

Some Problems on Semi C-Reducible Finsler Manifold

T.S. Chauhan

Associate Professor, Department of Maths, Bareilly College, Bareilly(U.P.), India

Abstract: Present paper deals to the study of semi C-reducible Finsler manifold. In this paper, we have obtained some important theorems on C-reducible Finsler manifold and semi C-reducible Finsler manifold.

Keywords: (h)hv-torsion tensor, angular metric tensor, C-2 like, C-reducible, semi C-reducible Finsler manifold.

1. Semi C-Reducible Finsler Manifold

A Finsler manifold F^n is said to be C-2 like Finsler manifold, if the (h)hv-torsion tensor C_{ijk} satisfies the following condition

$$(1.1) \quad C_{ijk} = (1/C^2)C_i C_j C_k.$$

Wherein

$$(1.2) \quad g^{jk} C_{ijk} = C_i$$

is the contracted torsion tensor.

A Finsler manifold F^n is said to be C-reducible Finsler manifold, if the (h)hv-torsion tensor C_{ijk} satisfies the following condition

$$(1.3) \quad C_{ijk} = \{1/(n+1)\}(h_{ij}C_k + h_{jk}C_i + h_{ki}C_j).$$

Wherein

$$(1.4) \quad h_{ij} = g_{ij} - l_i l_j$$

is angular metric tensor.

A Finsler manifold F^n is said to be semi C-reducible Finsler manifold, if the (h)hv-torsion tensor C_{ijk} satisfies the following condition

$$(1.5) \quad C_{ijk} = \{p/(n+1)\}(h_{ij}C_k + h_{jk}C_i + h_{ki}C_j) + (q/C^2)C_i C_j C_k.$$

In this regard, we have the following theorems:

Theorem 1.1:

In the C-reducible Finsler manifold, if the angular metric tensor is symmetric then (h)hv-torsion tensor is also symmetric with respect to first two indices.

Proof:

Interchanging the indices i and j in equation (1.3), we obtain

$$(1.6) \quad C_{jik} = \{1/(n+1)\}(h_{ji}C_k + h_{ik}C_j + h_{kj}C_i)$$

If the angular metric tensor h_{ij} is symmetric then the equation (1.6) becomes

$$(1.7) \quad C_{jik} = \{1/(n+1)\}(h_{ij}C_k + h_{ki}C_j + h_{jk}C_i)$$

By virtue of equations (1.3) and (1.7), we get

$$(1.8) \quad C_{ijk} = C_{jik}$$

Hence, the (h)hv-torsion tensor is symmetric with respect to first two indices in the C-reducible Finsler manifold.

Theorem 1.2:

In the C-reducible Finsler manifold, if the angular metric tensor is symmetric then (h)hv-torsion tensor is also symmetric with respect to last two indices.

Proof:

Interchanging the indices j and k in equation (1.3), we get

$$(1.9) \quad C_{ikj} = \{1/(n+1)\}(h_{ik}C_j + h_{kj}C_i + h_{ji}C_k)$$

If the angular metric tensor h_{ij} is symmetric then the equation (1.9) becomes

$$(1.10) \quad C_{ikj} = \{1/(n+1)\}(h_{ki}C_j + h_{jk}C_i + h_{ij}C_k)$$

From equations (1.3) and (1.10), we obtain

$$(1.11) \quad C_{ijk} = C_{ikj}$$

Hence, the (h)hv-torsion tensor is symmetric with respect to last two indices in the C-reducible Finsler manifold.

Theorem 1.3:

If the (h)hv-torsion tensor is symmetric with respect to first two indices in C-2 like and C-reducible Finsler manifold then (h)hv-torsion tensor is also symmetric with respect to first two indices in semi C-reducible Finsler manifold.

Proof:

By virtue of equations (1.1), (1.3), (1.5) and (1.8), we obtain the required result.

Theorem 1.4:

If the (h)hv-torsion tensor is symmetric with respect to last two indices in C-2 like and C-reducible Finsler manifold then (h)hv-torsion tensor is also symmetric with respect to last two indices in semi C-reducible Finsler manifold.

Proof:

By virtue of equations (1.1), (1.3), (1.5) and (1.11), we obtain the required result.

References

- [1] D. Bao, S.S. Chern & Z. Shen : An introduction to Riemann–Finsler Geometry, Springer-Verlag, (2000).

- [2] H. Yasuda & H. Shimada : On Randers spaces of scalar curvature, Rep. on Math. Phys., p.(347-360), 11, (1977).
- [3] M. Matsumoto: Finsler spaces with the hv-curvature tensor Ph_{ijk} of a special form, Rep. on Math. Phys's.
- [4] M. Matsumoto : On C-reducible Finsler spaces, Tensor, N.S., 24, p. (29-37), (1972).
- [5] M. Matsumoto : Randers spaces of constant curvature, Reports on Math. Phys., p. (249-261), 28, (1989).
- [6] S. Bacsó & M. Matsumoto : On Finsler spaces of Douglas type, A generalization of the notion of Berwald space, Publ. Math. Debrecen, p. (385-406), 51, (1997).
- [7] Z. Shen: Lectures on Finsler Geometry, World Scientific, Singapore, 307 pages, (2001).

Author Profile



T.S. Chauhan (Tarkeshwar Singh Chauhan) received Ph.D. and D.Sc. degrees in Mathematics from M.J.P.R.U., Bareilly in 1992 and 2008 respectively. He has been working in Maths deptt., Bareilly College, Bareilly since 1990 and now he is an Associate Professor. Under his guidance nearly 25 candidates have been awarded Ph.D. degree. Several papers and books are published in different branches in different publications under him.

