Some Problems on Semi C-Reducible Finsler Manifold

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Abstract: Present paper deals to the study of semi C-reducible Finsler manifold. In this paper, we have obtained some important theorems on C-reducible Finsler manifold and semi C-reducible Finsler manifold.

Keywords: (h)hv-torsion tensor, angular metric tensor, C-2 like, C-reducible, semi C-reducible Finsler manifold.

1. Semi C-Reducible Finsler Manifold

A Finsler manifold $F^n$ is said to be C-2 like Finsler manifold, if the (h)hv-torsion tensor $C_{ijk}$ satisfies the following condition

$$(1.1) \quad C_{ijk} = (1/C^2)C_iC_jC_k.$$ 

Wherein

$$(1.2) \quad g^{ik}C_{ijk} = C_i$$

is the contracted torsion tensor.

A Finsler manifold $F^n$ is said to be C-reducible Finsler manifold, if the (h)hv-torsion tensor $C_{ijk}$ satisfies the following condition

$$(1.3) \quad C_{ijk} = \{(l/(n+1))\lfloor h_{ik}C_j + h_{jk}C_i + h_{ij}C_k \rceil + \lfloor q/C^2\rfloor C_iC_jC_k \}.$$ 

Wherein

$$(1.4) \quad h_{ij} = g_{ij} - l_{ij}$$

is angular metric tensor.

A Finsler manifold $F^n$ is said to be semi C-reducible Finsler manifold, if the (h)hv-torsion tensor $C_{ijk}$ satisfies the following condition

$$(1.5) \quad C_{ijk} = \{(p/(n+1))\lfloor h_{ik}C_j + h_{jk}C_i + h_{ij}C_k \rceil + (q/C^2)C_iC_jC_k \}.$$ 

In this regard, we have the following theorems:

**Theorem 1.1:**

In the C-reducible Finsler manifold, if the angular metric tensor is symmetric then (h)hv-torsion tensor is also symmetric with respect to first two indices.

**Proof:**

Interchanging the indices i and j in equation (1.3), we obtain

$$(1.6) \quad C_{ijk} = \{(l/(n+1))\lfloor h_{ij}C_k + h_{jk}C_i + h_{ki}C_j \rceil + \lfloor q/C^2\rfloor C_iC_jC_k \}.$$ 

If the angular metric tensor $h_{ij}$ is symmetric then the equation (1.6) becomes

$$(1.7) \quad C_{ijk} = \{(l/(n+1))\lfloor h_{ik}C_j + h_{jk}C_i + h_{ij}C_k \rceil \}.$$ 

By virtue of equations (1.3) and (1.7), we get

$$(1.8) \quad C_{ijk} = C_{jik}.$$ 

Hence, the (h)hv-torsion tensor is symmetric with respect to first two indices in the C-reducible Finsler manifold.

**Theorem 1.2:**

In the C-reducible Finsler manifold, if the angular metric tensor is symmetric then (h)hv-torsion tensor is also symmetric with respect to last two indices.

**Proof:**

Interchanging the indices j and k in equation (1.3), we get

$$(1.9) \quad C_{ijk} = \{(l/(n+1))\lfloor h_{ik}C_j + h_{jk}C_i + h_{ij}C_k \rceil \}.$$ 

If the angular metric tensor $h_{ij}$ is symmetric then the equation (1.9) becomes

$$(1.10) \quad C_{ijk} = \{(l/(n+1))\lfloor h_{ij}C_k + h_{jk}C_i + h_{ki}C_j \rceil \}.$$ 

From equations (1.3) and (1.10), we obtain

$$(1.11) \quad C_{ijk} = C_{jik}.$$ 

Hence, the (h)hv-torsion tensor is symmetric with respect to last two indices in the C-reducible Finsler manifold.

**Theorem 1.3:**

If the (h)hv-torsion tensor is symmetric with respect to first two indices in C-2 like and C-reducible Finsler manifold then (h)hv-torsion tensor is also symmetric with respect to first two indices in semi C-reducible Finsler manifold.

**Proof:**

By virtue of equations (1.1), (1.3), (1.5) and (1.8), we obtain the required result.

**Theorem 1.4:**

If the (h)hv-torsion tensor is symmetric with respect to last two indices in C-2 like and C-reducible Finsler manifold then (h)hv-torsion tensor is also symmetric with respect to last two indices in semi C-reducible Finsler manifold.

**Proof:**

By virtue of equations (1.1), (1.3), (1.5) and (1.11), we obtain the required result.

References


**Author Profile**

T.S. Chauhan (Tarkeshwar Singh Chauhan) received Ph.D. and D.Sc. degrees in Mathematics from M.J.P.R.U., Bareilly in 1992 and 2008 respectively. He has been working in Maths dept., Bareilly College, Bareilly since 1990 and now he is an Associate Professor. Under his guidance nearly 25 candidates have been awarded Ph.D. degree. Several papers and books are published in different branches in different publications under him.