

# A Study of Different Type of H-Curvature Tensors in Sasakian Manifold

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**Abstract:** Purpose of the present paper is to study of different H-Curvature tensors in Sasakian manifold. In section 1 is devoted for introduction. Section 2 deals to the study of H-Projective curvature tensor, H-Conformal curvature tensor, H-Conharmonic curvature tensor, H-Concircular tensor in Sasakian manifold.

**Keywords:** H-Projective curvature tensor, H-Conformal curvature tensor, H-Conharmonic curvature tensor, H-Concircular tensor

## 1. Introduction

An n-dimensional Sasakian manifold  $M^n$  is an odd dimensional Riemannian space, which admits a unit killing vector field  $\eta^\lambda$  satisfying:

$$(1.1) \quad \eta_{k,i,j} = \eta_j g_{ik} - \eta_k g_{ij}$$

Wherein a comma (,) followed by index denotes the operation of covariant differentiation with regard to the fundamental tensor  $g_{ij}$  of the Riemannian space.

$$(1.2) \quad R^h_{ijk} = \partial_i \{^h_j k\} - \partial_j \{^h_i k\} + \{^h_i l\} \{^l_j k\} - \{^h_j l\} \{^l_i k\}$$

Whereas the Ricci tensor and the scalar curvature are respectively given by

$$(1.3) \quad R_{jk} = R^i_{ijk},$$

$$(1.4) \quad R = R_{jk} g^{jk}$$

and

$$(1.5) \quad \partial_i = (\partial/\partial x^i)$$

A tensor  $S_{ij}$  is defined as

$$(1.6) \quad S_{ij} = -F^a_i R_{aj}$$

then we have

$$(1.7) \quad S_{ij} = -S_{ji}$$

and

$$(1.8) \quad F^a_i S_{aj} = -S_{ia} F^a_j.$$

## 2. H-Curvature Tensor

H-Projective curvature tensor in the Sasakian manifold is defined as [5]:

$$(2.1) \quad P^h_{ijk} = R^h_{ijk} + \{1/(n+2)\}(R_{ik} \delta^h_j - R_{jk} \delta^h_i + S_{ik} F^h_j - S_{jk} F^h_i + 2S_{ij} F^h_k).$$

### Definition 2.1:

A Sasakian manifold is called H-Projective Recurrent if it satisfies the following condition

$$(2.2) \quad \nabla_1 P^h_{ijk} = \lambda_1 P^h_{ijk}.$$

Wherein  $\lambda_1$  is H-Projective recurrent vector.

### Definition 2.2:

A Sasakian manifold is said to be H-Projective symmetric if it satisfies the following condition

$$(2.3) \quad \nabla_1 P^h_{ijk} = 0.$$

### Definition 2.3:

A Sasakian manifold is termed as H-Projectively flat if

$$(2.4) \quad P^h_{ijk} = 0.$$

In this regard, we have the following theorem:

### Theorem 2.1:

If Sasakian manifold is H-projectively flat then the Ricci tensor holds the relation  $R_{ij} = -\{2/(n+2)\}(S_{in} F^n_j + F S_{ij})$ .

### Proof:

Transvecting equation (2.1) by  $g_{hm}$ , we get

$$(2.5) \quad P_{ijkm} = R_{ijkm} + \{1/(n+2)\}(g_{jm} R_{ik} - g_{im} R_{jk} + S_{ik} F_{jm} - S_{jk} F_{im} + 2S_{ij} F_{km}).$$

Transvecting equation (2.5) with  $g^{km}$  and using equation (1.8), we get

$$(2.6) \quad P_{ij} = R_{ij} + \{2/(n+2)\}(S_{in} F^n_j + F S_{ij}).$$

If a Sasakian manifold is H-projectively flat then equation (2.6) becomes reduced in the form

$$(2.7) \quad R_{ij} = -\{2/(n+2)\}(S_{in} F^n_j + F S_{ij}).$$

H-Concircular tensor is given by

$$(2.8) \quad C^h_{ijk} = R^h_{ijk} + \{R/n(n+2)\}(g_{ik} \delta^h_j - g_{jk} \delta^h_i + S_{ik} F^h_j - S_{jk} F^h_i + 2F_{ij} F^h_k).$$

### Definition 2.4:

A Sasakian manifold is called Sasakian manifold with recurrent H-Concircular curvature tensor, if it satisfies

$$(2.9) \quad \nabla_1 C^h_{ijk} = \lambda_1 C^h_{ijk}$$

for some non-zero recurrence vector  $\lambda_1$ .

### Definition 2.5:

A Sasakian manifold is said to be H-Concircular symmetric if it satisfies the following condition

$$(2.10) \quad \nabla_1 C^h_{ijk} = 0.$$

### Definition 2.6:

A Sasakian manifold is termed as H-Concircular flat if

$$(2.11) \quad C^h_{ijk} = 0.$$

H-Conharmonic curvature tensor is given by

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$$(2.12) \quad T_{ijk}^h = R_{ijk}^h + \{1/(n+4)\}(R_{ik}\delta_j^h - R_{jk}\delta_i^h + g_{ik}R_j^h - g_{jk}R_i^h + S_{ik}F_j^h - S_{jk}F_i^h + F_{ik}S_j^h - F_{jk}S_i^h + 2S_{ij}F_k^h + 2F_{ij}S_k^h)$$

**Definition 2.7:**

A Sasakian space satisfying the following condition

$$(2.13) \quad \nabla_1 T_{ijk}^h = \lambda_1 T_{ijk}^h$$

for some non-zero recurrence vector  $\lambda_1$ , will be called a Sasakian manifold with recurrent H-Conharmonic curvature tensor.

**Definition 2.8:**

A Sasakian manifold is said to be H-Conharmonic symmetric if it satisfies the following condition

$$(2.14) \quad \nabla_1 T_{ijk}^h = 0.$$

**Definition 2.9:**

A Sasakian manifold is termed as H-Conharmonic flat if

$$(2.15) \quad T_{ijk}^h = 0.$$

H-Conformal curvature tensor in the Sasakian manifold is given by

$$(2.16) \quad B_{ijk}^h = R_{ijk}^h + \{1/(n+4)\}(R_{ik}\delta_j^h - R_{jk}\delta_i^h + g_{ik}R_j^h - g_{jk}R_i^h + S_{ik}F_j^h - S_{jk}F_i^h + F_{ik}S_j^h - F_{jk}S_i^h + 2S_{ij}F_k^h + 2F_{ij}S_k^h) - \{R/(n+2)(n+4)\}(g_{ik}\delta_j^h - g_{jk}\delta_i^h + F_{ik}F_j^h - F_{jk}F_i^h + 2F_{ij}F_k^h).$$

**Definition 2.10:**

A Sasakian space satisfying the relation

$$(2.17) \quad \nabla_1 B_{ijk}^h = \lambda_1 B_{ijk}^h$$

is termed as Sasakian space with recurrent H-Conformal curvature tensor.

**Definition 2.11:**

A Sasakian manifold is said to be H-Conformal symmetric if it satisfies the following condition

$$(2.18) \quad \nabla_1 B_{ijk}^h = 0.$$

**Definition 2.12:**

A Sasakian manifold is termed as H-Conformal flat if

$$(2.19) \quad B_{ijk}^h = 0.$$

From equations (2.12) and (2.16), we get

$$(2.20) \quad B_{ijk}^h = T_{ijk}^h - \{R/(n+2)(n+4)\}(g_{ik}\delta_j^h - g_{jk}\delta_i^h + F_{ik}F_j^h - F_{jk}F_i^h + 2F_{ij}F_k^h).$$

By virtue of equations (2.8) and (2.20), we obtain

$$(2.21) \quad B_{ijk}^h = T_{ijk}^h + \{n/(n+4)\}(R_{ijk}^h = C_{ijk}^h),$$

In this regard, we have the following theorems:

**Theorem 2.2:**

In a Sasakian manifold is H-projective curvature tensor, H-concircular tensor and H-conharmonic curvature tensor are flat then H-conformal curvature tensor is also flat.

**Proof:**

By virtue of equations (2.4), (2.11), (2.15), (2.19) and (2.21).

**Theorem 2.3:**

In a Sasakian manifold is H-projective curvature tensor, H-concircular tensor and H-conformal curvature tensor are flat then H-conharmonic curvature tensor is also flat.

**Proof:**

By virtue of equations (2.4), (2.11), (2.15), (2.19) and (2.21).

**Theorem 2.4:**

In a Sasakian manifold is H-projective curvature tensor, H-conformal tensor and H-conharmonic curvature tensor are flat then H-concircular curvature tensor is also flat.

**Proof:**

By virtue of equations (2.4), (2.11), (2.15), (2.19) and (2.21).

**Theorem 2.5:**

In a Sasakian manifold is H-conformal curvature tensor, H-concircular tensor and H-conharmonic curvature tensor are flat then H-projective curvature tensor is also flat.

**Proof:**

By virtue of equations (2.4), (2.11), (2.15), (2.19) and (2.21).

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