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Adaptive Step Firefly Algorithm Based on Population Diversity

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Abstract: ***As a new swarm intelligence optimization method, firefly algorithm shows good performance on many complex optimization problems. However, due to the fixed parameters of FA, it is difficult to adapt to environmental changing during the iteration process, and FA easily lose its diversity and lead to premature convergence. In this paper, an adaptive step firefly algorithm based on population diversity called DASFA is proposed to improve the performance of FA.The DASFA designed an adaptive step which is decreasing as the search process and regulated by population diversity, it could help the algorithm maintains high diversity to getting out of the local optimal and finding the optimal value eventually. Experiments are conducted on ten classic benchmark functions, the results show that DASFA achieves better performance than FA and some its variants.***

Keywords: firefly algorithm, adaptive step, population diversity, swarm intelligence

1. Introduction

Firefly algorithm (FA)is a new swarm intelligence algorithm which inspired by social behavior of fireflies in the nature and developed by Xin-She Yang[1]. Preliminary studies showed that the FA outperforms the genetic algorithm and particle swarm optimization[2], and it has been shown that FA is very efficient in dealing with multimodal, global optimization problems. FA has attracted much attention and has been applied to many applications, such as economic dispatch problems[3], image processing [4], the job shop scheduling problem[5], antenna array design [6], etc. However, FA is same as other swarm intelligence algorithms, it exists the phenomenon of premature, slow rate of convergence and the deficiency of local search ability, which affect the performance of FA. How to balance the contradiction between exploration and exploitation is the primary problem for FA. Exploration means to search various solutions to explore the search space on the integrity area, while exploitation means to focus on the search in a local region to find better solution.

The appropriate control parameter setting is one way to resolve this conflict, while the main parameters of FA are fixed, which is difficult to cope with complex situations. Therefore, it is necessity for adapting parameters during a search process. Many scholars have done much research in these fields. Fister et al. [7]proposed a memetic firefly algorithm (MFA), which the parameters of α and β are dynamically adjusted with the search process. Experimental results show that the MFA performance than FA. Gandomi et al.[8]introduce chaos into FA to increase its global search mobility for robust global optimization. Various chaotic maps are utilized to tune the attractive movement of the fireflies in the algorithm. The results suggest that some chaotic FAs can clearly outperform FA.Yu et al. [9] proposed a variable step size FA (VSSFA), which a dynamical method is used to update the parameter α. In 2016, Yu et al. [10]presented a nonlinear time-varying step strategy for FA (NTSFA), which used a nonlinear decreasing and time-varying step-size to balance FA's ability of exploration and exploitation. Computational results show that VSSFA and NTSFA achieve better solutions than FA on a set of low-dimensional benchmark functions. However, in our experiments (see later), results show that both of them can hardly obtain valid solutions for some high-dimensional problems (D = 15, 30, etc).

On the other hand, population diversity plays a crucial role for measuring the ability of exploration and exploitation implicitly in the swarm intelligence algorithm. Diversity means the difference between individuals, when premature convergence occurs, the individuals tend to be consistent and the diversity of the population is very low, it is hard to jump from the local optimal. How to maintain a high population diversity is also a question worth considering. Yu et al. [11]proposed a modified FA which used a diversity threshold value to guide the algorithm to alternate between exploring and exploiting behavior. Experiments showed that the proposed algorithm can improve the performance of the basic FA. However, how to determine the diversity threshold value is a problem to be solved.

In this paper, we proposed an improved FA tries to balance the exploration and exploitation during the search process. The new approach is called DASFA, which presents an adaptive step strategy with population diversity. Experimental results show that DASFA performs better than FA, VSSFA, and NTSFA. The rest of the paper is organized as follows. Section 2 presents the basic FA. The proposed approach DASFA is described in Section 3. Section 4 is dedicated to the experimental results and discussion. The last section concludes the paper and gives the prospect of future work.

2. Firefly algorithm

The FA is simulate cluster behavior of the fireflies attract each other. In order to simplify the discussion, FA is based upon three idealized rules as following [1]:

1) All fireflies are unsexing so that one firefly is attracted to other fireflies regardless of their sexes. The brightness of a firefly is determined by the scenario of the objective function, the better position has the higher brightness.

2) Attractiveness is proportional to their brightness, thus for any two fireflies, the less brighter one will be attracted to the brighter one and the attractiveness decreases as their distance increases.
3) All fireflies move to brighter firefly.If there is no brighter one than a particular firefly, it will move randomly.

According to the above rules,the main parameters which decide the efficiency of FA are the variations of light intensity and attractiveness between neighboring fireflies. Light intensity decreases with the distance from its source,and light is also absorbed in the media, so the light instensity should vary with the distance r^2.Therefore,the attractiveness can be approximated as the following form:

$$\mathrm{I(r)} = I_0 e^{-\gamma r^2} \quad (1)$$

where I_0 denotes the light intensity of the source and γ is the light absorption coefficient of propagation media which can be taken as a constant.

The firefly's attractiveness is proportional to the brightness,the attractiveness β can be defined in similar pattern as I(r):

$$\beta(\mathrm{r}) = \beta_0 e^{-\gamma r^2} \quad (2)$$

where β_0 is the attractiveness at r =0.

The distance between any two fireflies x_i and x_j is express as the Euclidean distance in equation(3):

$$r_{ij} = \|x_i - x_j\| = \sqrt{\sum_{k=1}^{D}(x_{i,k} - x_{j,k})^2} \quad (3)$$

where D is the problem dimension and $x_{i,k}$ is the Kth component of the firefly x_i.

The firefly x_i is attracted to another more brighter firefly x_j,the movement of x_i firefly is defined by:

$$x_{i+1} = x_i + \beta_0 e^{-\gamma r_{ij}^2}\left(x_j - x_i\right) + \alpha\left(rand - \frac{1}{2}\right) \quad (4)$$

where the second term is due to the attraction, while the third term is randomization with α being the randomization parameter. **rand** is a random number generator uniformly distributed in [0,1]. For most cases in our implementation,we can take β_0= 1, γ=1 and $\alpha \in$[0,1]. As described above mentioned, FA can be summarized as the pseudo code shown in **code 1**. *Objective function* $f(\boldsymbol{x}), \boldsymbol{x} = (x_1, x_2, \dots, x_D)^T$

1. *Generate initial population of fireflies* $\boldsymbol{x}_i$ *(i = 1,2, ..., n)*
2. *Initialize* β_0, γ *and* α
3. *Light intensity* I_i *at* $\boldsymbol{x}_i$ *is determined by f(*$\boldsymbol{x}_i$*)*
4. ***while*** *(t <MaxGen)*
5. ***for*** *i = 1 : n all n fireflies*
6. ***for*** *j = 1 : n all n fireflies*
7. ***if*** *(*$I_j > I_i$*)*
8. *Move firefly i towards j in d-dimension(Apply Eq(4))*
9. ***end if***
10. *Attractiveness varies with distance r via* $exp(-\gamma r^2)$
11. *Evaluate new solutions and update light intensity*
12. ***end for*** *j*
13. ***end for*** *i*
14. *Rank the fireflies and find the current best*
15. ***end while***
16. *Postprocess results and visualization*

Code 1：Pseudo code of FA

3. Proposed Approach

Typically, exploration and exploitation are implicitly measured using population diversity. When individuals are distributed over the whole search space, the population has high diversity.Otherwise, it has low diversity.The population diversity of the swarm can be computed as[12]:

$$\mathrm{Div(t)} = \frac{1}{NL}\sum_{i=1}^{N}\sqrt{\sum_{j=1}^{D}(x_{ij}(t) - \overline{x_j(t)})^2} \quad (5)$$

where **N** is the population size, **L** is the length of longest the diagonal in the search space, **D** is the dimensionality of the problem, $x_{ij}(t)$ is the j-th value of the i-th firefly and $\overline{x_j(t)}$ is the average of the j-dimension over all fireflies, that is

$$\overline{x_j(t)} = \frac{\sum_{i=1}^{N} x_{ij}(t)}{N} \quad (6)$$

Swarm intelligence algorithms have to use stochastic components to a certain degree [13], it is the random step for FA. Randomness increases the diversity of population which enables FA to have the ability to find more promising solutions and jump out of local optimum. However, too much randomness may slow down the convergence of FA, while too little randomness may lead FA into the local optima. A proper step size which are changed according to some form of feedback from the search process is very important for balance exploration and exploitation. In general, a larger step is needed to accelerate the search for the region where the optimum value is located in the early stage of the algorithm, while the latter the smaller step is adopted to enhance ability of the local search. Therefore, we designed a new adaptive step, which are changed and related to population diversity during the running. The step α can be calculated as following:

$$\alpha(\mathrm{t}) = \mathrm{ceil}\left(\frac{\mathrm{D}}{10}\right) * \exp\left(-2 * \frac{\mathrm{t}}{\mathrm{MaxGen}}\right) * 1/\mathrm{Div(t)} \quad (7)$$

where **D** is the dimensions of problem, **ceil** is a function round up to an integer, **MaxGen** denotes the maximum iteration number and Div(t) is the population diversity as equation(5) defined.

The proposed new step in this paper decreases with the iterative process of the algorithm and population diversity plays a regulatory role. When the population diversity is small, it means the difference between individuals is very weak and the algorithm may fall into the local optimum, the lager step will be adopted to increase the population diversityand jump out of local optimum. When it has high population diversity, reducing the step length appropriately to avoid missing the optimal value. It would be beneficial to balance the ability of global exploration and local exploitation. In addition, the **ceil** function is introduced to improve its applicability, especially for the high-dimensional problem. Due to the large search space for multidimensional problems, larger steps are required to perform the search procedure of the algorithm.

4. Experiments and results

In this section, numeric experiments are desigened to study the performance of DASFA. For comparison, FA, VSSFA, NTSFA and DASFA are tested on ten benchmark functions and the experimental results were analyzed.

4.1 Benchmark functions

Ten benchmark functions including unimodal and multimodal functions which have been extensively in literatures[1,9-11]. All test functions are minimization problems and listed in Table 1.

Table 1: Benchmark functions used in the experiments,where D is the problem dimension

Name	Function	Search Range	Global Optimun
Sphere	$f_1 = \sum_{i=1}^{D} x_i^2$	$[-100,100]^D$	0
Schwefel 2.22	$f_2 = \sum_{i=1}^{D} \lvert x_i \rvert + \prod_{i=1}^{D} x_i$	$[-10, 10]^D$	0
Schwefel 1.2	$f_3 = \sum_{i=1}^{D} (\sum_{j=1}^{i} x_j)^2$	$[-100,100]^D$	0
Rosenbrock	$f_4 = \sum_{i=1}^{D-1} [100(x_{i+1} - x_i^2)^2 + (1 - x_i^2)^2]$	$[-10, 10]^D$	0
Step	$f_5 = \sum_{i=1}^{D} \lfloor x_i + 0.5 \rfloor^2$	$[-100,100]^D$	0
Quartic with noise	$f_6 = \sum_{i=1}^{D} i \cdot x_i^4 + random$	$[-1.28,1.28]^D$	0
Schwefel 2.26	$f_7 = \sum_{i=1}^{D} (-x_i \cdot \sin(\sqrt{\lvert x_i \rvert})) + D \cdot 418.9829$	$[-500,500]^D$	0
Rastrigin	$f_8 = \sum_{i=1}^{D} [x_i^2 - 10\cos(2\pi x_i) + 10]$	$[-5.12,5.12]^D$	0
Rastrigin 2.0	$f_9 = \sum_{i=1}^{D} [y_i^2 - 10\cos(2\pi y_i) + 10]$ Where $y_i = \begin{cases} x_i & \lvert x_i \rvert < 0.5 \\ \frac{round(2x_i)}{2} & \lvert x_i \rvert \geq 0.5 \end{cases}$	$[-5.12,5.12]^D$	0
Ackley	$f_{10} = -20\exp(-0.2\sqrt{\frac{1}{D}\sum_{i=1}^{D} x_i^2})$	$[-32,32]^D$	0

$$- \exp\left(\frac{1}{D}\sum_{i=1}^{D} \cos(2\pi x_i)\right) + 20 + e$$

4.2 Settings for the experiments

In the experiments, the same population size, the maximum iterations number and problems dimension are used for all algorithms, and they are set to 40, 2000 and 30. Other parameters are consistent with the original paper. In order to eliminate the impact imposed by stochastic, all the experiments are run 30 times in MATLAB 2011b independently and the results are recorded.

4.3 Results and discussion

Table 2 summarizes the computational results of the FA, VSSFA, NTSFA, and DASFA, where “Mean” denotes the mean best fitnessvalue,”Best” represents the better of the best fitness value, "Worst”denotesthe worst of the best fitness value, and “Stdev” is the standard deviation of the best fitness value.It can be seen that DASFA achieves better results than FA,VSSFA and NTSFA on all test functions except for f4, f6 and f9. It is worth mentioning that DASFA obtains the theoretical optimal value on f5. In addition, FA, VSSFA and NTSFA have terrible results for all test function, except for a few special cases such as VSSFA on f6. For f4 and f9, DASFA achieves the better mean, worst and stdev value, while FA got the best value.This means that DASFA can get reasonable results and has more stable performance. For f6, DASFA find the suboptimum solutions, although the gap is not obvious,VSSFA can search more accurate results.

Due to a small search domain for the Quartic with noise function, the small step may better for finding the optimal value. In summary, the results demonstrated that DASFA has the strong ability of global exploration and local exploitation than other algorithms.

Table 2: Computational results achieved by FA, VSSFA, NTSFA and DASFA

Functions	Algorithms	Mean	Worst	Best	Stdev
f1	FA	6.52E+04	7.74E+04	3.71E+04	9.77E+03
	VSSFA	6.45E+04	7.93E+04	4.15E+04	8.42E+03
	NTSFA	6.75E+04	8.11E+04	5.18E+04	7.49E+03
	DASFA	**4.99E-06**	**2.06E-05**	**2.28E-09**	**5.64E-06**
f2	FA	4.68E-01	1.94E+00	8.26E-04	5.17E-01
	VSSFA	3.04E-01	9.86E-01	1.60E-05	3.07E-01
	NTSFA	7.55E+01	1.51E+02	6.89E-05	4.06E+01
	DSCFA	**3.42E-03**	**1.28E-02**	**1.24E-05**	**2.57E-03**
f3	FA	1.27E+05	2.10E+05	6.34E+04	3.81E+04

	VSSFA	9.69E+04	1.31E+05	6.39E+04	1.94E+04
	NTSFA	1.21E+05	2.00E+05	6.75E+04	3.03E+04
	DASFA	**1.30E-03**	**1.03E-02**	**4.72E-08**	**2.13E-03**
f4	FA	2.62E+05	5.26E+05	5.75E+00	1.87E+05
	VSSFA	6.27E+00	4.57E+01	**1.17E-09**	9.67E+00
	NTSFA	2.64E+05	5.66E+05	1.57E+01	1.45E+05
	DASFA	**2.53E-05**	**1.00E-04**	1.67E-07	**3.19E-05**
f5	FA	6.43E+04	7.81E+04	3.71E+04	8.26E+03
	VSSFA	6.36E+04	7.34E+04	4.53E+04	7.08E+03
	NTSFA	6.22E+04	7.38E+04	4.70E+04	6.69E+03
	DASFA	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**
f6	FA	1.56E-02	5.14E-02	1.22E-03	1.28E-02
	VSSFA	**2.06E-04**	**7.17E-04**	**6.80E-06**	**1.53E-04**
	NTSFA	2.18E-03	7.94E-03	3.89E-04	1.62E-03
	DASFA	8.04E-04	2.69E-03	5.45E-05	5.26E-04
f7	FA	1.04E+04	1.12E+04	9.02E+03	4.77E+02
	VSSFA	1.02E+04	1.11E+04	9.32E+03	**4.40E+02**
	NTSFA	1.03E+04	1.11E+04	9.27E+03	1.03E+04
	DASFA	**5.51E+03**	**6.56E+03**	**4.24E+03**	5.79E+02
f8	FA	1.10E+01	3.01E+01	1.15E-05	1.10E+01
	VSSFA	7.10E-02	2.50E-01	3.62E-04	6.34E-02
	NTSFA	1.12E+01	4.03E+01	7.23E-11	1.35E+01
	DASFA	**3.74E-05**	**1.57E-04**	**3.12E-11**	**4.75E-05**
f9	FA	2.52E+01	3.00E+01	**7.29E-08**	9.91E+00
	VSSFA	3.17E+00	3.00E+01	1.63E-05	8.95E+00
	NTSFA	2.61E+01	3.00E+01	8.42E-08	8.46E+00
	DASFA	**6.08E-05**	**5.94E-04**	5.19E-07	**1.30E-04**
f10	FA	1.35E+00	1.44E+00	1.17E+00	6.32E-02
	VSSFA	8.81E-01	1.21E+00	3.76E-01	2.15E-01
	NTSFA	1.37E+00	1.47E+00	1.25E+00	5.77E-02
	DASFA	**1.83E-04**	**1.09E-03**	**5.13E-06**	**2.12E-04**

Figure 1 shows convergence graphs of DASFA and the other FA variants. As we can see, DASFA converges much faster than the other three algorithms on all test function except for f6 during the whole search process. It implies that DASFA has better performance of exploration and high diversity to get rid of trapping into local optimum and approach the global optima finally. For f4, VSSFA converges faster and more accurately than all other FAs (DASFA included).The reason is because the small step for VSSFA is more suitable for a small search domain, while high diversity can not lead the algorithm to focus on local search so that it is difficult to achieve the desired results.

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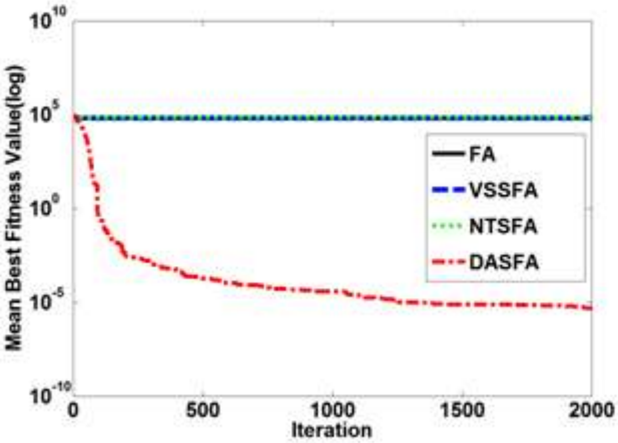
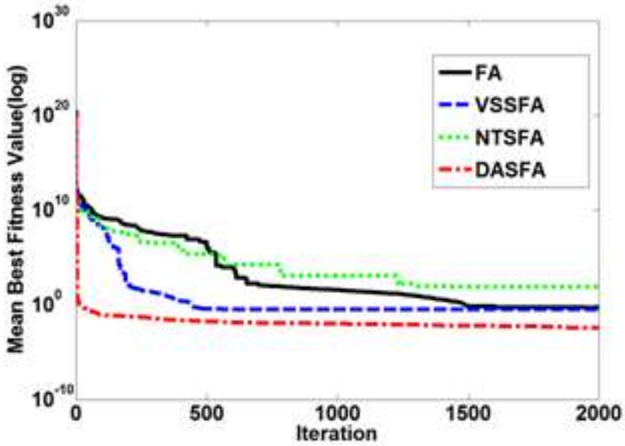


(a)f1



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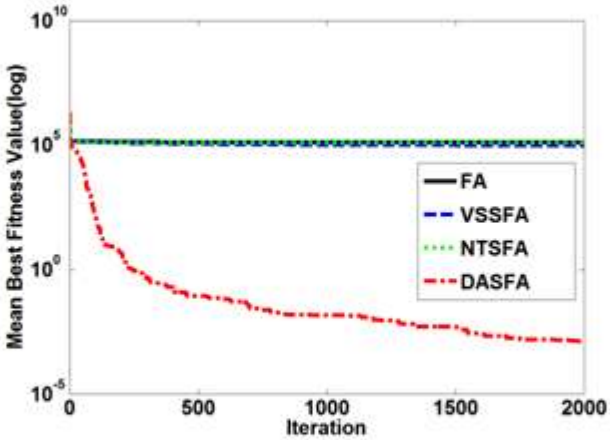

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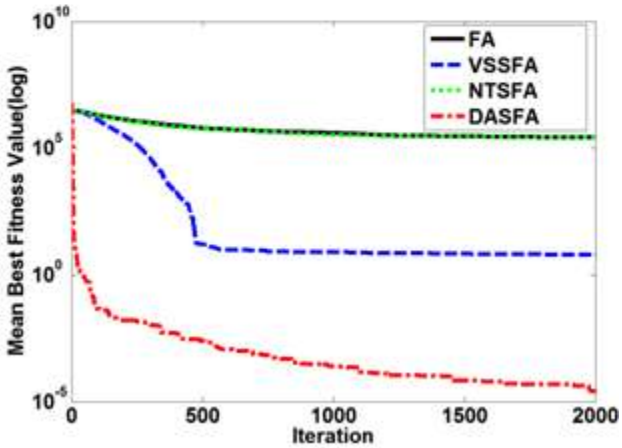

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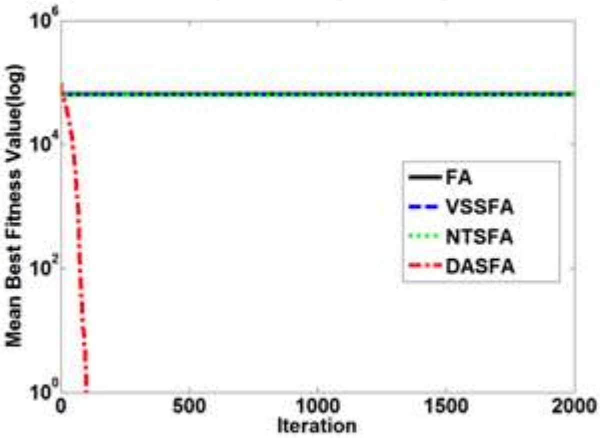

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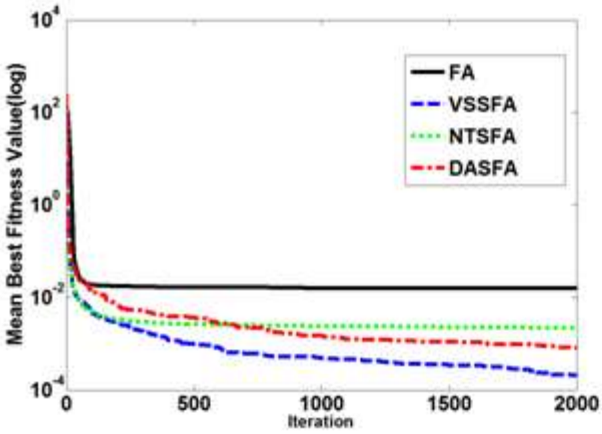

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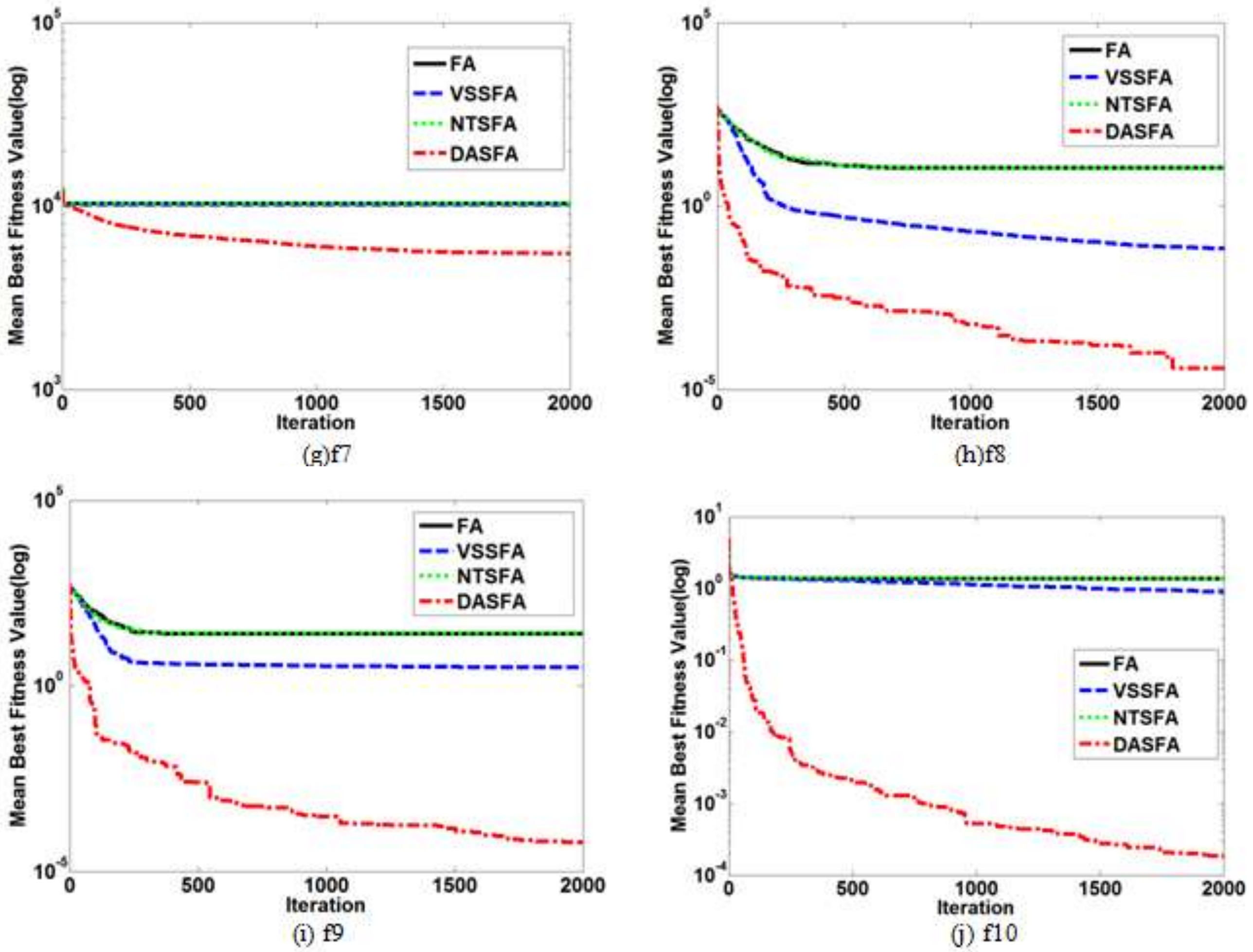

Figure 1: The convergence curves of different algorithms on benchmark functions

4.4 Experiment on different dimension problems

In order to investigate the performance of DASFA on different dimensional problems, D=5, 15, 30, and 50 were tested in the following experiments. The experimental environment agrees with the previous experiment except the problem dimension. Table 3 summarizes the computational results achieved by FA,VSSFA, NTSFA and DASFA on different dimension. The best results are indicated in bold. The results show that DASFA performs than other three algorithms on D is 5, 15, 30 and 50 except for VSSFA on D is 30 and 50 for function f6.It can be demonstrated that FA, VSSFA and NTSFA can hardly obtain valid solutions for some high-dimensional problems. Moreover, keeping high population diversity is particularly necessary to the search process for the algorithm as the results shown.

Table 3: Computational results achieved byFA, VSSFA, NTSFA and DASFA on different dimensions

Dimension	Functions / Algorithms	f1 mean	f2 mean	f3 mean	f4 mean	f5 mean
D=5	FA	4.44E+03	4.86E-01	4.28E+03	3.68E+00	4.01E+03
	VSSFA	4.13E+03	2.56E-01	3.65E+03	2.34E+00	4.12E+03
	NTSFA	4.13E+03	1.27E+00	4.41E+03	1.60E+03	4.14E+03
	DASFA	**4.65E-03**	**3.27E-03**	**4.48E-05**	**1.44E-04**	**1.50E+01**
D=15	FA	2.61E+04	4.94E-01	2.97E+04	7.90E+00	2.72E+04
	VSSFA	2.68E+04	2.73E-01	2.50E+04	2.79E+00	2.58E+04
	NTSFA	2.74E+04	1.43E+01	3.76E+04	6.28E+04	2.68E+04
	DASFA	**5.76E-06**	**8.45E-04**	**2.32E-04**	**1.46E-05**	**0.00E+00**
D=30	FA	6.52E+04	4.68E-01	1.27E+05	2.62E+05	6.43E+04
	VSSFA	6.45E+04	3.04E-01	9.69E+04	6.27E+00	6.36E+04
	NTSFA	6.75E+04	7.55E+01	1.21E+05	2.64E+05	6.22E+04

	DASFA	**4.99E-06**	**3.42E-03**	**1.30E-03**	**2.53E-05**	**0.00E+00**
D=50	FA	1.23E+05	4.18E-01	2.88E+05	2.41E+01	1.26E+05
	VSSFA	1.22E+05	3.03E-01	2.51E+05	1.50E+00	1.22E+05
	NTSFA	1.23E+05	1.38E+02	3.15E+05	4.93E+05	1.23E+05
	DASFA	**1.20E-05**	**5.41E-03**	**1.16E-02**	**9.45E-05**	**0.00E+00**

Table 3: Computational results achieved byFA, VSSFA, NTSFA and DASFA on different dimensions (Continued)

Dimension	Functions / Algorithms	f6 mean	f7 mean	f8 mean	f9 mean	f10 mean
D=5	FA	7.67E-04	1.20E+03	6.60E+00	5.20E+00	5.33E-01
	VSSFA	2.14E-03	1.14E+03	2.18E+00	4.76E+00	2.62E-01
	NTSFA	1.16E-03	1.12E+03	6.91E+00	5.22E+00	7.16E-01
	DASFA	**2.39E-04**	**1.04E+03**	**1.88E-05**	**7.17E-06**	**1.25E-03**
D=15	FA	1.79E-02	4.67E+03	2.45E+00	1.17E+01	1.08E+00
	VSSFA	1.38E-03	4.58E+03	3.18E-01	4.21E+00	7.19E-01
	NTSFA	7.31E-03	4.70E+03	1.33E+01	1.46E+01	1.20E+00
	DASFA	**4.68E-04**	**3.49E+03**	**1.54E-05**	**1.43E-05**	**1.90E-04**
D=30	FA	1.56E-02	1.04E+04	1.10E+01	2.52E+01	1.35E+00
	VSSFA	**2.06E-04**	1.02E+04	7.10E-02	3.17E+00	8.81E-01
	NTSFA	2.18E-03	1.03E+04	1.12E+01	2.61E+01	1.37E+00
	DASFA	8.04E-04	**5.51E+03**	**3.74E-05**	**6.08E-05**	**1.83E-04**
D=50	FA	5.14E-03	1.80E+04	6.73E-01	2.05E+01	1.35E+00
	VSSFA	**2.56E-04**	1.81E+04	5.28E-02	1.79E+00	8.52E-01
	NTSFA	1.23E-03	1.82E+04	5.63E+00	3.41E+01	1.43E+00
	DASFA	1.26E-03	**1.90E+03**	**2.78E-04**	**1.32E-04**	**2.22E-04**

5. Conclusion

To improve the performance of FA, an adaptive step firefly algorithm based on population diversity (DASFA) was proposed in this paper. The DASFA adoptsan adaptive step which is decreasing as the search process and regulated by population diversity, it could help the algorithm maintains high diversity to getting out of the local optimal and finding the optimal value eventually. Experiments are conducted on ten classic benchmark functions, the results show that DASFA achieves better performance with faster convergence rate and precision than FA, VSSFA and NTSFA on the majority of test functions. Moreover, it also performs well in different dimensions of the problem. However, DASFA fails to achieve the better results on certain issues such as function f6, how to enhance the local search ability of the algorithm is a problem we need to solve in the future.

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