International Journal of Science and Research (IJSR)
ISSN (Online): 2319-7064
Index Copernicus Value (2015): 78.96 | Impact Factor (2015): 6.391

Some Problems on Semi C-Reducible Finsler Manifold

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Abstract: ***Present paper deals to the study of semi C-reducible Finsler manifold. In this paper, we have obtained some important theorems on C-reducible Finsler manifold and semi C-reducible Finsler manifold.***

Keywords: (h)hv-torsion tensor, angular metric tensor, C-2 like, C-reducible, semi C-reducible Finsler manifold.

1. Semi C-Reducible Finsler Manifold

A Finsler manifold F^n is said to be C-2 like Finsler manifold, if the (h)hv-torsion tensor C_{ijk} satisfies the following condition

(1.1) $C_{ijk} = (1/C^2)C_iC_jC_k$.

Wherein
(1.2) $g^{jk} C_{ijk} = C_i$
is the contracted torsion tensor.

A Finsler manifold F^n is said to be C-reducible Finsler manifold, if the (h)hv-torsion tensor C_{ijk} satisfies the following condition
(1.3) $C_{ijk} = \{1/(n+1)\}(h_{ij}C_k + h_{jk}C_i + h_{ki}C_j)$.
Wherein
(1.4) $h_{ij} = g_{ij} - l_il_j$
is angular metric tensor.

A Finsler manifold F^n is said to be semi C-reducible Finsler manifold, if the (h)hv-torsion tensor C_{ijk} satisfies the following condition
(1.5) $C_{ijk} = \{p/(n+1)\}(h_{ij}C_k+ h_{jk}C_i+ h_{ki}C_j) + (q/C^2)C_iC_jC_k$.
In this regard, we have the following theorems:

Theorem 1.1:

In the C-reducible Finsler manifold, if the angular metric tensor is symmetric then (h)hv-torsion tensor is also symmetric with respect to first two indices.

Proof:

Interchanging the indices i and j in equation (1.3), we obtain
(1.6) $C_{jik} = \{1/(n+1)\}(h_{ji}C_k + h_{ik}C_j + h_{kj}C_i)$

If the angular metric tensor h_{ij} is symmetric then the equation (1.6) becomes
(1.7) $C_{jik} = \{1/(n+1)\}(h_{ij}C_k + h_{ki}C_j + h_{jk}C_i)$

By virtue of equations (1.3) and (1.7), we get
(1.8) $C_{ijk} = C_{jik}$

Hence, the (h)hv-torsion tensor is symmetric with respect to first two indices in the C-reducible Finsler manifold.

Theorem 1.2:

In the C-reducible Finsler manifold, if the angular metric tensor is symmetric then (h)hv-torsion tensor is also symmetric with respect to last two indices.

Proof:
Interchanging the indices j and k in equation (1.3), we get
(1.9) $C_{ikj} = \{1/(n+1)\}(h_{ik}C_j + h_{kj}C_i + h_{ji}C_k)$

If the angular metric tensor h_{ij} is symmetric then the equation (1.9) becomes
(1.10) $C_{ikj} = \{1/(n+1)\}(h_{ki}C_j + h_{jk}C_i + h_{ij}C_k)$

From equations (1.3) and (1.10), we obtain
(1.11) $C_{ijk} = C_{ikj}$

Hence, the (h)hv-torsion tensor is symmetric with respect to last two indices in the C-reducible Finsler manifold.

Theorem 1.3:

If the (h)hv-torsion tensor is symmetric with respect to first two indices in C-2 like and C-reducible Finsler manifold then (h)hv-torsion tensor is also symmetric with respect to first two indices in semi C-reducible Finsler manifold.

Proof:

By virtue of equations (1.1), (1.3), (1.5) and (1.8), we obtain the required result.

Theorem 1.4:

If the (h)hv-torsion tensor is symmetric with respect to last two indices in C-2 like and C-reducible Finsler manifold then (h)hv-torsion tensor is also symmetric with respect to last two indices in semi C-reducible Finsler manifold.

Proof:

By virtue of equations (1.1), (1.3), (1.5) and (1.11), we obtain the required result.

References

[1] D. Bao, S.S. Chern & Z. Shen : An introduction to Riemann–Finsler Geometry, Springer-Verlag, (2000).

[2] H. Yasuda & H. Shimada : On Randers spaces of scalar curvature, Rep. on Math.Phys.,p.(347-360),11, (1977).
[3] M.Matsumoto:Finsler spaces with the hv-curvature tensor Phijk of a special form, Rep. on Math. Phy's.
[4] M. Matsumoto : On C-reducible Finsler spaces, Tensor, N.S., 24, p. (29-37), (1972).
[5] M. Matsumoto : Randers spaces of constant curvature, Reports on Math. Phys., p. (249-261), 28, (1989).
[6] S. Bacso & M. Matsumoto : On Finsler spaces of Douglas type, A generalization of the notion of Berwald space, Publ. Math. Debrecen, p. (385-406), 51, (1997).
[7] Z. Shen: Lectures on Finsler Geometry, World Scientific, Singapore, 307 pages, (2001).

Author Profile

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