, and so B⊆A.
Similarly $(IB:_R IA)=(B:_R A)$, then $(B:_R A)=\lambda_R(x)$.
Which implies that A⊆ B. Therefore A = B.
Thus X is purely- fully cancellation fuzzy modules.

References

[1] Glimer.R.W,. "Multiplication ideal Theory" Marcel Dekkes New Yourk, (1972)
[2] Anderson.D.D,. and D.F,. Anderson "Some remarks on Cancellation ideal " Math. Goponica 29(6), (1984), 879-886.
[3] Ali.S Mijbass "On Cancellation modules" M.Sc. Thesis.University of Baghdad (1992).
[4] Buthna.N. Shihab "Restricted cancellation modules" M.Sc,..Thesis university of Tikrit (2000).
[5] Dr. Layla S.M, Buthyna N. Sh. And Thaier S. Rasheed "Relatively cancellation modules" M.Sc. Thesis University of Baghdad (2005).
[6] Hatam Y. Khalaf and Hadi G.Rashad "Fully Cancellation fuzzy modules" M Sc. Thesis University of Baghdad.
[7] Zada L A., "Fuzzy information and Control (8), (1965), 338-353.
[8] Bothaynah N.Shihab and Heba M.A Judi " Puerly –fully cancellation modules" International Journal of Applied Mathematics and Statistical Sciences(IJAMSS) Vol(5). (2016)., 79-96.