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Introducing the Use of Markov Chain Model for Informed Decision Making for Investors in a Stock Market Environment

Gongsin Isaac Esbond[1], Fumilayo W. O. Saporu[2]

[1]Department of Mathematics and Statistics, University of Maiduguri, Nigeria

[2] National Mathematical Centre, Kwali, FCT-Abuja, Nigeria

Abstract: ***A 5-state Markov chain model is used to model the behaviour of daily stock price on the floor of the stock market. A simple test for the time homogeneity of the arising transition probability matrix is proposed. Criteria that can aid informed decisions on trading for short and long term investors are developed from the properties of the model. Its use is illustrated with data from Ecobank, Nigeria for short term investors. The results show that it is best to buy stock when price is at high depreciation state, and sell within two days when next the price rises to the high appreciation state. The use of these criteria is recommended.***

Keywords: Markov Chain Model, Regular Markov Chain, Expected First Passage Times, Stock Exchange Price, Time Homogeneity Test, Fundamental Matrix

1. Introduction

In today's business world, it is expedient for companies to source for funds from the public. One of the possible ways for companies to raise funds from the public is through the sales of shares which create room for investors to be part owners of the particular company on one hand and for the particular company to increase its capital base to enlarge its operation on the other. Through ages investors have purchased shares of prospective companies with the view to getting returns on their investments. Some investors have had to purchase shares from more than one company to include in their portfolio. One important factor in determining a particular company's share to include in an investment portfolio is its performance at the stock market. A rise in the price of a company's stock signifies a good performance of the particular company's share. This implies a good turnover for the company and consequently a rise in the dividend paid to share holders. Companies making more profit do offer more as returns on investment compared to companies making less or no profit [1]. Furthermore, companies whose shares do well at the stock market are attractive to would be investors.

Investors in stock can be categorized into two, namely; short term investors and long term investors. The former comprises of those who trade daily in stocks through buying and selling, while the latter are those who purchase stocks with the aim of being paid dividends when due. Each category has a peculiar problem regarding the purchase of stocks. While the former's problem may concern the direction and magnitude of daily change in stock prices, the latter's is the long term change in stock prices of the prospective companies and the amount of dividends they can drive. Hence developing a decision tool that investors can use in gauging the stock market is appropriate.

2. Literature Review

Stock market is a highly volatile market which generates arguments as to whether or not stock prices can be predicted [2]. By volatility, it means that stock prices fluctuate unexpectedly, with an investor making money when prices go up and losing in the event of a down turn.

Several statistical approaches have been employed to predict the behaviour of stocks. These range from artificial neural networks (AAN) [3], Data mining and Regression [4], autoregressive integrated moving average (ARIMA) [5], weighted Markov chain [6], to Markov chain analysis. Markov chain test is applied to individual stock prices as well as market indices at various frequencies for different time periods; see for example [7] - [9] and [2]. [10] specifically tested for the stationarity (or time homogeneity) as well as the order of dependence of the Markov chain using daily and weekly returns of 200 individual stocks from 1963 to 1968 in a 3-state Markov Chain.

[11] looked at fluctuations in stock prices as a random walk; [12] showed empirically that stock prices satisfy the principle of lack of memory, implying past price fluctuations cannot be used to predict future prices; while [13], through the principle of efficient market hypothesis (EMH), explained that the future behaviour of stock prices can only be explained using current information.

A number of recent papers described the use of Markov chains in predicting stock price fluctuations, these include [14] - [16], and [1] , to mention a few. In all these, little or no consideration has been given to the test of time homogeneity of the transition probability matrix.

3. The Problem

As mentioned earlier there are two types of investors trading on the floor of the stock market; short and long term investors. The problem of these stock investors is the ability to predict with some level of precision, the future behaviour of either the market or the particular stock. It is thus useful for any potential investor to be able to obtain better information regarding the strength of a particular company's stock. The primary purpose of this paper therefore is to find criteria, based on a Markov chain model that can facilitate daily trading decisions for these two types of investors. A simple test for the time homogeneity of the transition probability matrix is also derived; this is new.

4. The Model

A model attempts to capture the essence of the underlying process. Here it is trading in a stock market environment. The essence of the trading process can be put succinctly as follows Shares or Stock prices of companies are traded on the floor of the stoke exchange on each working day. At the close of the day, the prices can either experience a rise or fall. Thus the problem of investment in stocks reduces to finding the stock of a company which gains most frequently and is able to bounce back to making gains in a short run after a drift.

A 3-state Markov chain model has already been used in modelling the behaviour of day closing percent change in stock prices. Here a 5-state Markov chain model is proposed so that finer details of the trading process are captured. The percentage change in stock price, denoted by $\boldsymbol{C}$, at the close of trading on each working day is categorize into states of the Markov chain model as shown Table 4.1 below.

Table 4.1: Categorisation of The States of the Markov Chain Model

$\boldsymbol{C}$	$C > 1$	$0 < C \leq 1$	$\boldsymbol{C = 0}$	$-\boldsymbol{1} \leq \boldsymbol{C} < 0$	$\boldsymbol{C} < -1$
State	HA	LA	U	LD	HD

$\boldsymbol{C}$: Percentage change in stock price, HA: Higher Appreciation, LA: Lower Appreciation, U: Unchanged, LD: Lower Depreciation, HD: Higher Depreciation

The usual assumption of Markov chain model obtains. These are stated below:

1) There is Markov dependence. That is, the future time (t+1), probability behaviour of the process is uniquely determined once the state of the system at the present time, t is given.
2) The Markov chain is time homogeneous. That is, the one step transition probabilities are independent of time.

5. The Data

The data for this study are the day closing percent change in the stock price of Ecobank Nig. for six years, January 4 2010 to December 31, 2015, constituting 1485 trading days, are obtained from financial times online with FT.com.htm. The nature of the model determines how the data are processed for application. Here it is a 5-state Markov chain model that is under consideration. Hence the data are processed into a 5x5 matrix of the number of transitions for each of the twelve months of the year. Simplifying assumptions, that, the time between two successive trading days are equal and that holidays and strike periods are ignored, are made in processing the data.

The data were so processed for each of the six years under consideration. Because of the scantiness of the data, they were subsequently pooled over each month of the year respectively for the six years and shown in Table 5.1 as a monthly data. Also shown in Table 5.1 is the single overall pooled data.

Table 5.1: Matrices of Number of Transitions for 2010 - 2015

			Tomorrow					
			HA	***LA***	***U***	***LD***	***HD***	Total
January	Today	***HA***	1	1	2	5	4	13
		LA	3	10	6	13	1	33
		U	3	5	6	8	2	24
		LD	2	13	8	13	3	39
		HD	3	4	0	5	2	14
			Tomorrow					
			HA	***LA***	***U***	***LD***	***HD***	Total
February	Today	***HA***	0	3	1	5	0	9
		LA	4	15	4	13	1	37
		U	0	9	12	6	1	28
		LD	4	11	7	13	3	38
		HD	2	1	0	2	0	5
			Tomorrow					
			HA	***LA***	***U***	***LD***	***HD***	Total
March	Today	***HA***	1	2	0	5	2	10
		LA	6	16	6	18	1	47
		U	1	7	11	6	2	27
		LD	2	18	8	12	1	41
		HD	0	2	3	1	0	6
			Tomorrow					
			HA	***LA***	***U***	***LD***	***HD***	Total
April	Today	***HA***	3	1	4	3	2	13
		LA	4	12	4	13	1	34
		U	2	5	4	2	1	14
		LD	2	18	5	22	2	49
		HD	2	1	0	2	2	7
			Tomorrow					
			HA	***LA***	***U***	***LD***	***HD***	Total
May	Today	***HA***	6	1	0	3	3	13
		LA	4	9	4	13	2	32
		U	0	6	6	6	2	20
		LD	3	12	8	15	4	42
		HD	2	3	1	5	1	12
			Tomorrow					
			HA	***LA***	***U***	***LD***	***HD***	Total
June	Today	***HA***	1	6	2	3	3	15
		LA	2	12	6	17	2	39
		U	2	3	2	6	3	16
		LD	3	12	6	14	6	41
		HD	6	4	1	3	3	17
			Tomorrow					
			HA	***LA***	***U***	***LD***	***HD***	Total
July	Today	***HA***	0	2	1	4	3	10
		LA	2	17	3	18	4	44
		U	1	3	2	5	3	14
		LD	4	14	8	17	4	47
		HD	2	7	0	4	2	15
			Tomorrow					
			HA	***LA***	***U***	***LD***	***HD***	Total
August	Today	***HA***	2	4	0	1	2	9
		LA	1	10	6	15	9	41
		U	0	3	1	7	2	13
		LD	2	21	6	14	1	44
		HD	3	6	0	6	2	17
			Tomorrow					
								Total
September	Today	***HA***	2	2	1	1	3	9
		LA	3	10	7	17	4	41
		U	1	5	2	6	1	15
		LD	3	21	4	23	2	53
		HD	0	5	0	4	0	9
			Tomorrow					
			HA	***LA***	***U***	***LD***	***HD***	Total
October	Today	***HA***	0	4	3	5	3	15

		LA	4	14	8	10	4	40
		U	1	6	5	5	2	19
		LD	5	13	4	19	0	41
		HD	4	2	2	1	1	10
			Tomorrow					
			HA	***LA***	***U***	***LD***	***HD***	Total
November	Today	***HA***	0	2	1	4	5	12
		LA	4	13	3	16	5	41
		U	1	3	2	5	4	15
		LD	4	14	5	13	4	40
		HD	2	8	2	2	2	16
			Tomorrow					
			HA	***LA***	***U***	***LD***	***HD***	Total
December	Today	***HA***	3	2	0	2	6	13
		LA	3	20	4	17	3	47
		U	1	2	1	3	0	7
		LD	4	21	3	10	2	40
		HD	3	4	0	6	0	13
			Tomorrow					
			HA	***LA***	***U***	***LD***	***HD***	Total
Overall	Today	***HA***	19	30	15	41	36	141
		LA	40	158	61	180	37	476
		U	13	57	52	67	23	212
		LD	38	188	72	185	32	515
		HD	29	47	9	41	15	141
			Tomorrow					
			HA	***LA***	***U***	***LD***	***HD***	Total
March /April Combined	Today	***HA***	4	3	4	8	4	23
		LA	10	28	10	31	2	81
		U	3	12	15	8	3	41
		LD	4	36	13	34	3	90
		HD	2	3	3	3	2	13

6. Test for the Assumptions of the Model

In modelling, simplifying assumptions are made so that the model is tractable. The assumption of time homogeneity of the Markov chain in this instance is one of such. This is rarely tested. It is shown below how this can be done using a simple test.

The assumption of time homogeneity of the Markov chain implies that we need to test that the one-step transition probability matrix is same over time, that is, the months of the year. Before doing this, the following notations are defined.
$\boldsymbol{N_t}$ denotes the matrix of the number of one step transitions at time t = 1, 2, ... , 12 with elements ${}_t n_{ij}$ which are the number of one step transitions from state i to j; $\boldsymbol{i, j = 1, 2, \dots, 5}$ and ${}_t n_{i.}$ is the i[th] row total of $\boldsymbol{N_t}$. These are presented in Table 5.1 above.

$\boldsymbol{P_t}$ denotes the matrix of one step transition probabilities at time t, with elements ${}_t p_{ij}$ and ${}_t P^n$ denotes the matrix of n[th]-step transition probability with elements ${}_t p_{ij}{}^{(n)}$, n = 1,2,3….

The estimate of ${}_t p_{ij}$ is obtained using the equation

$$ {}_t\hat{p}_{ij} = \frac{{}_t n_{ij}}{{}_t n_{i.}}, t = 1,\dots,12; i, j = 1,\dots,5 \qquad (6.1)$$

The combined estimate is obtained using the equation

$$\hat{p}_{ij} = \frac{\sum_{t=1}^{12} {}_t n_{ij}}{\sum_{t=1}^{12} {}_t n_{i.}}, i, j = 1, 2, \dots, 5 \qquad (6.2)$$

It is noted that the outcome for any row $\boldsymbol{i = 1, 2, \dots, 5}$ is multinomial with parameters $\boldsymbol{n_{i.}}$ and $\boldsymbol{p_{ij}}$, $\boldsymbol{j = 1, 2, \dots, 5}$. Consequently for any t and $\boldsymbol{i}$ the hypothesis

$$H_0 \ : {}_t p_{ij} = \hat{p}_{ij}$$

$$H_1 : \ {}_t p_{ij} \neq \hat{p}_{ij} \qquad j = 1,2,\dots,5$$

can be tested using the test statistic

$$w(t) = \sum_{j=1}^{5} \frac{({}_t n_{ij} - {}_t n_{i.}\hat{p}_{ij})^2}{{}_t n_{i.}\hat{p}_{ij}} \qquad (6.3)$$

$\boldsymbol{w(t)}$ has a $\boldsymbol{\chi^2}$ distribution with 4 degrees of freedom. Also for any transition probability matrix at month $\boldsymbol{t}$,

$$H_0 \ : {}_t p_{ij} = \hat{p}_{ij}$$

$$H_1 : \ {}_t p_{ij} \neq \hat{p}_{ij} \qquad i, j = 1,2,\dots,5$$

can be tested using the test statistic

$$Q(t) = \sum_{i=1}^{5} \sum_{j=1}^{5} \frac{({}_t n_{ij} - {}_t n_{i.}\hat{p}_{ij})^2}{{}_t n_{i.}\hat{p}_{ij}} \qquad (6.4)$$

$\boldsymbol{Q(t)}$ has a $\boldsymbol{\chi^2}$ distribution with 20 degrees of freedom.
It is now clear that in testing the hypothesis of time homogeneity of the transition probability matrix, we set up the joint hypotheses.

$$H_0 \ : {}_t p_{ij} = \hat{p}_{ij} \qquad i, j = 1,2,\dots,5$$

$$H_1 : \ {}_t p_{ij} \neq \hat{p}_{ij} \qquad t = 1,2,\dots,12$$

and use the test statistic

$$H = \sum_{t=1}^{12} \sum_{i=1}^{5} \sum_{j=1}^{5} \frac{({}_t n_{ij} - {}_t n_{i.}\hat{p}_{ij})^2}{{}_t n_{i.}\hat{p}_{ij}} \qquad (6.5)$$

H has a $\boldsymbol{\chi^2}$ distribution with 220 degrees of freedom.

It should be noted that 20 degrees of freedom is subtracted from the sum of the degrees of freedom for the monthly test in order to arrive at 220. This is because each $\boldsymbol{\hat{p}_{ij}}$ is a combined estimate of the whole data.

6.1 Application to Data

The overall combined estimate of the transition probability matrix was first computed using equations (6.2) and the result is shown in Table 6.1.

Table 6.1: Estimated Overall (2010–2015) Transition Probability Matrix

		Tomorrow				
		HA	LA	U	LD	HD
Today	HA	0.1348	0.2128	0.1064	0.2908	0.2553
	LA	0.0840	0.3319	0.1282	0.3782	0.0777
	U	0.0613	0.2689	0.2453	0.316	0.1085
	LD	0.0738	0.365	0.1398	0.3592	0.0621
	HD	0.2057	0.3333	0.0638	0.2908	0.1064

The compatibility of the estimates for each row of the overall transition probability matrix with corresponding values for each row of every month was tested using the test statistic, $\boldsymbol{w(t)}$, given in equation (6.3) and the results are tabulated separately for each month of the year in Table 6.2

Table 6.2: Row-wise Test for Estimated Monthly Transition Probability Matrices

Sub (i)	$\boldsymbol{w(t)}$	d. f	p-value	Sig	Remark
January	2.26	4	0.688	sig	Homogeneous
	1.82	4	0.769	,,	,,
	2.08	4	0.721	,,	,,
	1.78	4	0.777	,,	,,
	1.38	4	0.848	,,	,,
February	6.30	4	0.178	,,	,,
	2.27	4	0.687	,,	,,
	8.12	4	0.087	,,	,,
	1.84	4	0.764	,,	,,
	2.24	4	0.692	,,	,,
March	2.79	4	0.594	,,	,,
	3.00	4	0.557	,,	,,
	4.21	4	0.379	,,	,,
	3.30	4	0.508	,,	,,
	20.1	4	0.001	ns	Heterogeneous
April	9.38	4	0.052	sig	Homogeneous
	18.7	4	0.001	ns	Heterogeneous
	2.79	4	0.593	sig	Homogeneous
	35.1	4	0.000	ns	Heterogeneous
	8.94	4	0.063	sig	Homogeneous
May	13.0	4	0.011	ns	Heterogeneous
	1.05	4	0.902	sig	Homogeneous
	1.57	4	0.814	,,	,,
	2.24	4	0.692	,,	,,
	1.12	4	0.890	,,	,,
June	3.69	4	0.449	,,	,,
	1.46	4	0.833	,,	,,
	3.49	4	0.479	,,	,,
	5.32	4	0.256	,,	,,
	3.84	4	0.428	,,	,,
July	1.85	4	0.764	,,	,,
	2.62	4	0.624	,,	,,
	2.30	4	0.681	,,	,,
	1.37	4	0.849	,,	,,
	2.27	4	0.686	,,	,,
August	4.78	4	0.311	sig	Homogeneous
	13.4	4	0.009	ns	Heterogeneous
	4.65	4	0.325	sig	Homogeneous
	3.31	4	0.508	,,	,,
	1.42	4	0.841	,,	,,
September	1.73	4	0.785	,,	,,
	1.95	4	0.746	,,	,,
	1.58	4	0.812	,,	,,
	3.25	4	0.516	,,	,,
	5.45	4	0.244	,,	,,
October	3.74	4	0.443	,,	,,
	3.76	4	0.439	,,	,,
	0.37	4	0.985	,,	,,
	5.86	4	0.210	,,	,,
	6.53	4	0.163	,,	,,
November	3.10	4	0.542	,,	,,
	2.13	4	0.712	,,	,,
	4.51	4	0.341	,,	,,
	1.51	4	0.824	,,	,,
	4.34	4	0.362	,,	,,
December	5.49	4	0.241	,,	,,
	2.30	4	0.681	,,	,,
	2.11	4	0.716	,,	,,
	5.80	4	0.215	,,	,,
	3.58	4	0.466	,,	,,
April/March Combined	2.83	4	0.586	,,	,,
	4.49	4	0.343	,,	,,
	4.98	4	0.290	,,	,,
	2.65	4	0.618	,,	,,
	6.69	4	0.153	,,	,,

sig = significant ns = not significant

The tenability of the overall combined estimate of transition probability matrix was next tested for each month using the test statistic, Q(t), given in equation (6.4) and the results are shown in Table 6.3.

Table 6.3: Test of Homogeneity of Estimated Monthly Transition Probability Matrices

Subsample (t)	Q(t)	d. f	p-value	Sig	Remark
January	9.314	20	0.979	sig	Homogeneous
February	20.77	20	0.411	,,	Homogeneous
March	33.38	20	0.031	ns	Heterogeneous
April	74.90	20	0.000	,,	Heterogeneous
May	18.99	20	0.522	sig	Homogeneous
June	17.80	20	0.600	,,	,,
July	10.41	20	0.960	,,	,,
August	27.57	20	0.120	,,	,,
September	13.96	20	0.833	,,	,,
October	20.26	20	0.442	,,	,,
November	15.59	20	0.742	,,	,,
December	19.28	20	0.504	,,	,,
March/April Combined	21.65	20	0.360	sig	Homogeneous

sig = significant ns = not significant

Finally the homogeneity test was conducted using the test statistic H, given in equation (6.5) and the result is shown in table 6.4

Table 6.4: Overall Test for Homogeneity of Estimated Transition Probability Matrix

	H	d. f	p-value	Sig	Remark
12 Months	282.2	220	0.003	sig	Heterogeneous
Modified	195.6	200	0.575	ns	Homogeneous

Modified means combining March and April as one month to have eleven months overall

6.2 Discussion of Test Results

The result in Table 6.2 show quite clearly that only in few rows in the months of March and April were the estimates of the overall transition probability matrix rejected. It was noticed that for these months the data were particularly scanty. This could have been the possible reason for the rejections recorded. Consequently the data for the two months were combined and the row-wise test repeated. The results obtained now show no rejections in any of the rows. The result in Table 6.3 shows that separately for each month of the year and for March and April combined, estimate of the overall transition probability matrix is not rejected. The result of the joint test shown in Table 6.4 also indicates the same. Consequently it can be concluded for these test results that the transition probability matrix is reasonably time homogenous over the months of the year.

7. Markov Chain Properties

There are some Markov chain properties that can be useful in providing the basis for an informed decision during trading on the floor of the stock market for any of the two types of investors described above. These are examined below for the purpose of developing such criteria.

7.1 nth – Step Transition Probability Matrix

The estimated n^{th} – step transition probability matrix is given

$$P^n = \begin{pmatrix} 0.1348 & 0.2128 & 0.1064 & 0.2907 & 0.2553 \\ 0.0840 & 0.3319 & 0.1282 & 0.3782 & 0.0777 \\ 0.0613 & 0.2689 & 0.2453 & 0.3160 & 0.1085 \\ 0.0738 & 0.3650 & 0.1398 & 0.3593 & 0.0621 \\ 0.2057 & 0.3333 & 0.0638 & 0.2908 & 0.1064 \end{pmatrix}^n$$

Of interest here are cases when n = 1, 2,…,5. These are computed and shown in Table 7.1.

Table 7.1: Estimated n^{th} – Step Transition Probability Matrices

			Tomorrow				
		State	HA	LA	U	LD	HD
P	Today	HA	0.1348	0.2128	0.1064	0.2907	0.2553
		LA	0.0840	0.3319	0.1282	0.3782	0.0777
		U	0.0613	0.2689	0.2453	0.3160	0.1085
		LD	0.0738	0.3650	0.1398	0.3593	0.0621
		HD	0.2057	0.3333	0.0638	0.2908	0.1064
P^2	Today	HA	0.1165	0.3191	0.1247	0.3320	0.1077
		LA	0.0910	0.3264	0.1408	0.3489	0.0929
		U	0.0915	0.3198	0.1523	0.3421	0.0943
		LD	0.0885	0.3263	0.1431	0.3508	0.0913
		HD	0.1030	0.3132	0.1277	0.3414	0.1147
P^3	Today	HA	0.0968	0.3213	0.1372	0.3446	0.1001
		LA	0.0932	0.3239	0.1407	0.3468	0.0954
		U	0.0932	0.3229	0.1419	0.3460	0.0960
		LD	0.0928	0.3241	0.1412	0.3469	0.0950
		HD	0.0968	0.3230	0.1375	0.3448	0.0979
P^4	Today	HA	0.0945	0.3233	0.1397	0.3459	0.0966
		LA	0.0936	0.3236	0.1405	0.3464	0.0959
		U	0.0937	0.3234	0.1406	0.3463	0.0960
		LD	0.0935	0.3236	0.1406	0.3464	0.0958
		HD	0.0942	0.3232	0.1399	0.3461	0.0966
P^5	Today	HA	0.0939	0.3234	0.1403	0.3463	0.0961
		LA	0.0937	0.3235	0.1405	0.3463	0.0960
		U	0.0937	0.3235	0.1405	0.3463	0.0960
		LD	0.0937	0.3235	0.1405	0.3463	0.0960
		HD	0.0938	0.3235	0.1403	0.3463	0.0961

7.2 Vectors of Limiting Probability Distribution and Mean Recurrence Times

From the n^{th}-step transition probability matrices given in Table 7.1 above, it is clear that the underlying Markov chain is regular and therefore has a limiting probability distribution vector, say, $\boldsymbol{x} = (x_1, x_2, x_3, x_4, x_5)$. This can be estimated [17] by solving the system of equations given by

$$\boldsymbol{xP} = \boldsymbol{x} \qquad (7.1)$$

This was solved using R-Statistical package and the result is shown in Table 7.2a below.

Table 7.2 (a): Limiting Probability Distribution

State	HA	LA	U	LD	HD
$\hat{\boldsymbol{x}}$	0.0938	0.3235	0.1404	0.3463	0.0960

By interpretation, the estimate $\hat{\boldsymbol{x}}$ gives the proportion of time spent in each of the states of the Markov chain in the long run. Another useful concept is the recurrence time. It is defined as the length of time taken to return to state $\boldsymbol{S_i}$ for the first time given it started from state $\boldsymbol{S_i}$. Also because we are dealing with a regular Markov chain, the vector for the mean recurrence time denoted by **R** has components estimated by the inverse of the corresponding components of the vector $\hat{\boldsymbol{x}}$.This again is tabulated in Table 7.2b below

Table 7.2 (b): Mean Recurrence Times

State	HA	LA	U	LD	HD
$\hat{\boldsymbol{R}}$	11	3	7	3	10

Both estimates $\hat{\boldsymbol{x}}$ and $\hat{\boldsymbol{R}}$ will be useful in the discussion of criteria set out below.

7.3 Expected First Passage Times

Another useful concept is the mean first passage time.

Definition: If an ergodic Markov chain is started in $\boldsymbol{S_i}$, the expected (average) number of steps to reach state $\boldsymbol{S_j}$ for the first time is called the expected first passage time.

It should be noted that expected first passage time becomes the mean recurrence time when $\boldsymbol{i} = \boldsymbol{j}$ in the above definition; both concepts are closely related.
The fundamental matrix of a regular Markov chain [17] is given by

$$\boldsymbol{Z} = (\boldsymbol{I} - \boldsymbol{P} + \boldsymbol{X})^{-1} \qquad (7.2)$$

where $\boldsymbol{I}$ is the identity matrix, $\boldsymbol{P}$ is the transition probability matrix and $\boldsymbol{X}$ is a matrix whose rows are the limiting probability distribution vectors $\boldsymbol{x_i}$ obtained from $\boldsymbol{P}$.

Also the mean first passage matrix **M** for the ergodic chain is determinable [17] from $\boldsymbol{Z}$, and its $(\boldsymbol{i},\boldsymbol{j})^{th}$ component is

$$\boldsymbol{m_{ij}} = \frac{\boldsymbol{z_{jj} - z_{ij}}}{\boldsymbol{x_j}} \qquad (7.3)$$

where $\boldsymbol{z_{ij}}$ is the $(\boldsymbol{i},\boldsymbol{j})^{th}$ component of $\boldsymbol{Z}$ and $\boldsymbol{x_j}$ the $\boldsymbol{j}^{th}$ element of the vector $\boldsymbol{x_i}$.

Using equation 7.3, the estimated values for the matrix of expected first passage times $\boldsymbol{M}$ are computed and tabulated in Table 7.3.

Table 7.3: Matrix of Estimated Expected First Passage Times

		First Passage to				
		HA	LA	U	LD	HD
First Passage from	HA	0	3	8	3	9
	LA	12	0	8	3	11
	U	12	3	0	3	11
	LD	12	3	8	0	11
	HD	10	3	9	3	0

8. Criteria for Choice of Stock by Investors

The problem of stock investors is primarily in the predictive ability of the behaviour of the market. Thus the consideration of the magnitude of gain or loss as embedded in the formation of the Markov chain model described above is of immense importance in reflecting the nature of this problem. Specifically, investors may be interested, for example, in how strong their intended stock can bounce back after a drift, how often their particular stock is able to maintain the top gaining state, how many trading days it takes to reach the top gaining state from less gaining or losing state. All these interests are distinct for each of the short-term and long-term investors because they have different aims of trading on the floor of the stock market. Consequently, the criteria for their informed choice of stock will be different. These are separately developed below.

8.1 Criteria for a Short-Term Investor

The concern of the short term investor is in the tempo of the daily or short-term behaviour of the day closing percent change in stock prices. He or she wants to buy and sale at the shortest possible time in order to make gain. Consequently, for example, his or her focus could be on the probability of a price movement from high to low appreciation or from low to high appreciation. This price movement can easily be gauged by the transition probability matrix. Consequently for an informed choice the estimated n^{th}-step transition probability matrix for n = 1, 2, …, 5 could be set as a useful criteria because it will give the probabilities of price movement in each of the five days. These are tabulated in Table 7.1

Other useful indices are given below

1) The estimated proportion of time spent in each state in the long run. These are given in Table 7.2a
2) The mean recurrence times. These are also given in Table 7.2b
3) The matrix of the expected first passage times. These are given in Table 7.3

8.2 Criteria for a Long-Term Investor

The major concern of a long-term investor is in the purchase of stock so that he or she can earn dividend when due. As dividends are paid yearly, emphasis is shifted here to yearly price change. Consequently, the transition probability matrix must first be prepared in yearly form before the same criteria developed above for short-term investors can be correspondingly used as an informed choice.

8.3 Illustration with Data

The Ecobank data available is processed in daily form so that it can only be used for illustration for short term investors. The highlight of the properties of its transition probability matrix as shown by the result above is that it becomes stable after three days. In the long run, about forty percent (40%) of the times spent in the states are spent in each of the appreciation and depreciation states and it takes about two weeks to return back to appreciation and depreciation states respectively starting from each of these states.

The results from the four criteria above presents the short term investor with simple tools that can be of help in taking decision while doing daily trading on the floor of the stock exchange market. Its use is illustrated as follows.

Step 1 ***Examine the estimated one-step transition probability matrix***

From the estimated first step transition probability matrix given in Table 6.1, it is clear that from the high depreciation state it is more likely to transit to an appreciation state the next trading day, and while in the high appreciation state, it is more likely to transit to a depreciation state the next trading day. This implies that it is best to buy the share when it is in high depreciation state.

Step 2 ***Examine the matrix of the estimated first passage times***

From the matrix of estimated expected first passage times given in Table 7.4, it is clear that from high appreciation state it will transit to the low depreciation state the third trading day. This implies we sell the shares quickly within two days when it is in the high appreciation state.

Step 3 ***Decision***

Buy stock when it is in the high depreciation state and sell within two days when next the price has risen to the high appreciation state.

9. Conclusion

It has been shown quiet clearly that by modelling the behaviour of stock price using Markov chain model, inference derivable from its properties can be set as criteria useful for short-term and long-term investors in taking informed decisions during trading on the floor of the stock market. Such use of the Markov chain model is consequently recommended for investors.

References

[1] A. Bairagi, S. A. CH. Kakaty (2015), “Analysis of Stock Market Price Behaviour: A Markov Chain Approach,” International Journal of Recent Scientific Research, Vol.6, Issue 10, pp 7061-7066.
[2] G. McQueen, S. Thorley (1991), “Are Stocks Returns Predictable? A Test Using Markov Chain,” The Journal of Finance, vol. 56, No 1, 49-60.
[3] S. Neewi, P.O Asagba, L.G Kabari (2013), “Predicting the Nigerian Stock Using Artificial Neural Network,” European Journal of Computer Science and Information, Vol.1, No. 1 pp 30-39.
[4] S.O Abdulsalam, K.S Adewole, R.G Jimoh (2011), “Stock Trend Prediction using Regression: A Data Mining Approach,” ARPN Journal of Systems and Software, Vol. 1, No 4, pp 154-157.
[5] A. A Adebiyi, A.O Adewunmi, C.K Ayo (2014), “Stock Price Prediction using ARIMA Model,” UKSim-AMSS 16th International Conference on Computer Modelling and Simulation, 2014.
[6] Z. Qing-Xin (2014), “Application of Weighted Markov Chain in Stock Price Forecasting,” Advanced Science and Technology Letters, Vol. 53 (ICM 2014), pp 241-244, [online] Available http://dx.doi.org/10.14257/astl.2014.53.50
[7] V Nierderhoffer, M. Osborne (1966), “Market Making Reversal on Stock Exchange,” Journal of American Statistical Association, 61,897-916.

[8] B.D. Fieltz (1971), "Stationarity of Random data: Some Implications for the Distribution of Stock Price Changes," Journal of Financial and Quantitative Analysis, Volume 6, pp 1025-1034
[9] B.D Fieltz (1975), "On Stationarity of Transition Probability Matrices of Common Stocks" Journal of Financial and Quantitative Analysis, Volume 10,pp 327-339.
[10] B.D Fieltz, T.N Bhargava (1973), "The Behaviour of Stock Price Relatives - A Markovian Analysis," Operations Research, Volume 21, No 6, pp 1183- 1199.
[11] L, Bachelier (1914), "Le jeu, la chance, et le hazard," Flammarion, Paris.
[12] E.F Fama (1965), "The Behaviour of Stock Market Prices," Journal of Business, 38, 34-105
[13] A Lendasse, E De Bodt, V Wertz, M Verleysen (2008), "Non Linear Financial Time Series Forecasting – Application to Bel 20 Stock Market Index," European Journal of Economics and Social Systems, 14(1), pp 81-91
[14] J.I Eseoghene (2011), "The Long Run Prospects of Stocks in the Nigerian Capital Market: A Markovian Analysis," JORIND (9) 1, June 2011, ISSN 1596-8303.
[15] E.O Christian, K.S Timothy (2014), "On Predicting the Long Run Behaviour of Nigerian Bank Stock Prices: A Markov Chain Approach," American Journal of Mathematics and Statistics, Vol.2, No 4, 212 - 215.
[16] M.A Raheem, P.O Ezeupe (2014), "A Three State Markov Approach to Predicting Asset Returns," Statistics and Information Modelling Research Group, Department of Engineering and Mathematics, Sheffield Hallam University, Sheffield S1 1WB, UK.
[17] C.M Grinstead, J.L Snell, Introduction to Probability, American Mathematical Society, 2 Revised Edition, July 1, 1997.